Off to compute $\frac{d\sigma}{d\Omega}$ for various physical processes.

$e^+e^- \longrightarrow \mu^+\mu^-$ last time

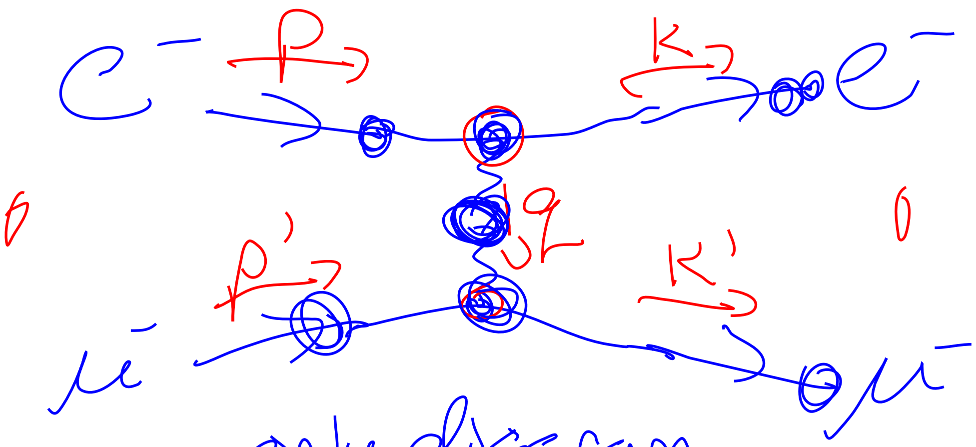
All the other things which happen in QED

$e^-\mu^- \to e^-\mu^-$ $\longrightarrow$ Almost $e^+e^- \to \mu^+\mu^-$ Crossing

$e^-e^- \to e^-e^-$ Møller scatt - Interference

$e^-e^+ \to e^-e^+$ Bhabha Scatt

$e^-\gamma \to e^-\gamma$ Compton Scatt - γ in in, final states



only diagram

Turn New → Old

$p \leftrightarrow p$ $p+p' \leftrightarrow p-k'$

$k \leftrightarrow -p'$ $s \leftrightarrow u$

$k' \leftrightarrow k$ $p-k \leftrightarrow p+p'$

$p' \leftrightarrow -k'$ $t \leftrightarrow s$

$u \leftrightarrow t$

$$\mathcal{M} = i\,\frac{-i\,g_{\mu\nu}}{q^2+i\epsilon}\,\bar{u}(k,s_f)\,ie\gamma^\mu u(p,s_i)\;\bar{u}(k',s_f')\,ie\gamma^\nu u(p',s_i')$$

$$\frac{1}{4}\sum_{s_i s_i' s_f s_f'} \mathcal{M}^*\mathcal{M} = \frac{e^4}{4(q^2)^2}\, g_{\mu\nu}\, g_{\alpha\beta} * \mathrm{Tr}\left[(\not{k}+m_e)\gamma^\mu(\not{p}+m_e)\gamma^\alpha\right] * \mathrm{Tr}\left[(\not{k}'+m_\mu)\gamma^\nu(\not{p}'+m_\mu)\gamma^\beta\right]$$

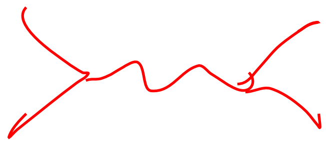
$q = p-k$

last time

$$\frac{e^4}{4(q^2)^2}\, g_{\mu\nu}\, g_{\alpha\beta}\, \mathrm{Tr}\left(({\not{p}}'-m_e)\gamma^\mu(\not{p}+m_e)\gamma^\alpha\right)\mathrm{Tr}\left((\not{k}+m_\mu)\gamma^\nu(\not{k}'-m_\mu)\gamma^\beta\right)$$

$q = p+p'$

Last Time:



$$\frac{1}{4}\sum \mathcal{M}^\dagger \mathcal{M} = 2e^4\left(\frac{t^2+u^2}{s^2}\right)$$

This time: $u \to s$

$s \to t$

$t \to u$

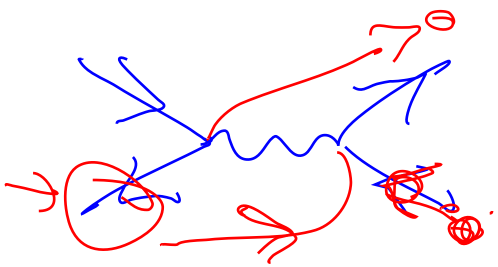
$$\boxed{\frac{1}{4}\sum \mathcal{M}^\dagger \mathcal{M} = 2e^4\left(\frac{u^2+s^2}{t^2}\right)}$$

Leave in m_μ^2: $\dfrac{8e^4}{(q^2)^2 \to t^2}\left(p\cdot k'\, p'\cdot k + p\cdot p'\, k\cdot k' - m_\mu^2\, p\cdot k\right)$ $(m_e = 0)$

How did it happen?

