write with finger

Higher order effects

Technical

Meaning

Tensors

$$\frac{G}{q^2} \to G \pi G \; \frac{1}{q^2 - \mu}$$

QED

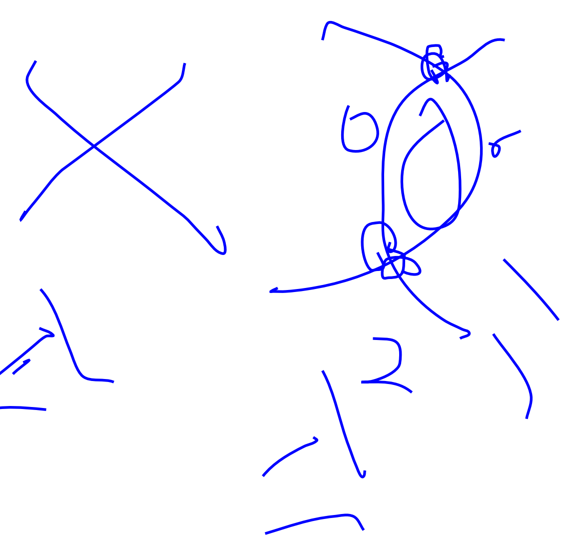
Mixed denom.

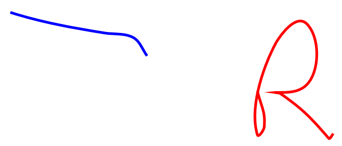
$$\frac{1}{(q^2 - m^2)((l+q)^2 - m^2)}$$

Poles - Wick

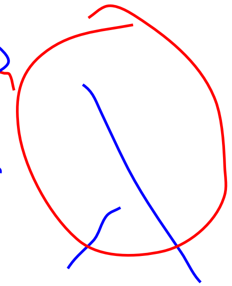
Divergences

$\lambda \varphi^4$





$\mathcal{M} =$



$$+ \frac{(-i\lambda)^2}{2} \int \frac{d^D l}{(2\pi)^D} \frac{i}{l^2 - m^2 + i\epsilon} \frac{i}{(l+q)^2 - m^2 + i\epsilon}$$

Trick: Feynman Param.

