We Have the Theory QED!
Let's Calculate some Stuff!

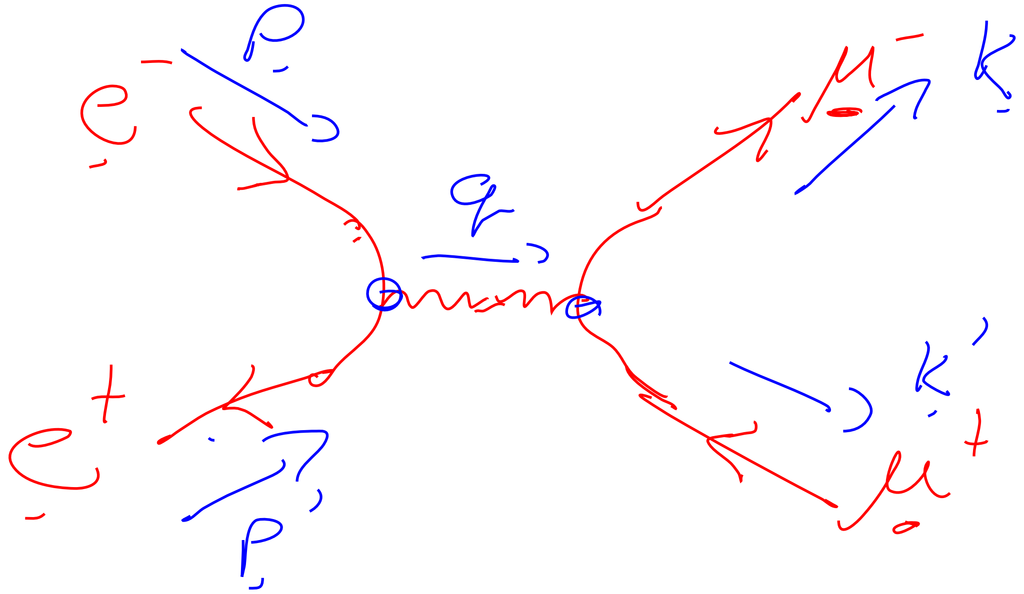
charge $q = -1$

Electron + Muon QED

$$\mathcal{L}(\psi_e, \psi_\mu, A^\mu) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_e\left[i\gamma^\mu(\partial_\mu + iqeA_\mu) - m_e\right]\psi_e$$
$$+ \bar{\psi}_\mu\left[i\gamma^\nu(\partial_\nu + iqeA_\nu) - m_\mu\right]\psi_\mu$$

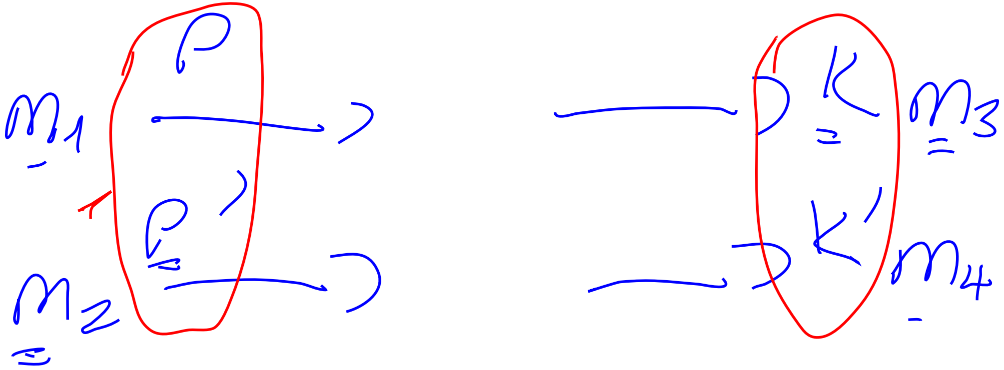
name, not Lorentz index

$e^- e^+ \longrightarrow \mu^- \mu^+$



$q = p + p'$

$q = k + k'$



$$p+p'-k-k'=0$$

- $p-k = k'-p'$
- $p-k' = k-p'$
- $p+p' = k+k'$

$$p\cdot p = m_e^2$$

$p\cdot p' \to$

$-p\cdot k \to$

$-p\cdot k' \to$

$$p\cdot(p+p'-k-k')=0$$

$$\boxed{s+t+u=m_1^2+m_2^2+m_3^2+m_4^2}$$

Exact

positive

$$\boxed{(p+p')^2 = s} = p^2+2p\cdot p'+p'^2 = (k+k')^2$$

$$= m_1^2+2p\cdot p'+m_2^2$$

$$\boxed{p\cdot p' = \frac{s-m_1^2-m_2^2}{2}}$$

usually negative

$$(p-k)^2 = t \qquad -p\cdot k = \frac{-t+m_1^2+m_3^2}{2}$$

$$(p-k')^2 = u \qquad -p\cdot k' = \frac{-u+m_1^2+m_4^2}{2}$$

us. negative

In the Limit m^2 small vs. s, $|p\cdot k|$

Ignore m^2 → simplifies (life, expr. etc.)

