Let's understand & calculate in gauge thy's.

$\mathcal{L}(\bar{\psi}, \psi, \Phi, \Phi^*, A_\mu)$ $\quad U(1)$ symmetry $\quad \psi \to e^{i\theta}\psi$

$\Phi \to e^{i\theta}\Phi$

$A_\mu \to A_\mu + \partial_\mu \theta$

$$\mathcal{L} = \bar{\psi}(i\not{D} - M)\psi + D_\mu \Phi^* D^\mu \Phi - V(\Phi) - \frac{1}{4}\boxed{F_{\mu\nu}F^{\mu\nu}} \to ??$$

$D_\mu = \partial_\mu - ieA_\mu$ $\qquad -\Phi^* D_\mu D^\mu \Phi$

Propagator of Free thy $\quad S(x) = -i\langle 0|T(\psi(x)\bar{\psi}(0))|0\rangle$

$$= \frac{1}{i\not{\partial} - M} \xrightarrow{p\text{-sp.}} \frac{1}{\not{p} - M} = \frac{\not{p}+M}{p^2 - M^2}$$

Propagator Φ : $G(x) = -i\langle 0|T(\Phi(x)\Phi^*(0))|0\rangle$

$$= \frac{1}{-\partial_\mu\partial^\mu} \xrightarrow{p} \frac{1}{p^2 - m^2}$$

Gauge field propagator?

$$\underline{G_{\mu\nu}(x)} = -i\langle 0|T(A_\mu(x)A_\nu(0))|0\rangle$$

$$\underline{(p)} \qquad \int d^4x\, e^{ip_\mu x^\mu}$$

$$\boxed{(\not p - m)\, S = \underline{\underline{1}} \qquad \underline{\quad} \;\; " \;\; \frac{1}{\not p - m}}$$

in coord. $(i\not\partial - m)S(x) = \underline{\underline{\delta^4(x)}}$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}\Big(\underbrace{\partial_\mu A_\nu - \partial_\nu A_\mu}_{\text{anti}}\Big)\Big(\underbrace{\partial^\mu A^\nu - \partial^\nu A^\mu}_{\text{anti on } \mu\leftrightarrow\nu}\Big)$$

$$= -\frac{1}{2}\partial_\mu A_\nu(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

Int by parts $$= \frac{1}{2}A_\nu(\partial_\mu\partial^\mu A^\nu - \partial_\mu\partial^\nu A^\mu) = \frac{1}{2}A_\mu\boxed{(g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)}A_\nu$$

What's inverse?

$$\mathcal{L}_{F^2} = \frac{1}{2} A_\mu \left(g^{\mu\nu} \partial_\alpha \partial^\alpha - \partial^\mu \partial^\nu\right) A_\nu$$

p-space:

$$\frac{1}{2} A_\mu \underbrace{\left(-g^{\mu\nu} p^2 + p^\mu p^\nu\right)}_{\text{Invert} \to \text{propagator}} A_\nu$$

$$G_{\mu\nu}(p) = \int_x \langle 0 | T\left(A_\mu(x) A_\nu(0)\right) | 0 \rangle e^{ip\cdot x}$$

$$\underbrace{\left[\underbrace{-g^{\mu\alpha} p^2 + p^\mu p^\alpha}\right] G_{\alpha\nu}}_{\text{But this is singular.}} = g^\mu_{\ \nu} \; !!$$

There Is None

p^μ pointing x-dir

$$\begin{array}{c} \\ \mu \end{array} \begin{bmatrix} p^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -p^2 & 0 \\ 0 & 0 & 0 & -p^2 \end{bmatrix} \begin{pmatrix} 0 \\ \vec{p} \\ 0 \\ 0 \end{pmatrix}$$