$$\frac{1}{AB} = \int_0^1 dx \, \frac{1}{\left(xA+(1-x)B\right)^2}$$

$$x l^2 - x m^2 + (1-x)(l+q)^2 - (1-x)^2 q^2$$

$$= \left(l+(1-x)q\right)^2 + \left(x(1-x)q^2 - m^2\right)$$

$$dl \to d\left(l+(1-x)q\right) = dl$$

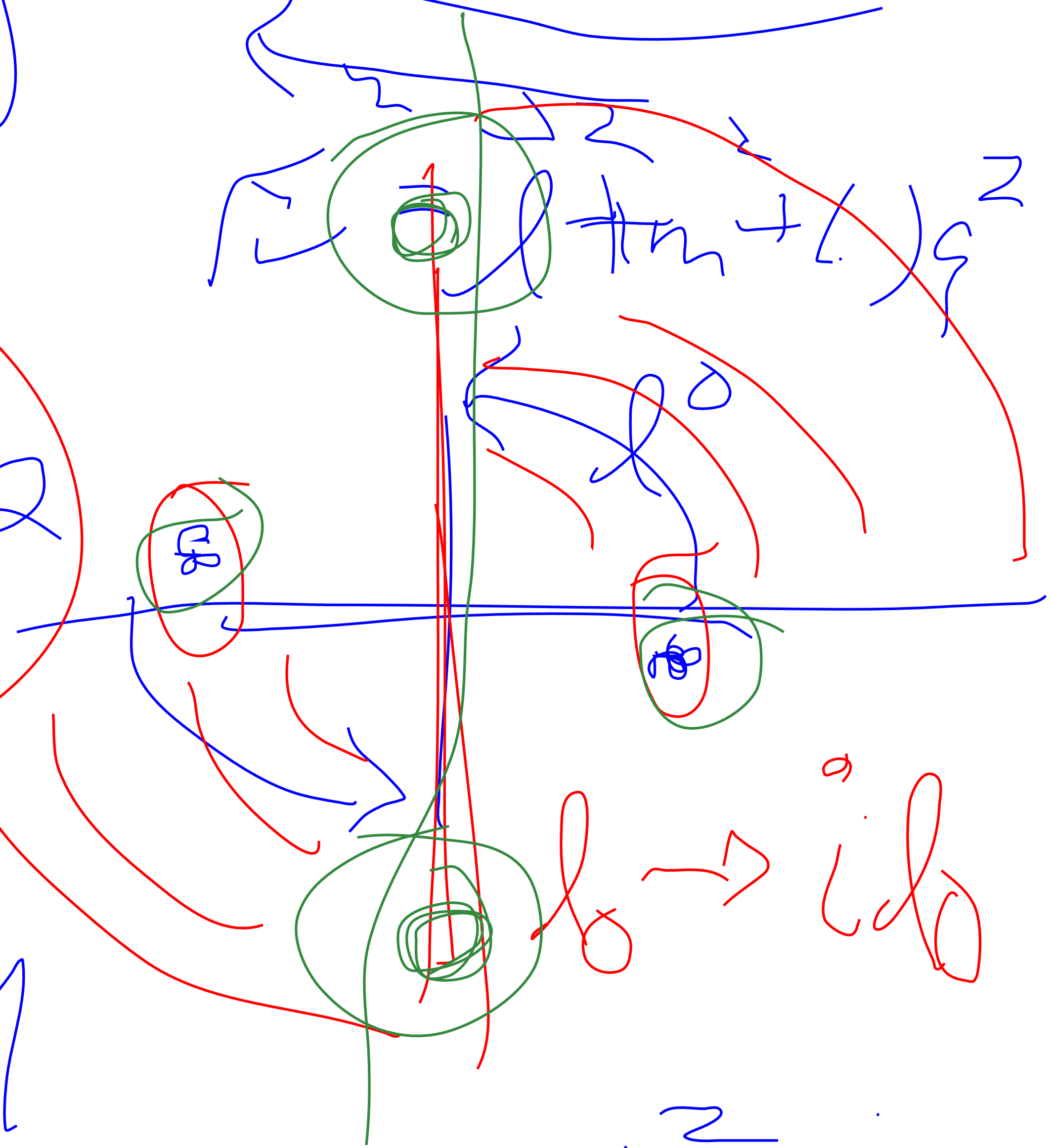
$$\int dx \int \frac{d^D \ell}{(2\pi)^D} \frac{1}{(\ell^2 - (\ldots))^2}$$

Solves Denom's

$$\ell_0^2 - (\vec{\ell}^{\,2} + m^2 + (\ldots) q^2)$$

$$i \int \frac{d\ell^D}{(2\pi)} \frac{1}{(\ell^2 - \xi^2 + i\epsilon)^2}$$

$1 - i\epsilon$



$\ell_0 \to i\ell_0$

$$\int dx \int \frac{d^D \ell_E}{(2\pi)^D} \frac{1}{(\ell_E^2 + m^2 x(1-x) q^2)^2}$$

$$-\frac{\lambda^2}{2}\int_0^1 dx \int^{\Lambda} \frac{d^4 l_E}{(2\pi)^4}\left(\frac{1}{l_E^2+M^2}\right)^2 \qquad M^2 = x(1-x)q^2$$

$$\frac{2}{2\,16\pi^2}\ \frac{1}{8\pi^2}\int \text{angles}\ \frac{}{2\pi^2}\int^{\Lambda} l^3 dl\,\frac{1}{(l^2+M^2)^2}$$

$$2l\,dl = d(l^2)$$

$$\frac{1}{16\pi^2}\int_0^{\Lambda^2}\frac{l^2\,dl^2}{(l^2+M^2)^2} = \log\frac{\Lambda^2}{M^2} - 1$$

$$\mathcal{M} = \lambda \Theta \frac{\lambda^2}{32\pi^2} \int_0^1 dx \left(\log \frac{\Lambda^2}{m^2 - x(1-x) q^2} - 1 \right)$$