Small Mass Approx Only

$$2p\cdot p' = s = 2k\cdot k'$$
$$2p\cdot k = -t = 2p'\cdot k'$$
$$2p\cdot k' = -u = 2p'\cdot k$$

$p^2 = 0$

$p'^2 = 0$

$k^2 = 0$

$k'^2 = 0$

$\boxed{s+t+u=0}$

$s > 0$

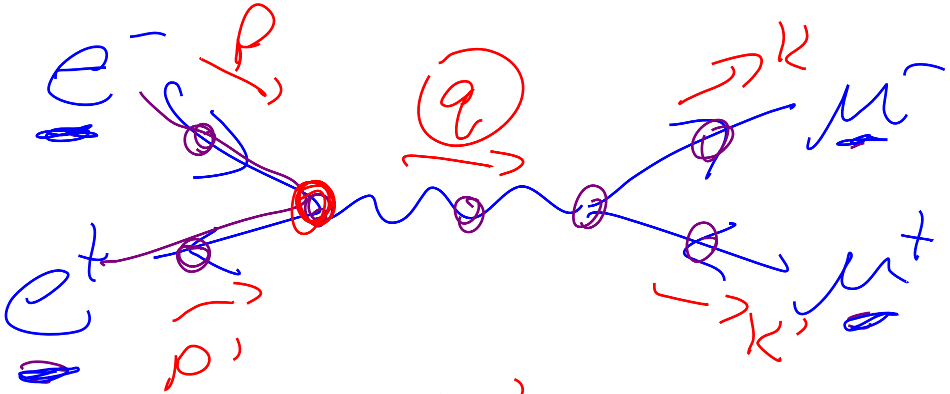
$t \leq 0$

$u \leq 0$

in this limit

Mandelstamm Variables

$\to 0 \quad t - m^2 < 0$



$q = p + p'$

$q^2 = s$

$q^\mu J_\mu = 0 \rightarrow \partial^\mu J_\mu = 0$

don't need this

$$\mathcal{M} = i\, \underbrace{\bar{v}(p', s_i')(ie\gamma^\mu)u(p, s_i)}_{\alpha\,\#}\; \frac{-i g_{\mu\nu} + \cancel{(\alpha-1)\frac{q_\mu q_\nu}{q^2}}}{q^2 + i\epsilon}$$

$$\times\; \underbrace{\bar{u}(k, s_f)(ie\gamma^\nu)v(k', s_f')}_{\alpha\,\#}$$

$q^2 = s$

$(\not{p} - m)u = 0$

$\bar{v}(\not{p}' + m) = 0$

what is $\bar{v}(p')(\not{p}' + m_e + \not{p} - m_e)u(p) = 0$

Address : $\left(\frac{1}{2}\right)^{\#} \sum_{\text{in } S} \sum_{\text{fin } S} \mathcal{M}^* \mathcal{M}$??

$$\left[\bar{v}(p)\gamma^\mu u(p)\right]^\dagger = u^\dagger(p)\,\gamma^{\mu\dagger}\,(\bar{v})^\dagger = u^\dagger \gamma^{\mu\dagger}\gamma^0 v$$

$\gamma^0\gamma^0$

$\bar{v} = v^\dagger \gamma^0$

$\gamma^0\gamma^{\mu\dagger}\gamma^0 = \gamma^\mu$

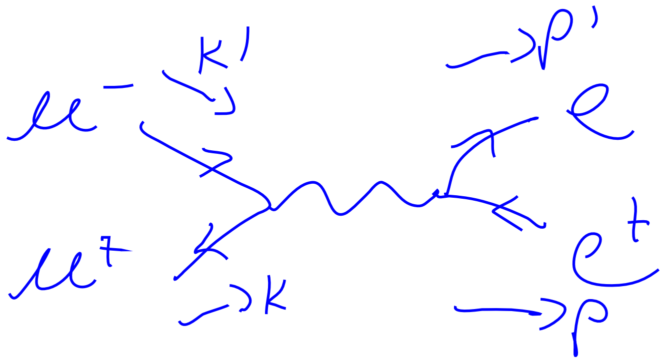
$(\bar{v})^\dagger = \gamma^{0\dagger} v = \gamma^0 v$

$u^\dagger\gamma^0 = \bar{u}$

$\bar{u}\gamma^\mu v$

$$\mathcal{M}^* = (-i)\,\bar{u}(p s_1)(ie)\gamma^\alpha v(p' s_2)\,\frac{+i\,g^{\alpha\beta}}{s+i\epsilon}\,\bar{v}(k' s_2)(ie)\gamma^\beta u(k, s_1)$$