ν

Problem: If $\boxed{A_\mu \propto P_\mu}$ then

$$A_\nu\left[g^{\mu\nu}p^2 - p^\mu p^\nu\right]A_\mu = A_\nu\left[p^\nu p^2 - p^\nu p^2\right]$$

$$= 0$$

A freedom

$A_\mu \propto \partial_\mu$ problem. As in $A_\mu \sim \partial_\mu \theta$

Shift $A_\mu \to A_\mu + \partial_\mu \theta$ nothing changes. E stays same

S stays same

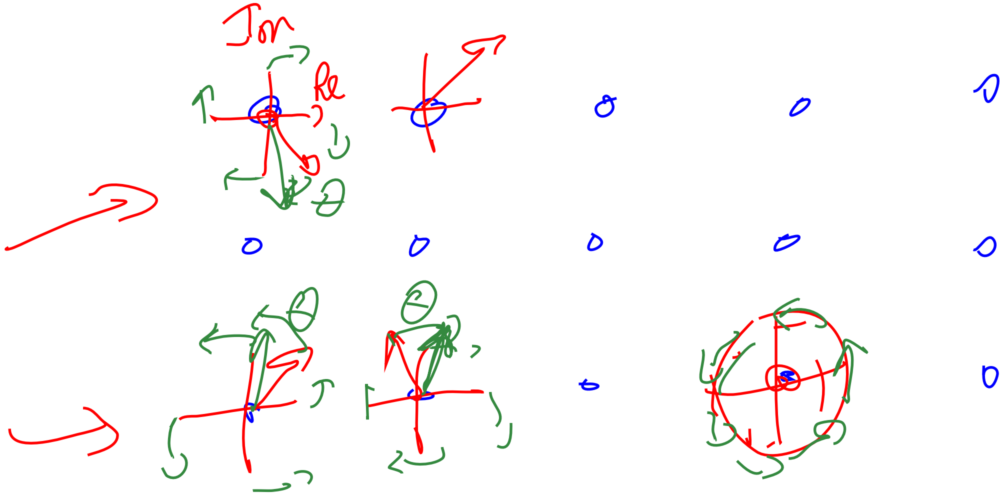
A_μ as big as you want $\propto \partial_\mu \theta$

No cost. I cannot find a propagator.

Not really physical

What does this mean??

Think of space (time) as lattice



At each pt. $\hat{\Phi}$ takes a value (in path $\int$)

At each pt, Φ takes value in 2D space $\mathbb{C}$

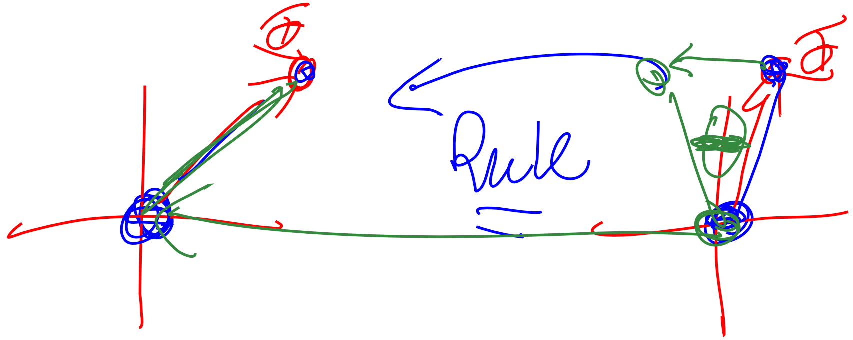
"Fiber space" Fiber Bundle

Gauge transform: Rotation of $\mathbb{C}$ plane at each pt.

$\Phi \to e^{i\theta}\Phi$ $\quad$ $\theta(\underline{x})$ different at each pt.

Change in coord choice for $\mathbb{C}(\underline{x})$

So what's a derivative?



$$\partial_\mu \Phi = \frac{\Phi(x+\Delta) - \Phi(x)}{\Delta}$$

Changes when I rotate

$$D_\mu \Phi = \frac{W\,\Phi(x+\Delta) - \Phi(x)}{\Delta}$$

$W = \exp[-ie\Delta A_\mu]$ ΔA_μ : How many Rad to rotate $\Phi(x+\Delta)$ to compare w $\Phi(x)$