1) $q^2 \to 0$

$$\log \frac{\Lambda^2}{m^2} - 1$$

2) $|q^2| \gg m^2$

$$\log \frac{\Lambda^2}{|q^2|} + 1$$

$$\log q^2/m^2$$

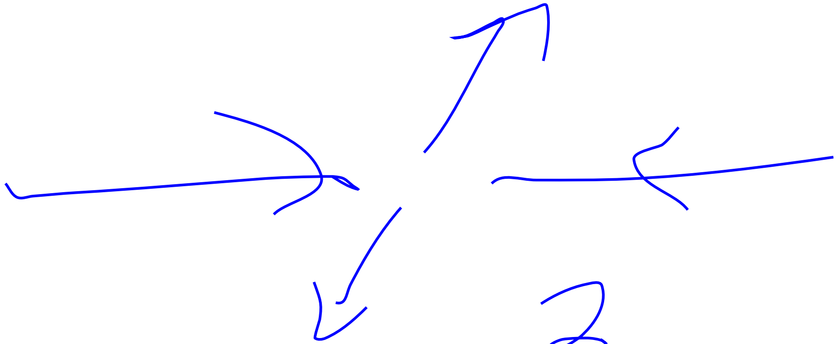
$$S\sigma = \frac{\lambda^2}{32\pi^2}\left(1 - \frac{k}{32\pi^2}\log\frac{\Lambda^2}{m^2}\right)$$

$S\sigma =$

g^2

λ_0 - in $\mathcal{L}$. Not measured

We "measure" λ in Expt.



λ_m = value found in expt.

$$\sigma_0 = \frac{\lambda^2}{32\pi s}$$

Choose favorite "μ"

$$\lambda(\mu) = \lambda_m$$

$$\sigma(q) = \frac{\lambda_m^2}{32\pi s}$$

$$\Delta m = \left(1 - \frac{\lambda_0}{32\pi^2}\left(\log\frac{\Lambda^2}{m^2} - 1\right)\right)$$

$$G_s = \left(1 - \frac{\lambda_0}{32\pi^2}\left(\log\frac{\Lambda^3}{X} - 1\right)\right)$$