$\longrightarrow$



$$\sum_{s_1 s_2 s_1' s_2'} 4\mathcal{M}^{*}\mathcal{M} = \frac{e^4 g_{\mu\nu} g_{\alpha\beta}}{(s+\cancel{i\varepsilon})(s-\cancel{i\varepsilon})}$$

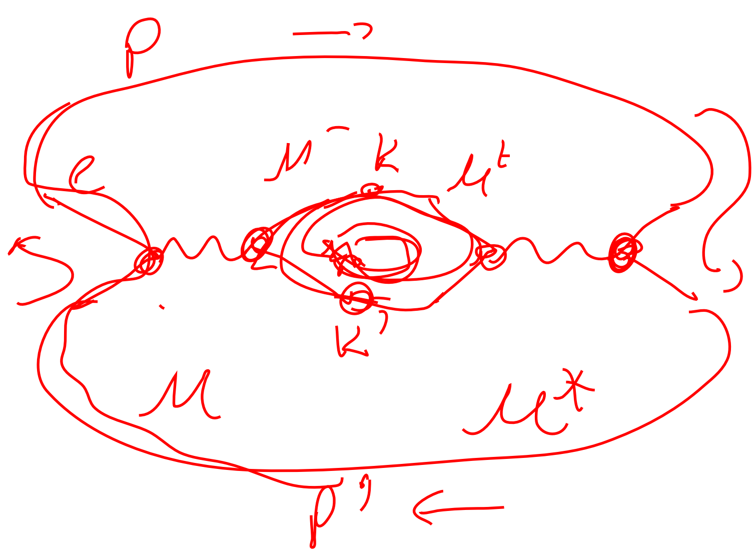
A $\quad \bar{v}(p's_2)\gamma^{\mu}u(p,s_1)\,\bar{u}(p s_1)\gamma^{\alpha}v(p',s_2)$

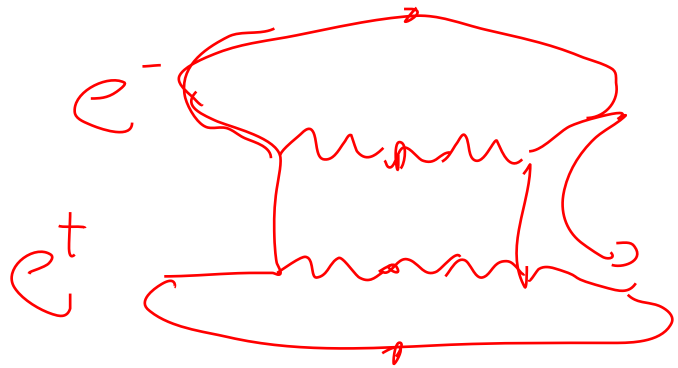
B $\quad \bar{u}(k,s_1')\gamma^{\nu}v(k's_2')\,\bar{v}(k's_2)\gamma^{\beta}u(k s_1')$

$\mathcal{M} \qquad \mathcal{M}^{*}$

$$\sum_{s_1 s_2} A = \mathrm{Tr}\left[\gamma^{\mu}\underbrace{\sum_{s} u\bar{u}(p s_1)}_{\not p + m_e}\gamma^{\alpha}\underbrace{\sum_{s_2} v\bar{v}(p' s_2)}_{\not p' m_e}\right]$$

$$\sum_{s_1' s_2'} B = \mathrm{Tr}\left[\gamma^{\nu}(\not k' - m_\mu)\gamma^{\beta}(\not k + m_\mu)\right]$$





$\mathrm{Tr}\,\gamma\not p\,\gamma\not p\,\gamma\not p\,\gamma\not p$

$$\mathrm{Tr}\,(\not{p}+m_e)\,\gamma^\alpha\,(\not{p}'-m_e)\,\gamma^\mu$$

$$= m_e^2\,\underbrace{\mathrm{Tr}\,\gamma^\alpha\gamma^\mu}_{4g^{\mu\alpha}} + m_e\,\mathrm{Tr}(\ldots) + \mathrm{Tr}\,\not{p}\,\gamma^\alpha\,\not{p}'\,\gamma^\mu$$

$$\mathrm{Tr}\,\not{p}\,\gamma^\alpha\,\not{p}'\,\gamma^\mu \to 4\left(p^\alpha p'^\mu + p'^\alpha p^\mu - g^{\mu\alpha}\,p\cdot p'\right)$$