$u\bar{u}(p) \to \not{p}+m$

$v\bar{v}(k) \to \not{k}-m$



$\bar{v} \to \bar{u}$

$v \to u$

$\not{p}'-m \to \not{k}+m = (-)(-\not{k}-m)$

$\not{p}+m \to (-)(\not{k}-m)$

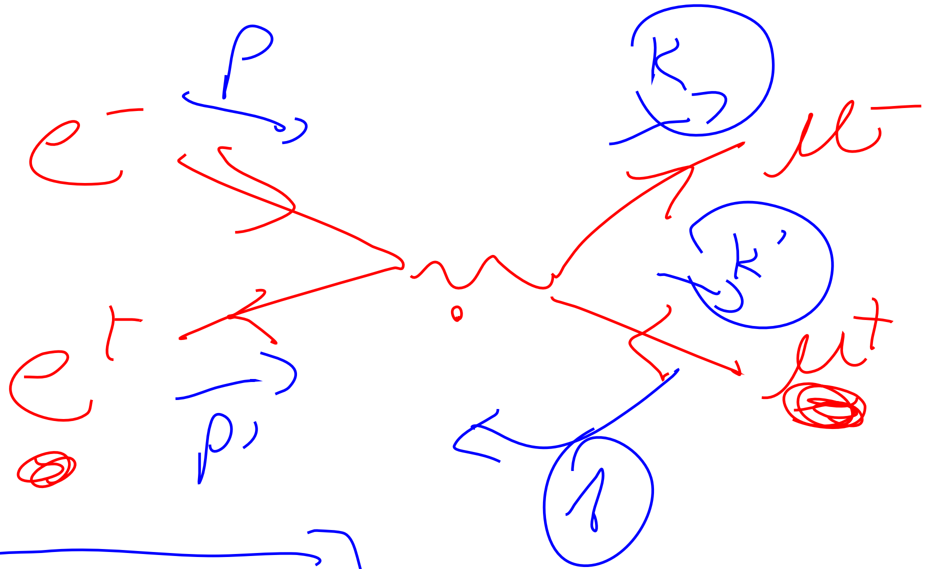
Crossing : Rule

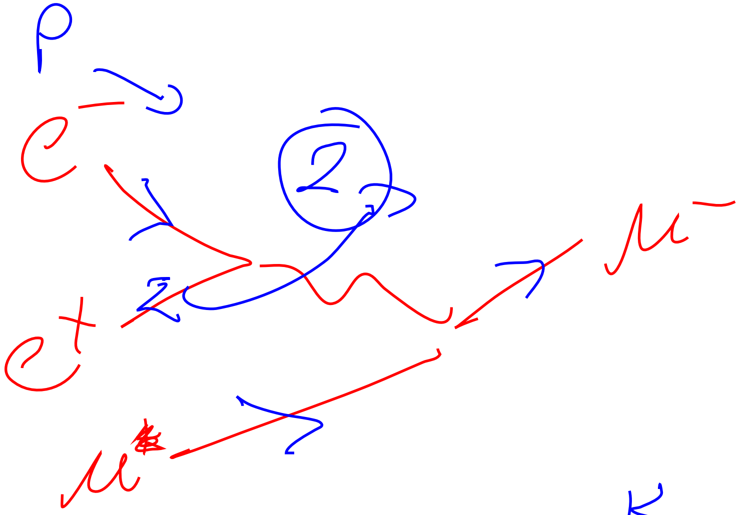
If processes are related by moving In part $\leftrightarrow$ Out Anti etc
In anti $\leftrightarrow$ Out Part

Then $\left(p_{in} \to k_{out} \right) \longrightarrow$ rearrange

ferm which switch between in, fin states

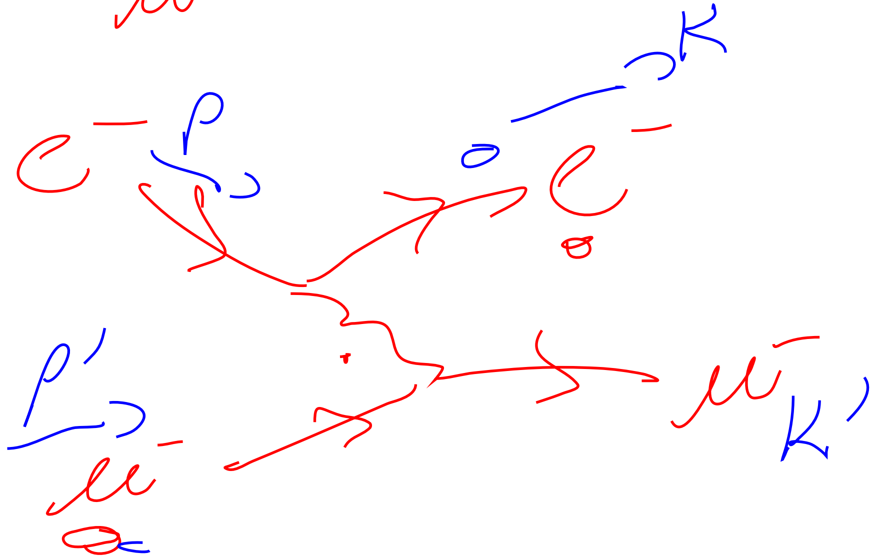
$(-) \longrightarrow$ New $|M|^2$.