$$X \rightarrow 9e^{-2}$$

$$G_s = \left(\Delta m + \frac{\lambda_m}{32\pi^2}\log\frac{X}{m^2}\right)$$

Renorm

Lagrangian coupling λ_0

1) not measured

2) differs by $\log \Lambda^2/m^2$ from what you do measure

3) dep on Λ

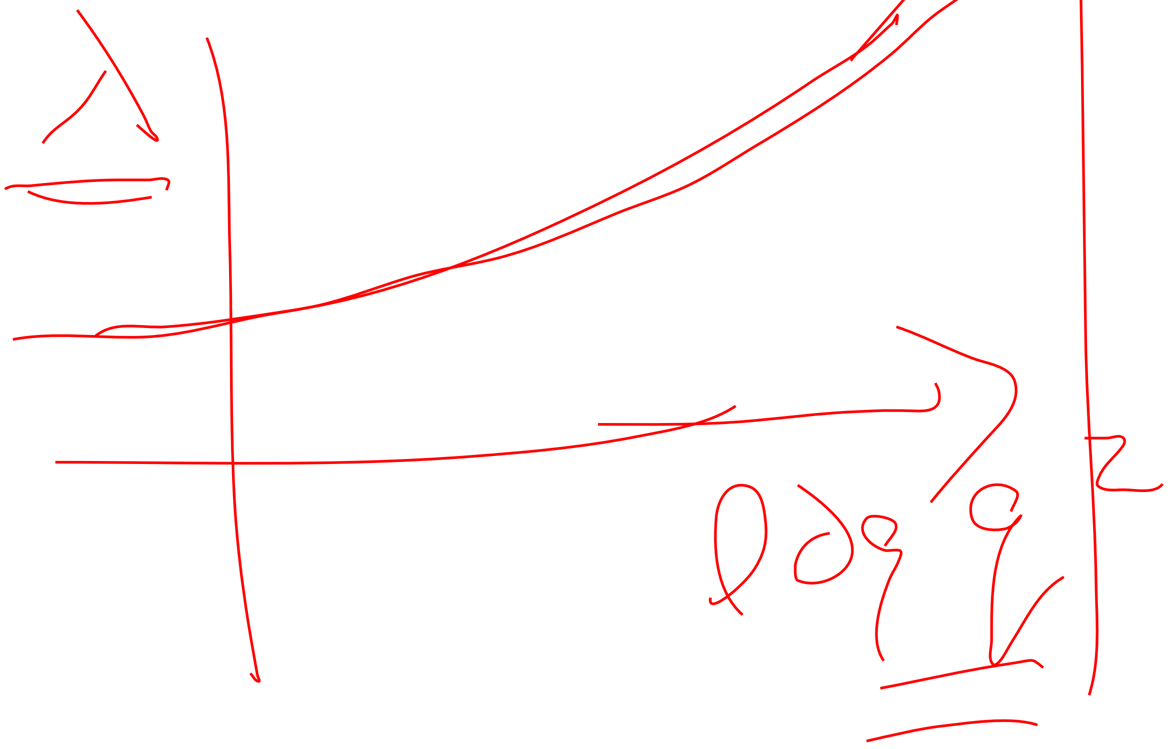
Measure λ_m in some xpt

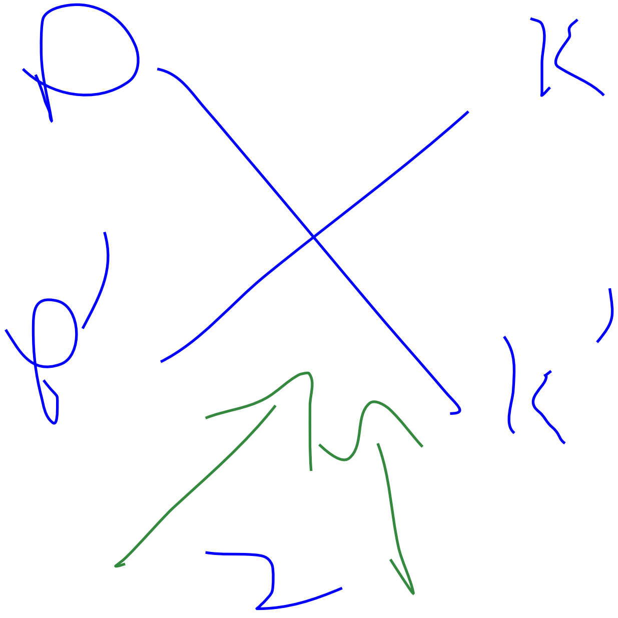
Evaluate – Process I want λ_0

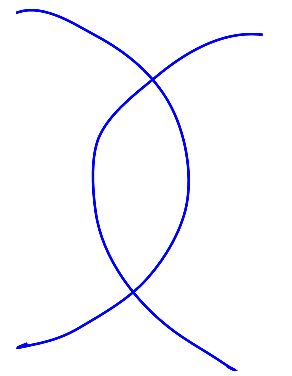
– Proc defining λ_m

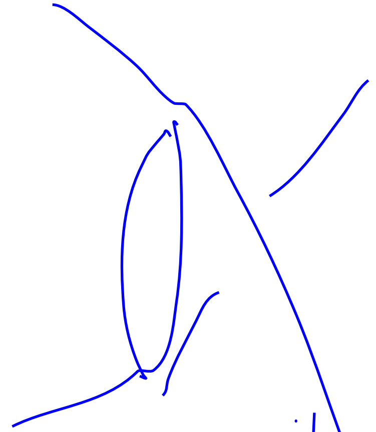
Solve in terms of λ_m – no Λ

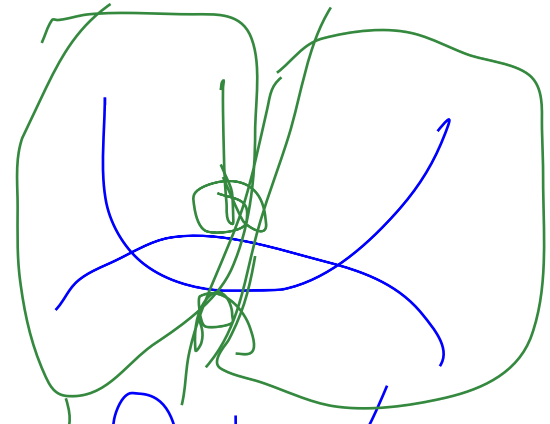
$$\lambda(\mu_2) = \lambda(\mu_1)\left[1 + \frac{\lambda}{32\pi^2}\log\frac{\mu_2^2}{\mu_1^2}\right]$$