$-i\Theta$

Wilson line

$A_\mu \to A_\mu + e\partial_\mu \Theta$

$\exp -i\int e A_\mu dx^\mu$

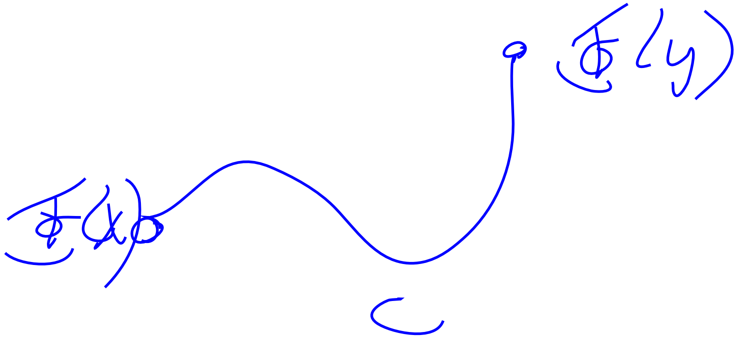
C_1 or C_2

A_μ is nontrivial if C_1, C_2 give diff't answers.

$\partial_\mu A_\nu - \partial_\nu A_\mu \neq 0$

A_μ is a comparator (Affine Connection)

telling how to rotate Φ when comparing diff't points

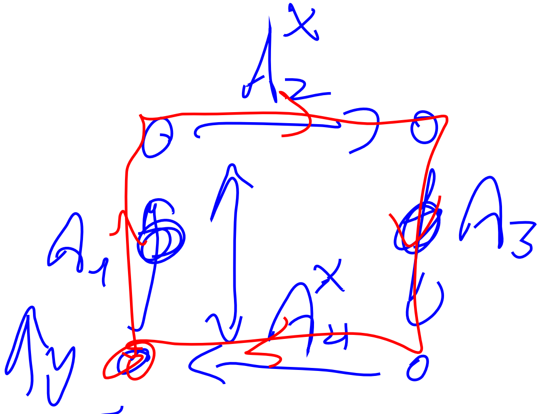


compare $\underbrace{\Phi(x)}_{e^{i\theta(x)}}$ with $\underbrace{\exp\left(-ie\int A_\mu dx^\mu\right)}_{e^{i(\theta(x)-\theta(y))}} \underbrace{\Phi(y)}_{e^{i\theta(y)}}$

$$\Phi(x) \to e^{i\theta(x)}\,\Phi(x)$$

$$\underline{A_\mu \to A_\mu + e\,\partial_\mu\theta}$$

What's Coord & What's Physical



$\Phi(x)$

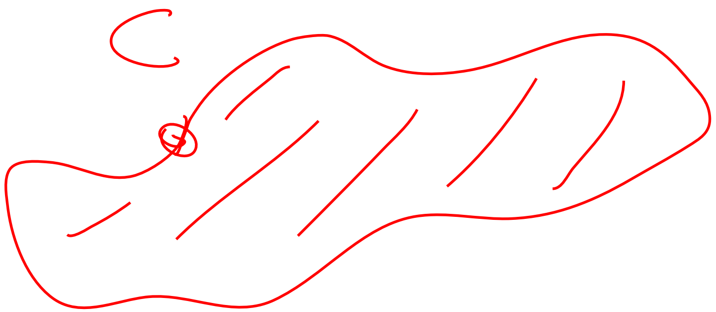
$\xrightarrow[x]{}$

$$e^{ieA_1}\,\underbrace{e^{ieA_2}\,e^{-ieA_3}}_{e^{ie(A_y(a\hat{x})-A_y(0))}}\,e^{-ieA_4}$$

$$e^{ie(A_1^y - A_3^y)} = e^{ie(A_y(0)-A_y(a\hat{x}))} = e^{-iea^2\partial_x A_y}$$

$$\times\, e^{+iea^2\partial_y A_x}$$

Total phase $e^{iea^2(\partial_x A_y - \partial_y A_x)}$



$\iint F_{\alpha\beta}\, dx^\alpha \wedge dx^\beta$ over surface → phase picked up.

$$A_\mu^1 \quad \Phi^1 \quad \psi_1$$

Are they same?