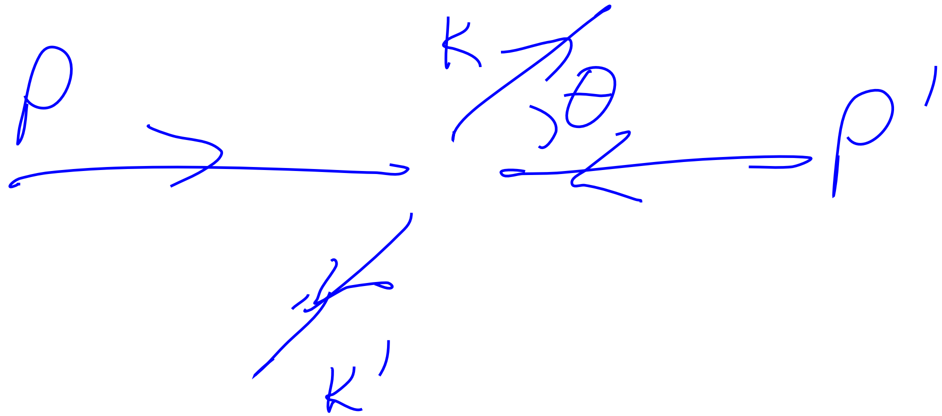


$$\boxed{\begin{aligned} p' &\to -k \\ k &\to k' \\ -k' &\to p' \end{aligned}}$$

$$(-1)^2 =$$



$$|\mathcal{M}|^2 = 2e^4 \, \frac{u^2+s^2}{t^2}$$



$$\begin{array}{ccccc} & t & x & y & z \\ p = (& E & 0 & 0 & E \;) \\ & E & p_x & p_y & p_z \end{array}$$

$$p' = (E \;\; 0 \;\; 0 \;\; -E)$$

$$k = (E \;\; E\sin\theta \;\; 0 \;\; E\cos\theta)$$

$$k' = (E \;\; -E\sin\theta \;\; 0 \;\; -E\cos\theta)$$

$$p+p' = (2E \;\; 0 \;\; 0 \;\; 0) \qquad \underline{s = 4E^2}$$

$$p-k = (0 \;\; -E\sin \;\; 0 \;\; E(1-\cos))$$

$$p-k' = (0 \;\; +E\sin \;\; 0 \;\; E(1+\cos))$$

$$t = -2E^2(1-\cos\theta)$$

$$u = -2E^2(1+\cos\theta)$$

$$2e^4\left(\frac{u^2+s^2}{t^2}\right) = \boxed{\frac{4+(1+\cos\theta)^2}{(1-\cos\theta)^2}\, 2e^4}$$

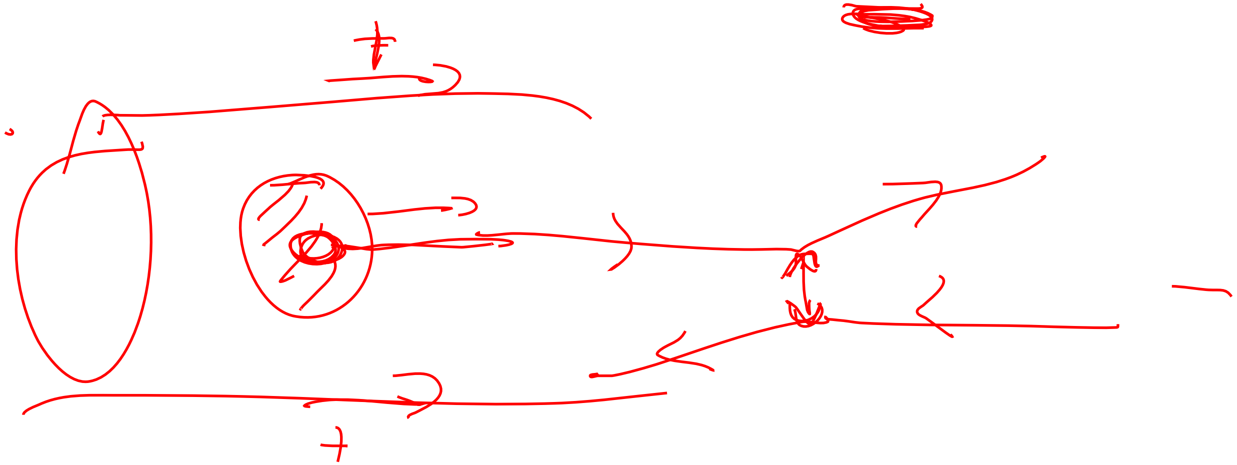
$$\sigma = \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi \; (\ldots)_2 \, e^4 \, \frac{4+(1+\cos\theta)^2}{(1-\cos\theta)^2}$$