$g_{\mu\nu}\,g_{\alpha\beta} \times (\text{this}) \longrightarrow \underline{\text{Algebra}}$

$$\frac{1}{4}\sum_{ss'ss'} \mathcal{M}^*\mathcal{M} = \frac{1}{4}\,\frac{e^4}{s^2}\,\boxed{g_e^2\,g_\mu^2}\cdot 4\left(p^\alpha p'^\mu + p^\mu p'^\alpha - (p\cdot p' + m_e^2)\,g^{\mu\alpha}\right)$$

$$\cdot\, 4\left(k_\alpha k'_\mu + k_\mu k'_\alpha - (k\cdot k' + m_\mu^2)\,g_{\mu\alpha}\right)$$

$$\Rightarrow \overline{|\mathcal{M}|^2}$$

$$p\cdot p' = \frac{s}{2} - m_e^2 \quad \text{etc.}$$

$$k\cdot k' = \frac{s}{2} - m_\mu^2$$

$$\rightarrow |\overline{\mathcal{M}}|^2 = \frac{8e^4}{s^2}\left(p\cdot k\, p'\cdot k' + p'\cdot k\, p\cdot k' + 2m_\mu^2\, p\cdot p'\right.$$

$$\left. + \, m_e^2,\ m_\mu^2 m_e^2 \text{ terms}\right)$$

But $\frac{m_e^2}{m_\mu^2} = \left(\frac{.511\text{ MeV}}{105\text{ MeV}}\right)^2 = \frac{1}{40{,}000}$ —

we have $\mathcal{O}(e^2/4\pi^2)$

$\sim 1/500$

corr. we haven't computed

Don't Listen to Bono (U2)

"Don't you, forget about me"

$$\sigma = \frac{1}{\underbrace{2E_p 2E_{p'}}_{s}\underbrace{|v_e - v_{e^+}|}_{2}} \int \frac{d^3k\, d^3k'}{(2\pi)^6 2E_k 2E_{k'}} (2\pi)^4 \delta^4(p+p'-k-k') \times$$

Get an Answer!

CM frame

$p^\mu = (E\ 0\ 0\ E)$

$p'^\mu = (E\ 0\ 0\ -E)$

$k = \sqrt{E^2 - m_\mu^2}$

$k^\mu = (E\ k\sin\theta\ 0\ k\cos\theta)$

$k'^\mu = (E\ -\ 0\ -\ )$

$\delta^3(\vec{p}+\vec{p}\,' - \vec{k} - \vec{k}') \, d\sigma \int d^3\vec{k}'$

$\int k^2 dk\, 2\pi\delta(2E - 2k^0) = \int k\, k^0 dk^0\, \delta(2E-2k^0) = \frac{k}{2}$

$k\,dk = d(k^2)$
$= d(k^2+m^2)$
$= d(E^2) = E\,dE$

$\frac{d\sigma}{d\cos\theta} \propto \left(1+\frac{m_\mu^2}{E^2}\right) + \left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\theta$