$$A_\mu^3 = A_\mu^1 + \partial_\mu \theta^2$$

$$\Phi_2 = e^{i\theta}\Phi_1$$

$$\psi_2 = e^{i\theta}\psi_1$$

physically equivalent

$$\Phi \quad e^{i\theta}\Phi$$

$$\psi \to e^{i\theta}\psi$$

$$A \quad A + \partial_\mu \theta$$

phys equiv

$$Z = \int \mathcal{D}(\psi\bar{\psi}\Phi\Phi^* A)\, e^{iS[\ldots]}$$

equivalent terms in $\int$ variables

$\{\psi\bar{\psi}\Phi\Phi^* A_\mu\}$ into equivalence classes

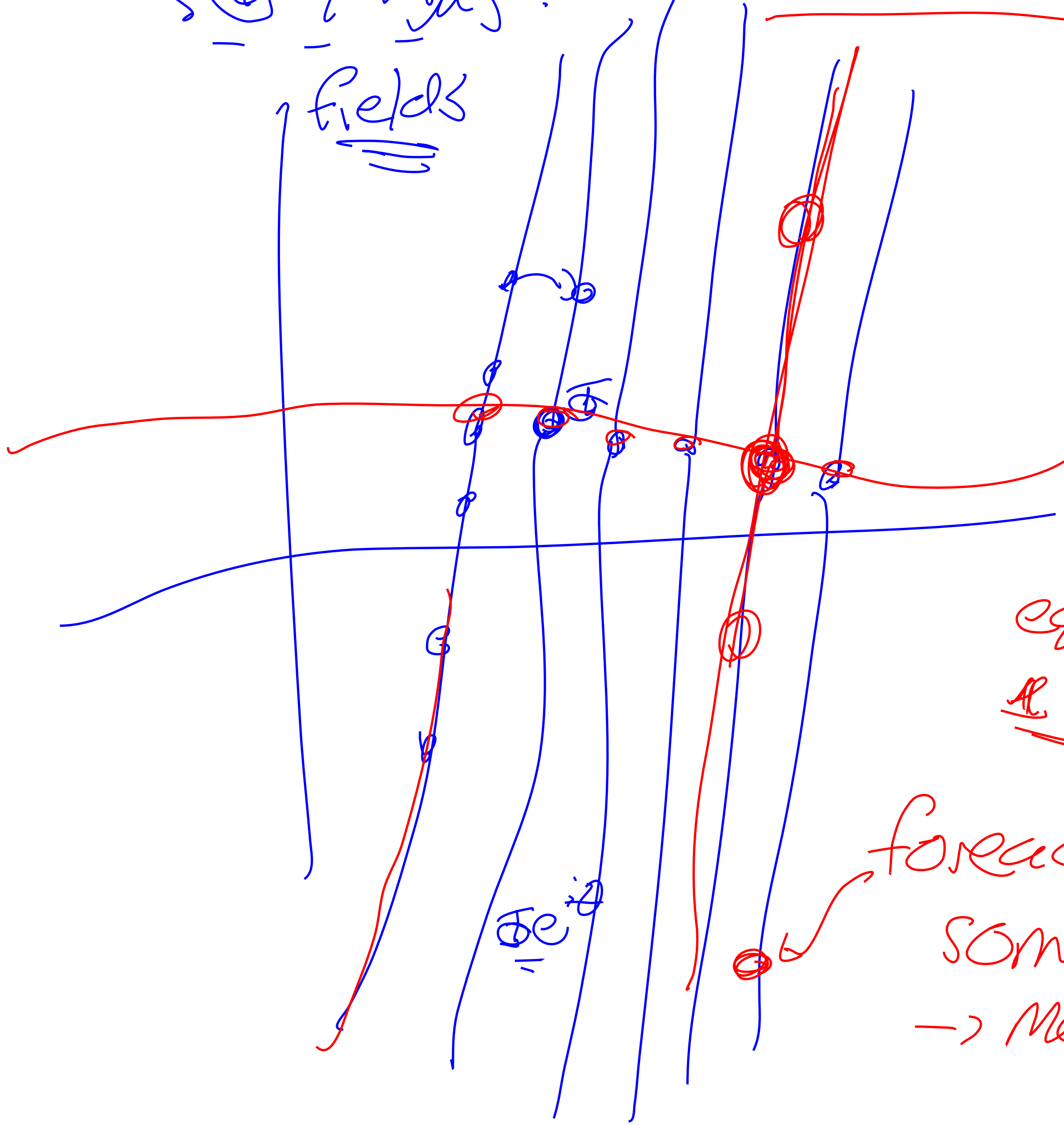
two are equiv. if $\boxed{\partial_\mu \theta(x)\, e^{i\theta}}$ (Gauge) trans. from 1 to other

Plan don't integrate All $(A_\mu \psi\bar{\psi}\Phi\Phi^*)$

Just get 1 rep. from each equivalence class.