$\theta \, d\theta$ $\qquad$ 2π $\qquad$ $\to \left(\frac{\theta^2}{2}\right)^2$

$$\int_{-1}^{1} d(\cos\theta) \, \frac{4+(1+\cos\theta)^2}{(1-\cos\theta)^2}$$

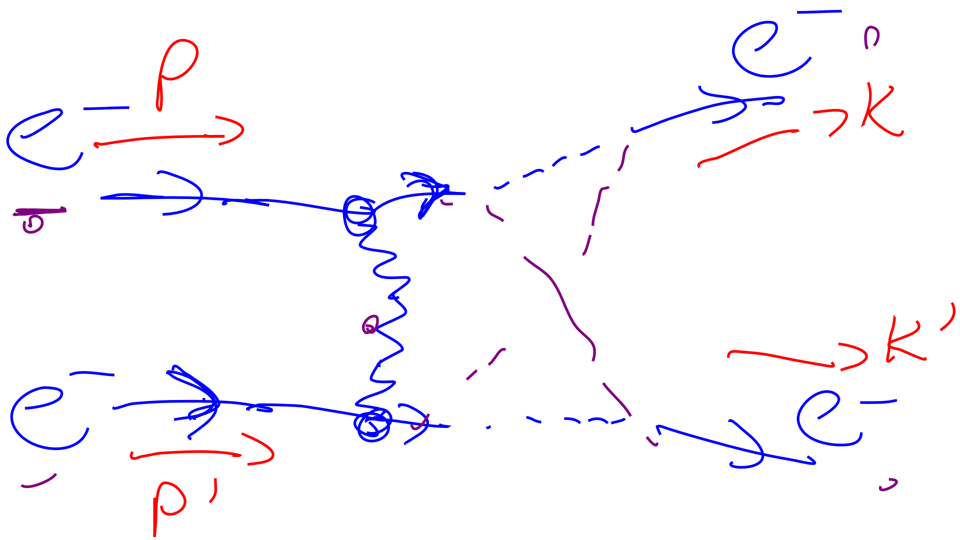
$$\int_0 \frac{\theta \, d\theta}{\theta^4}$$

Rutherford Scatt.