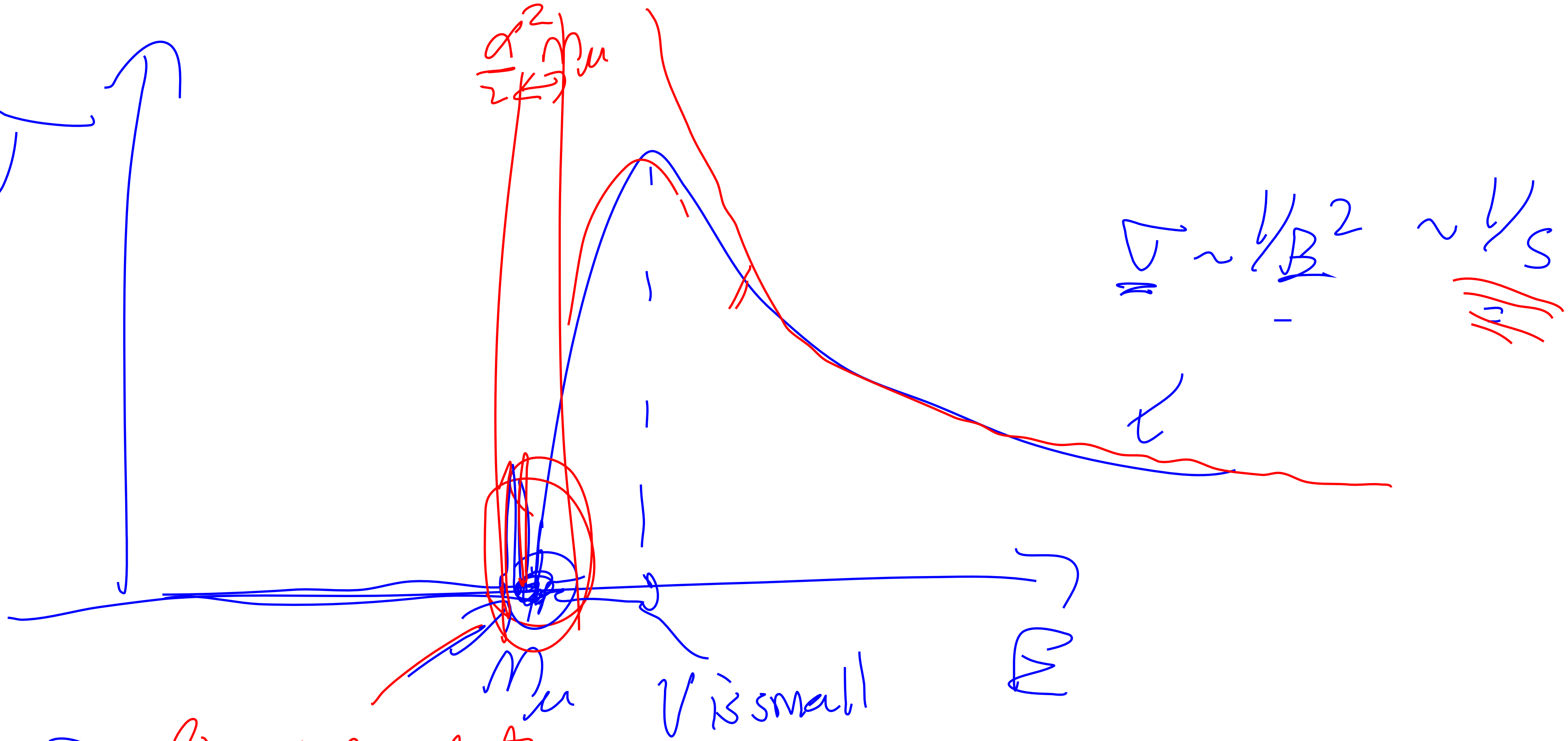
$\alpha = \frac{e^2}{4\pi}$

Max at $E = \frac{\sqrt{5}}{\sqrt{2}-1} m_\mu$

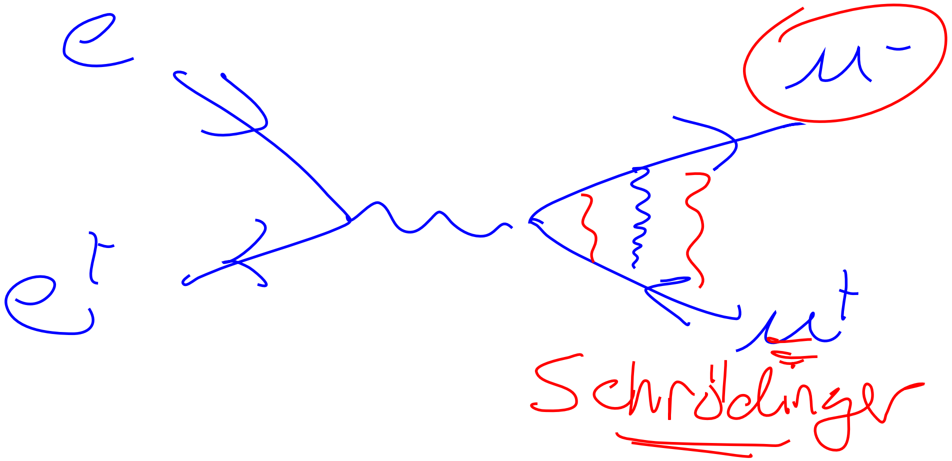
$\sigma_{tot} = \frac{4\pi\alpha^2}{3E_{cm}^2}\sqrt{1-m_\mu^2/E^2}\left(1+\frac{1}{2}\frac{m_\mu^2}{E^2}\right)$

$\boxed{E \geq m_\mu}$

$= \sqrt{s}/2$



→ disagreement w Exp.