Find something **not** same for "equivalent"

$\{\Phi \,\&\, A_\mu\}$. Pick one value



surf crosses each equiv class at time

$$\boxed{\partial_\mu A^\mu}$$

$A_\mu \to A_\mu + \partial_\mu \theta$

$\partial_\mu A^\mu \to \partial_\mu A^\mu + \partial_\mu \partial^\mu \theta$

new & distinct

for each Eq class there is some θ-choice $\to \partial_\mu A^\mu = 0$

$\to$ Measure $\partial_\mu A^\mu = \alpha(x)$ Pick $\theta = -(\partial_\mu \partial^\mu)^{-1} \alpha$

$A_\mu \to A_\mu + \partial_\mu \theta$. $\partial_\mu A^\mu \to \partial_\mu A^\mu + \partial_\mu \partial^\mu (\partial_\mu \partial^\mu)^{-1} \alpha$

Constructive way $\rightarrow \{A_\mu\ \psi\ \Phi\}$

to find one unique $A_\mu'\ \psi'\ \Phi'$ where $\boxed{\partial_\mu A^{\mu\prime} = 0}$

$\alpha = \partial_\mu A^\mu$

$A_\mu \to A_\mu + \partial_\mu \theta$

$\Phi \to e^{i\theta}\Phi$

$A^{\mu\prime}$

Pick $\underbrace{\theta = -\partial_\mu \partial^\mu \alpha}$

Impose $\partial_\mu A^\mu = 0$ in Path $\int$ $\rightarrow$ get each Equiv Class exactly 1 time