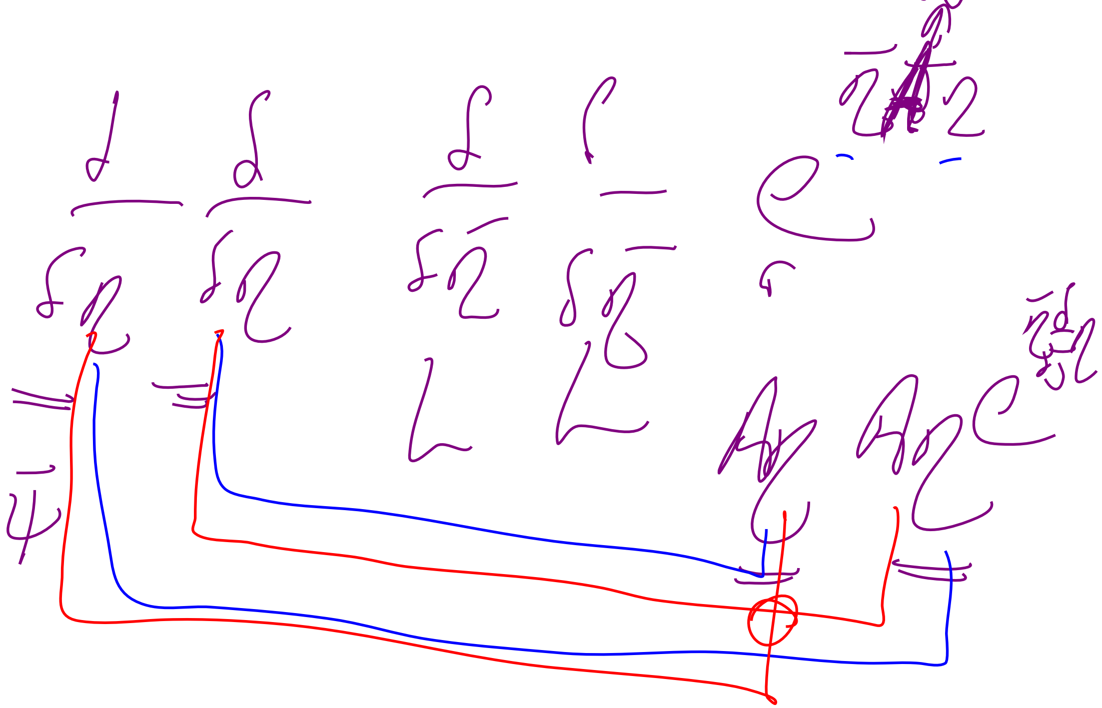


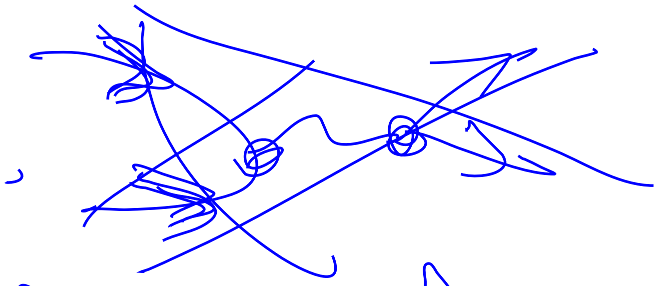
IR divergenze

Interference

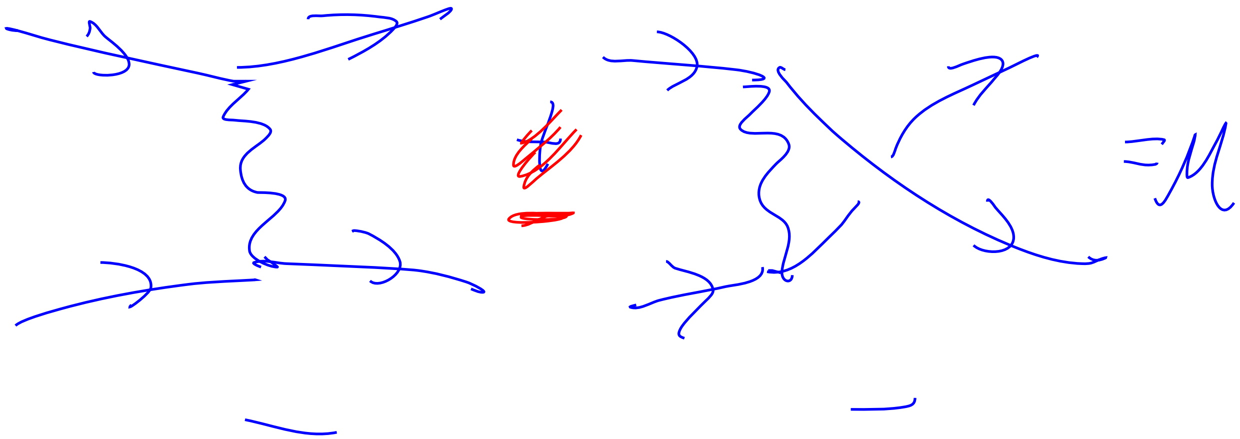


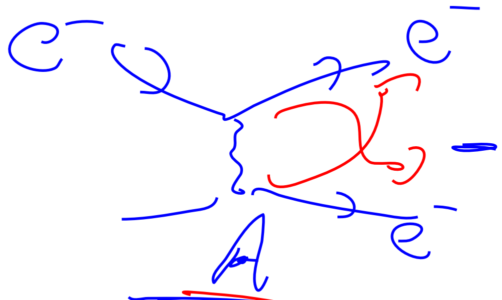
which choice is right? Both.

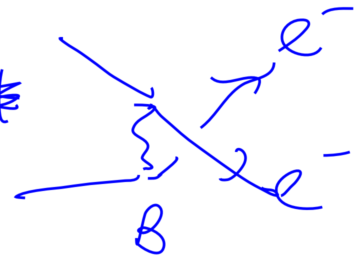




Draw Both Diagrams



$\mathcal{M} =$ 

$-$ 

Møller Scatt.

A: $i(ie)^2\, \bar{u}(k)\gamma^\mu u(p)\, \bar{u}(k')\gamma^\nu u(p')\, \dfrac{-i g_{\mu\nu}}{(p-k)^2}$

B: $\to i(ie)^2\, \bar{u}(k')\gamma^\mu u(p)\, \bar{u}(k)\gamma^\nu u(p')\, \dfrac{-i g_{\mu\nu}}{(p-k')^2}$

$$|\mathcal{M}|^2 = A^*A + B^*B - A^*B - B^*A$$

$$\boxed{A^*A = 2e^4\,\frac{u^2+s^2}{t^2}}$$

$$\boxed{B^*B = 2e^4\,\frac{t^2+s^2}{u^2}}$$

$k' \leftrightarrow k$ $\quad k \leftrightarrow k'$ $\quad s \to s$, $t \to u$, $u \to t$

Either

Final states are the same!

$$\Rightarrow \frac{1}{2}\int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi$$

or π

or $\pi/2$

$\Downarrow$

$$A^* B = -e^4 \quad \bar{u}(k')\gamma^\mu u(p)\ \bar{u}(k)\gamma^\nu u(p')\ \frac{g_{\mu\nu}}{(p-k')^2 = u}$$

$$\bar{u}(p)\gamma^\alpha u(k)\ \bar{u}(p')\gamma^\beta u(k')\ \frac{g_{\alpha\beta}}{(p-k)^2 = t}$$