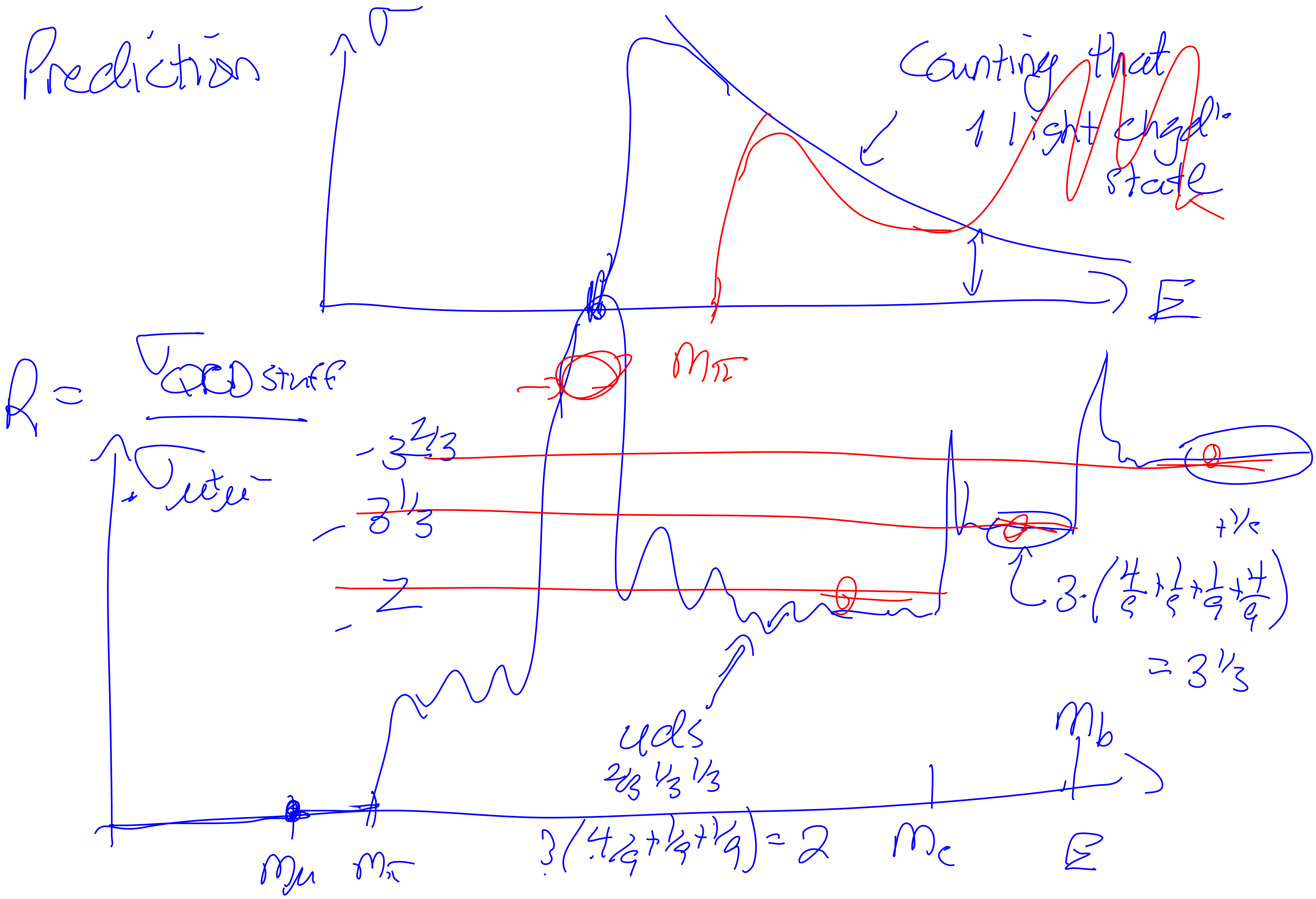
photon from one line to other

$\frac{\alpha}{v} \sim 1$ if $v \sim \alpha$

$v^2 \sim \alpha^2$

— $E_{bind} \sim \frac{\alpha^2}{2} m_\mu$

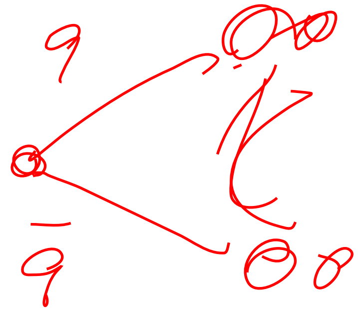
Prediction

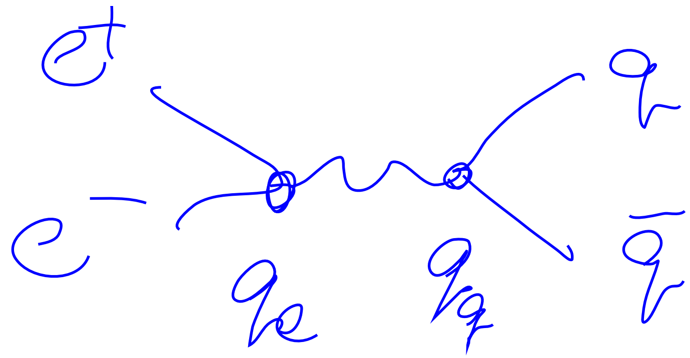


σ
Counting that 1 light chg'd state
E
m_π
$R = \frac{\sigma_{QCD\ stuff}}{\sigma_{\mu^+\mu^-}}$
$3\frac{2}{3}$
$3\frac{1}{3}$
2
$+\frac{1}{3}$
$3\cdot\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}\right)$
$= 3\frac{1}{3}$
uds
$\frac{2}{3}\ \frac{1}{3}\ \frac{1}{3}$
$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right) = 2$
m_μ
m_π
m_c
m_b
E

$$R = \frac{\sigma_{hadrons}}{\sigma_{muons}} = \boxed{\frac{\sum_{q\bar q} \sigma_{q\bar q} \times (q\bar q \to hadrons)}{\sigma_{\mu^+\mu^-}}}$$