$$\int \mathcal{D}(\Phi, \Phi^\dagger, \psi, \bar\psi, A_\mu)\, e^{iS[\dots]}$$

$$\delta(\partial_\mu A^\mu)$$

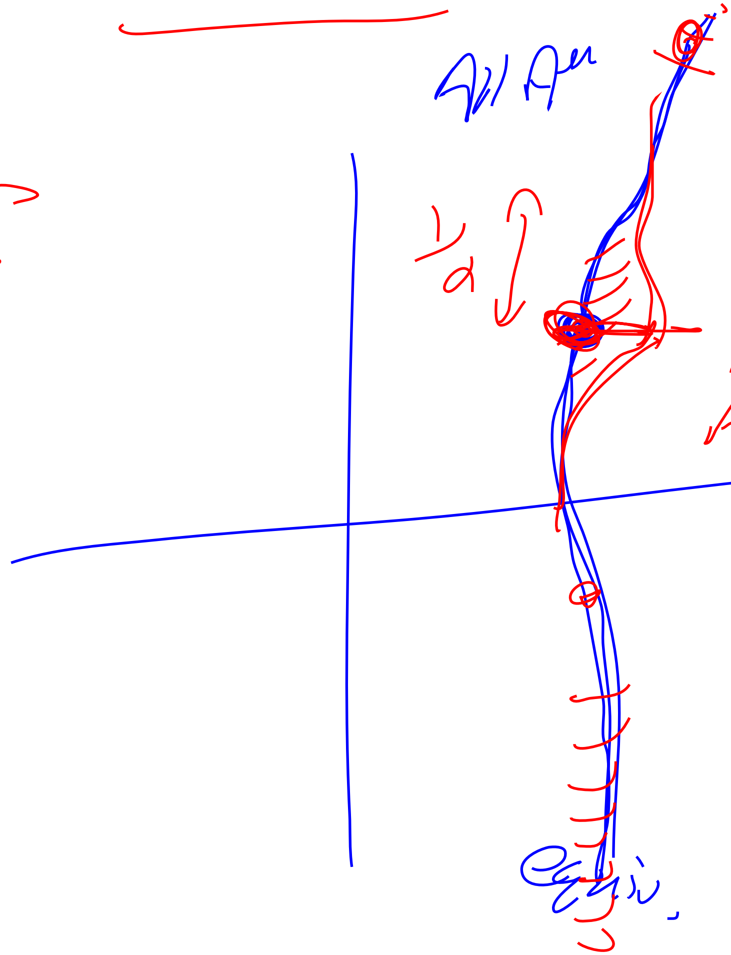
1 δ funz per space-time point

$$\delta(\partial_\mu A^\mu(x)) = \int d\lambda(x)\, \exp\left[i\lambda(x)\, \partial_\mu A^\mu - i \frac{1}{\alpha}\lambda^2 \right]$$

α param.

Or

$\equiv$



prefer $\partial_\mu A^\mu = 0$

Also allow $\partial_\mu A^\mu = \lambda$ smaller value

But I suppress it.

λ-int is Gaussian

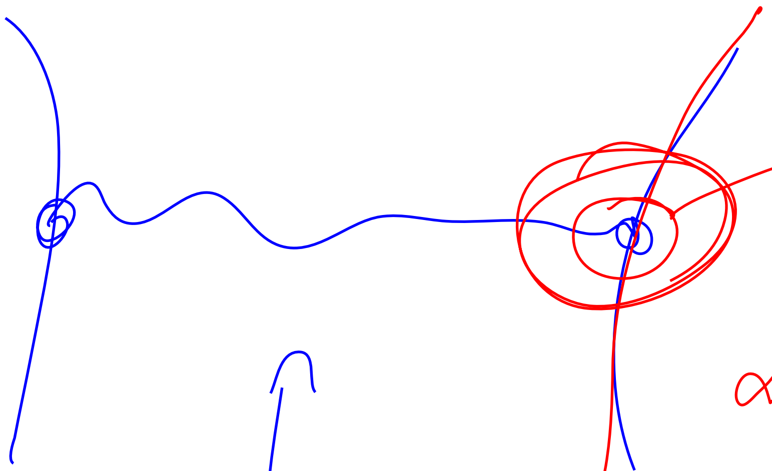
$$\int d\lambda \; e^{i\left(\frac{1}{2\alpha}\lambda^2 + \lambda\,\partial_\mu A^\mu \pm \frac{\alpha}{2}(\partial_\mu A^\mu)^2\right)}$$

$$\int d\lambda \; e^{\frac{i}{2\alpha}(\lambda + \alpha\,\partial_\mu A^\mu)^2} \; e^{\frac{i\alpha}{2}(\partial_\mu A^\mu)^2}$$

$$\exp \frac{i}{2}\int_x A_\mu\left[g^{\mu\nu}\partial_\lambda\partial^\lambda - \partial^\mu\partial^\nu + \alpha^{-1}\partial^\mu\partial^\nu\right]A_\nu$$

$$G_{\mu\nu} = \boxed{\frac{-1}{p^2+i\epsilon}\left[g_{\mu\nu} + (\alpha-1)\frac{p_\mu p_\nu}{p^2}\right]}$$

Bret Brt Bret Brt Bret



$\rightarrow A_\mu J^\mu$ And $\partial_\mu J^\mu = 0$

$\rightarrow p_\mu J^\mu = 0$

α ?? Don't Answers depend on what α I choose?

$$\frac{-ig^{\mu\nu} + i(1-\alpha)p^\mu p^\nu/p^2}{p^2+i\epsilon}$$

Anything ∝ p^μ or p^ν will not contribute

α → won't matter.

Pick α so calc is easy

Usually α = 1 $\quad G_{\mu\nu} = \frac{-ig_{\mu\nu}}{p^2+i\epsilon}$

Feynman Gauge

Some problems α = 0 is better

Landau Gauge

Check to make sure α doesn't matter.

But it never will.

$$7 \quad \frac{i}{\not{p}-m} = \frac{i(\not{p}+m)}{p^2-m^2}$$

$$\frac{i}{p^2-m^2}$$

$$\frac{-i\, g_{\mu\nu}}{p^2+i\epsilon}$$

$$\begin{bmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$