$$= \frac{-e^4 g_{\mu\nu} g_{\alpha\beta}}{tu} \underbrace{\bar{u}(k')\gamma^\mu u(p)\bar{u}(p)\gamma^\alpha u(k)\bar{u}(k)\gamma^\nu u(p')\bar{u}(p')\gamma^\beta u(k')}_{\mathrm{Tr}\left[(\not{k}'+m)\gamma^\mu(\not{p}+m)\gamma^\alpha(\not{k}+m)\gamma^\nu(\not{p}'+m)\gamma^\beta\right]}$$

$$\simeq -\mathrm{Tr}\, \not{k}' \underbrace{\gamma^\mu \not{p}\gamma^\alpha \not{k}\gamma_\mu}_{=-2\not{k}\gamma^\alpha\not{p}} \not{p}'\gamma_\alpha = -2\,\mathrm{Tr}\,\not{k}'\not{k}\underbrace{\gamma^\alpha\not{p}\not{p}'\gamma_\alpha}_{4p\cdot p'}$$

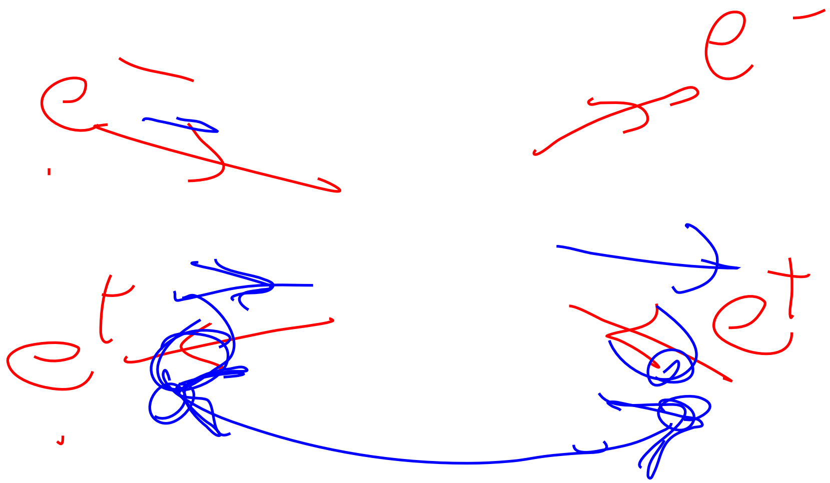
$$= -8\underbrace{\mathrm{Tr}\,\not{k}'\not{k}}_{4k\cdot k'}\, p\cdot p' = -32\, p\cdot p'\, k\cdot k' = -8s^2$$

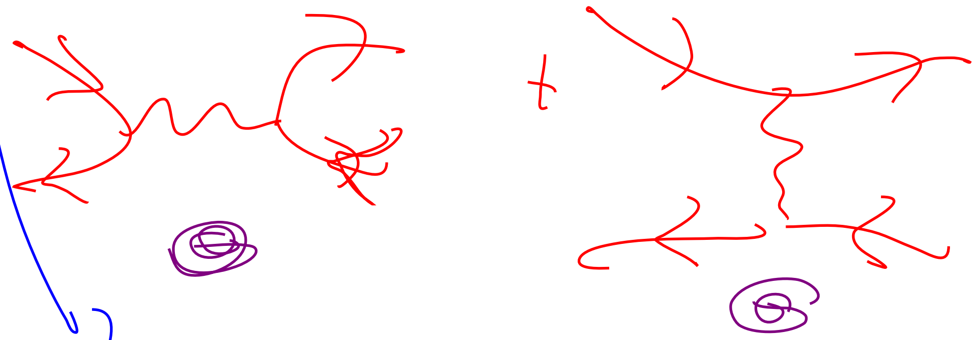
$$|M|^2 = 2e^4\left[\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} + 2\frac{s^2}{tu}\right]$$

that's it.

positive

Bhabha Scattering





$p \to p$

$k \to k$

$p' \to -k'$

$k' \to -p'$

$t \to t$

$s \to u$

$u \to s$

$(-1)^2$

$$|M|^2 = 2e^4\left[\frac{u^2+s^2}{t^2} + \frac{u^2+t^2}{s^2} + 2\frac{u^2}{st}\right]$$