$(q\bar q \to hadrons) \simeq 1$





$$\sigma_{q\bar q} = 3 \cdot q^2 \cdot \sigma_{\mu^+\mu^-}$$

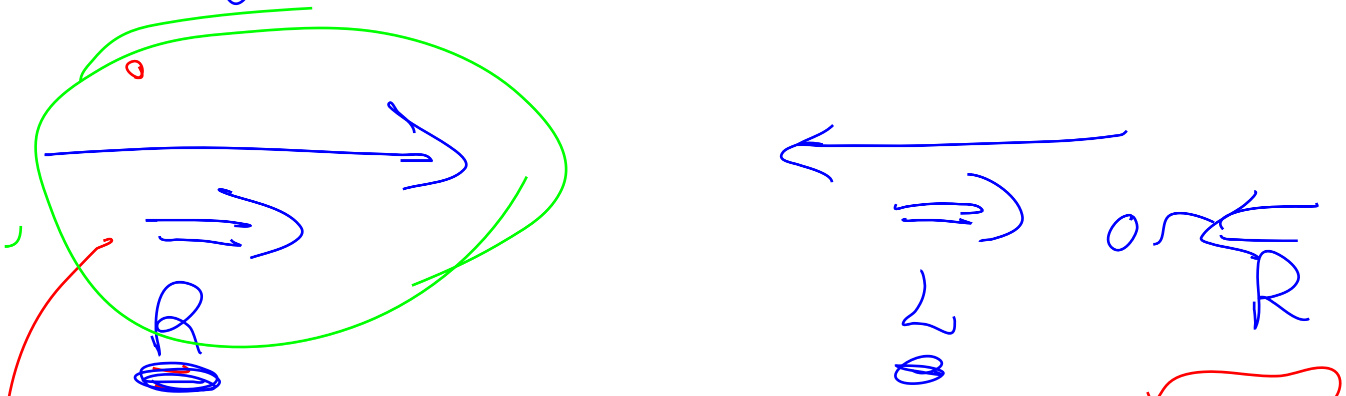
$$|M|^2 \propto q_e^2 \, q_q^2$$

- 3 colors
- 3 colors
- 3 colors

$$R \longrightarrow 3 \sum q_f^2$$

f = udsc...

Lazy - avg spins. Do Better - Polarized Beams.



$R - e^-$, $L - e^+$ } contrib.

R, R } $\to 0$

we have $\mathcal{M} = (\ldots)$ $\quad \bar v(p',s_2) P_L \gamma^\mu (P_R + \cancel{P_L}) u(p,s_R)$

$P_L u(p,s_R) = 0$

$P_R u(p,s_R) = u(p,s_R)$

$$P_R = \frac{1+\gamma_5}{2} = P_R^2$$

$$\gamma^\mu P_L = P_R \gamma^\mu$$

$$\gamma^\mu P_R = P_L \gamma^\mu$$

$$P_L = \frac{1-\gamma_5}{2}$$

$P_R u(p,s_R) = u(p,s_r)$

$P_R u(p,s_L) = 0$

$$\bar v \gamma^\mu P_R u = \underbrace{\bar v P_L}_{L} \gamma^\mu \underbrace{P_R u}_{R}$$

R - e^- only annih. a L - e^+ never R $\cdot$ e^+

$u_+ = \sqrt{m}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$ rest $\longrightarrow \sqrt{m}\begin{pmatrix}\sqrt{\gamma}\\0\\1/\sqrt{\gamma}\\0\end{pmatrix}$

$P_R = \begin{bmatrix}1&&&\\&1&&\\&&0&\\&&&0\end{bmatrix}$ $(1/2, 0)$

$u_- \longrightarrow \sqrt{m}\begin{pmatrix}0\\1/\sqrt{\gamma}\\0\\\sqrt{\gamma}\end{pmatrix}$