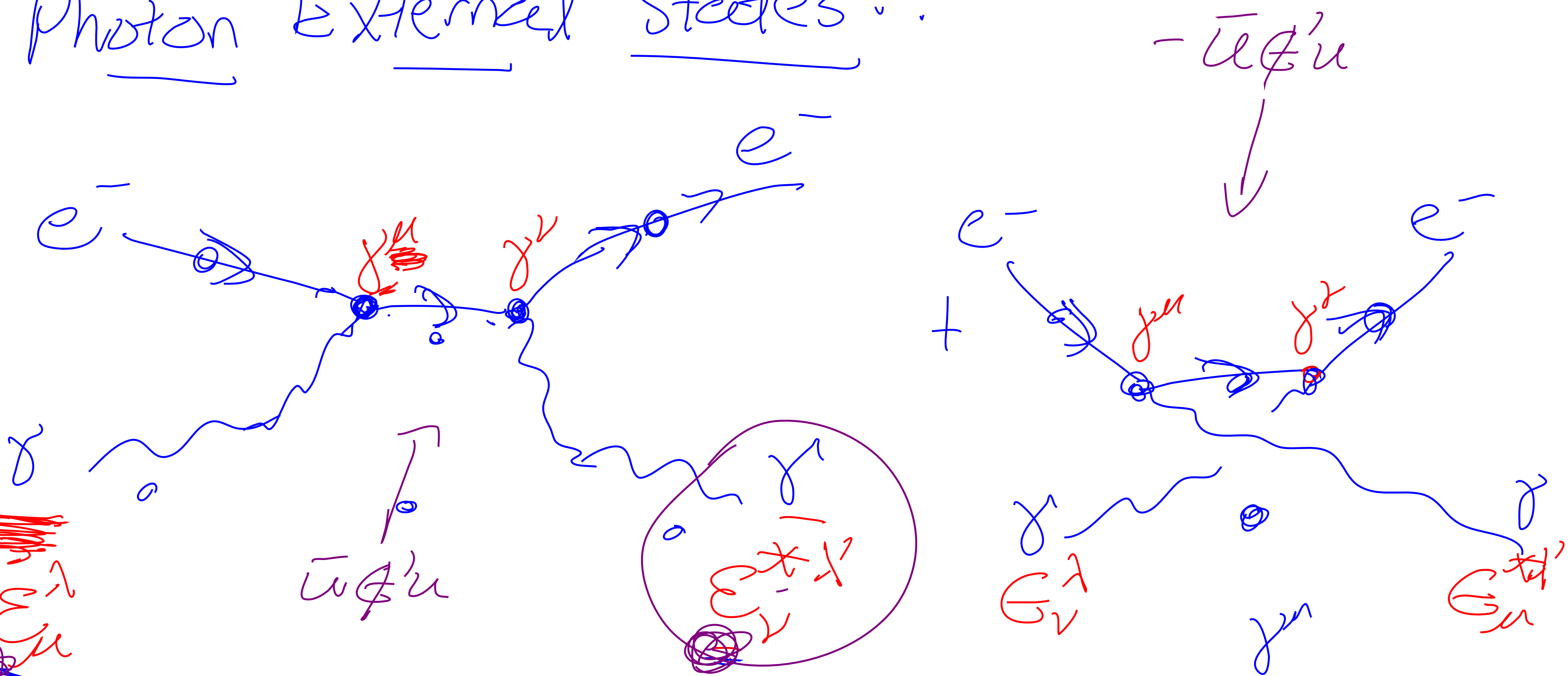
positive positive negative

$u<0$

$t<0$

$s>0$

Photon External States??



ϵ^λ_μ

polar. state

$\lambda = 1, 2$

$\lambda' = 1, 2$

$\not{\epsilon} \to \not{p}'$

$$\mathcal{M} = i(-ie)^2 \bar{u}(k) \not{\epsilon}^*_{\lambda'} \frac{i(\not{p}' + \not{p} + m)}{(p+p')^2 - m^2 + i\epsilon} \not{\epsilon}_\lambda u(p)$$

$$+ i(-ie)^2 \bar{u}(k) \not{\epsilon}_\lambda \frac{i(\not{p} - \not{k}' + m)}{(p-k')^2 - m^2 + i\epsilon} \not{\epsilon}^*_{\lambda'} u(p)$$

$\mathcal{M} = \underbrace{\epsilon_\mu}_{\text{polar. vector}} \underbrace{\mathcal{M}^{\mu}}_{\text{ev. else}}$

$A_\mu J^\mu$ $\quad p'_\mu J^\mu = \partial_\mu J^\mu$

$\epsilon_\mu \to p'_\mu$ then you get 0

Claim: $\boxed{p'_\mu \mathcal{M}^\mu = 0}$. Is it true?

$(p+p')(p+p') - m^2$

$(p+p')^2 - m^2$

Replace $\not\epsilon$ with $\not p'$

$$\mathcal{M} = i(ie)^2\, \bar{u}(k) \not\epsilon^{*} \frac{\cancel{\not p + \not p' + m}}{\cancel{(p+p')^2 - m^2}} \not p' u(p)$$

$$+\, i(ie)^2\, \bar{u}(k) \not p' \frac{\cancel{\not p - \not k' + m}}{\cancel{(p-k')^2 - m^2}} \not\epsilon^{*} u(p)$$

$(-)\cancel{(+\not p - \not k' + m)}$

$(\not p' - \not k + m)$

$= i(ie)^2 \bar{u} \not\epsilon' u$

$-i(ie)^2 \bar{u} \not\epsilon' u$

$$\underbrace{p+p'}=k+k'$$

$$p-k'=k-p'$$

$$k'-p=p'-k$$

$\cancel{\epsilon} \to \cancel{p'}$ get 0.

or $\cancel{\epsilon}^{*} \to \cancel{k'}$

ϵ_μ is what? Sit in frame w. $\vec{p}'$ is z axis

$$\begin{array}{ccccc} & E & p_x & p_y & p_z \\ \circ\; p' = & (1 & 0 & 0 & 1) \; E \end{array}$$

$$\epsilon^1 = (0\ 1\ 0\ 0)$$

$$\epsilon^2 = (0\ 0\ 1\ 0)$$

$$\underline{\epsilon}^R = (0\ 1\ i\ 0)/\sqrt{2}$$

$$\underline{\epsilon}^L = (0\ 1\ -i\ 0)/\sqrt{2}$$

- $p'_\mu p'^\mu = 0$
- $\bar{p}'_\mu \bar{p}'^\mu = 0$
- $\boxed{p'_\mu \bar{p}'^\mu = 1}$

$$\circ\; \bar{p}' = \frac{1}{2E}(1\ 0\ 0\ -1)$$

$$\mathcal{M} = \epsilon_\mu \mathcal{M}^\mu$$

$$\lambda = 1: \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\mathcal{M}^\dagger \mathcal{M} = \underbrace{\sum_\lambda \epsilon_\mu^\lambda \epsilon_\nu^{*\lambda}}_{} \mathcal{M}^\mu \mathcal{M}^{*\nu}$$

$$\lambda = 2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\sum_\lambda \epsilon_\mu^\lambda \epsilon_\nu^{*\lambda}$$

$$-g_{\mu\nu} = \begin{array}{c|cccc} & t & x & y & z \;\; \mu \\ t & \begin{bmatrix} \cancel{-1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \cancel{0} \, 1 \end{bmatrix} \\ x & \\ y & \\ z & \\ \nu & \end{array} = \boxed{\sum_\lambda \epsilon_\mu^\lambda \epsilon_\nu^{*\lambda} - P_\mu^\lambda P_\nu' - P_\mu'' P_\nu''}$$

$$\frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} \frac{1}{2} \qquad \frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \end{bmatrix} \frac{1}{2}$$

$$= - \begin{bmatrix} \frac{1}{2} & & & \frac{1}{2} \\ & & & \\ & & & \\ -\frac{1}{2} & & & -\frac{1}{2} \end{bmatrix} \;\; '' \;\; - \begin{bmatrix} \frac{1}{2} & & & -\frac{1}{2} \\ & & & \\ & & & \\ \frac{1}{2} & & & -\frac{1}{2} \end{bmatrix}$$

$$\Theta g_{\mu\nu} = \sum_\lambda \epsilon^*_\mu \epsilon_\nu + \underbrace{\bar{p}_\mu p'_\nu + p'_\mu \bar{p}'_\nu}$$

But $p'_\nu M^{*\nu} = 0$ harmless

$p'_\mu M^\mu = 0$

$$\boxed{\sum_\lambda \epsilon^*_\mu \epsilon_\nu \longrightarrow -g_{\mu\nu}}$$ why?

smart: $g_{\mu\nu} = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$

$$\frac{1}{4}\sum_{\text{spins}} \mathcal{M}^\dagger \mathcal{M} = \frac{e^4}{4}\, \bar{u}(k) \Big[\underbrace{\gamma^\nu \frac{i(\not p + \not k + m)}{s-m^2}\gamma^\mu}_{\cdot} + \gamma^\mu \frac{i(\not k' - \not p + m)}{u-m^2}\gamma^\nu \Big] u_p$$

$$\bar{u}_p \Big[\underbrace{\gamma_\mu \frac{(-i)(\not p + \not p' + m)}{s-m^2}\gamma_\nu}_{\cdot} + \gamma_\nu (-i) \frac{\not k' - \not p + m}{u-m^2}\gamma_\mu \Big] u_k$$

$\frac{1}{2}$ e^- spin

$\frac{1}{2}$ γ polas.

for $m_e \to 0$ $\quad \mathrm{Tr}\, \underbrace{\gamma_\nu \not k \gamma^\nu}_{-2\not k} (\not p + \not p')\, \underbrace{\gamma^\mu \not p \gamma_\mu}_{-2\not p} (\not p + \not p') \quad$ + 3 other terms

$$4\,\mathrm{Tr}\, \not k (\cancel{\not p} + \not p')\, \underbrace{\not p (\cancel{\not p}}_{\not p \not p = p^2 = m^2 = 0} + \not p') \quad + \text{3 other terms}$$

$$= 16(k\cdot p'\, p\cdot p' + k\cdot p'\, p\cdot p') = -8su \quad \cdots \quad \frac{1}{4}\sum \mathcal{M}^\dagger \mathcal{M} = -2e^4\left(\frac{u}{s} + \frac{s}{u}\right)$$

$u < 0 \quad s > 0$

work hard keep m_e^2

$$\overline{|\mathcal{M}|^2} = 2e^4\left[\frac{p\cdot k'}{p\cdot p'} + \frac{p\cdot p'}{k\cdot k'} + 2m^2\left(\frac{1}{p\cdot p'} - \frac{1}{k\cdot k'}\right) + m^4\left(\frac{1}{p\cdot p'} - \frac{1}{\cancel{p}\cdot k'}\right)^2\right]$$

Now we can do photons