$P_L = \begin{bmatrix}0&&&\\&0&&\\&&1&\\&&&1\end{bmatrix}$ $(0, 1/2)$

$(1/2, 1/2)$

e^- $\longrightarrow$ $\Rightarrow$ $\quad$ $\longleftarrow$ e^+ $\Rightarrow$

total spin? $\underset{1/2}{\Rightarrow} + \underset{1/2}{\Rightarrow} = 1$

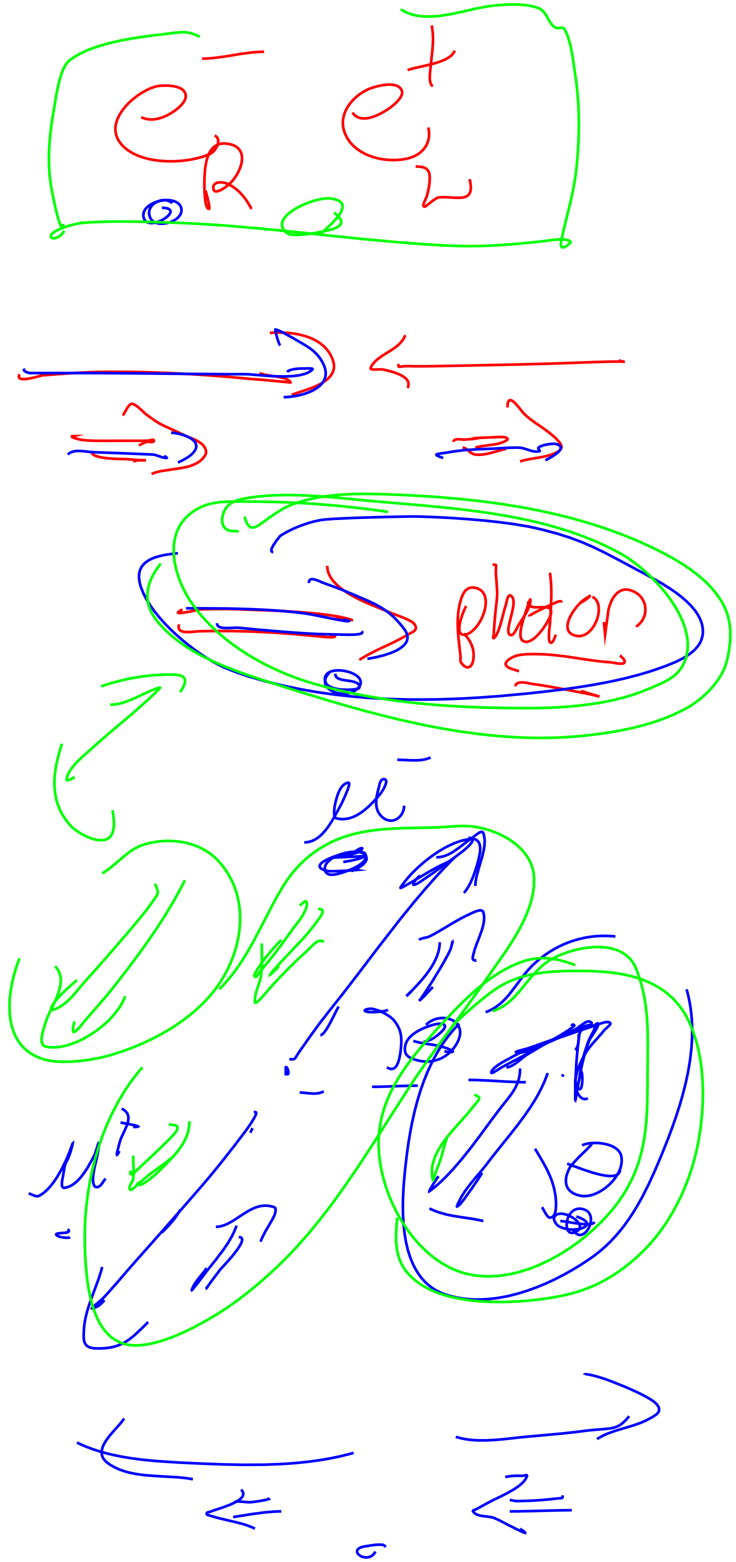
spin 1

photon

$\Rightarrow + \Leftarrow = 0$

not photon

$(1/2, 0) \otimes (1/2, 0) = (1, 0)$ or $(0, 0)$



Angular pattern $\to (1+\cos\theta)^2$

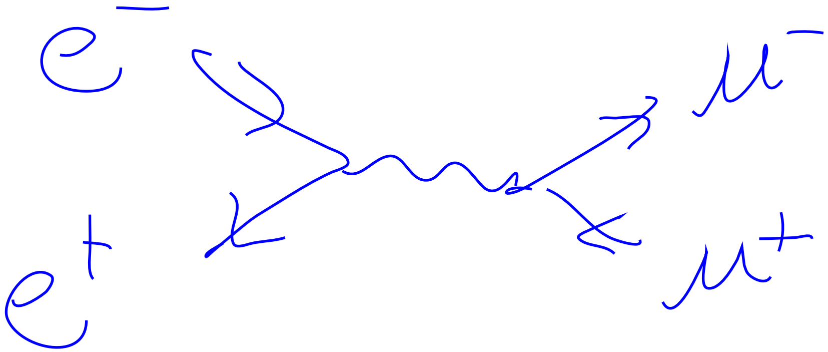
$R \to L$ $\qquad (1-\cos\theta)^2$

$1+\cos^2\theta$

$E \gg m_\mu$

$$\mathrm{Tr}\,\gamma^\mu\gamma^\nu\gamma^5 = 0$$

$$\boxed{\mathrm{Tr}\,\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\gamma^5 = -4i\,\epsilon^{\mu\nu\alpha\beta}}$$



$\Downarrow$ Crossing