$(p-k)^2 \qquad p-k' \qquad p+p'$

$q^2 \qquad\qquad t \qquad\qquad u \qquad\qquad s$

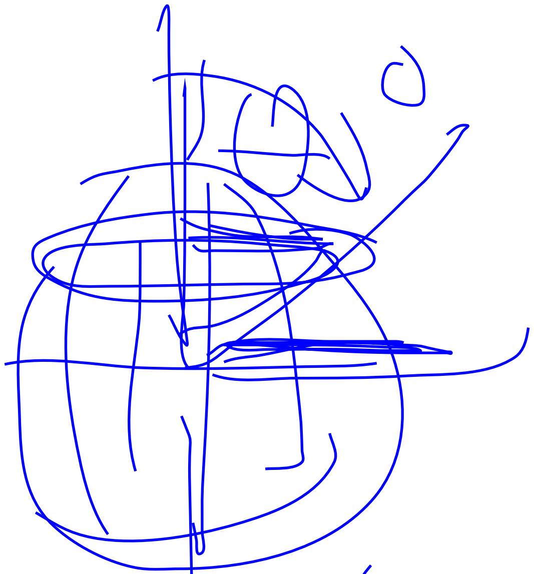
$$\lambda - \frac{\lambda^2}{32\pi^2}\left[\log\left(\frac{\Delta^2}{-t}\right) + \log\left(\frac{\Delta^2}{-u}\right) + \log\left(\frac{\Delta^2}{-s}\right)\right] + \lambda^3$$

$-q^2$

$D = N$: $\int_0^\pi d\theta \, A_{N-1} \sin^{N-2}\theta$

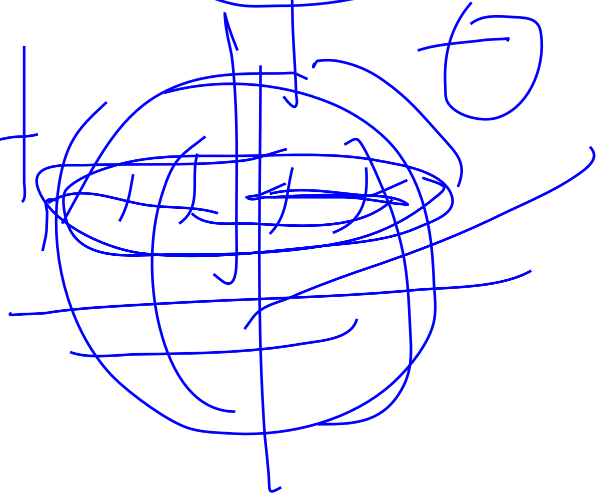
$2, \; 2\pi, \; 4\pi, \; 2\pi^2; \; \frac{8}{3}\pi^2; \; [illegible] \; \pi^3 \; [illegible]$

$D = 3$



$\int_0^\pi d\theta \; 2\pi \sin\theta = 4\pi$

$D = 4$



$\int_0^\pi d\theta \; 4\pi \sin^2\theta = 2\pi^2$

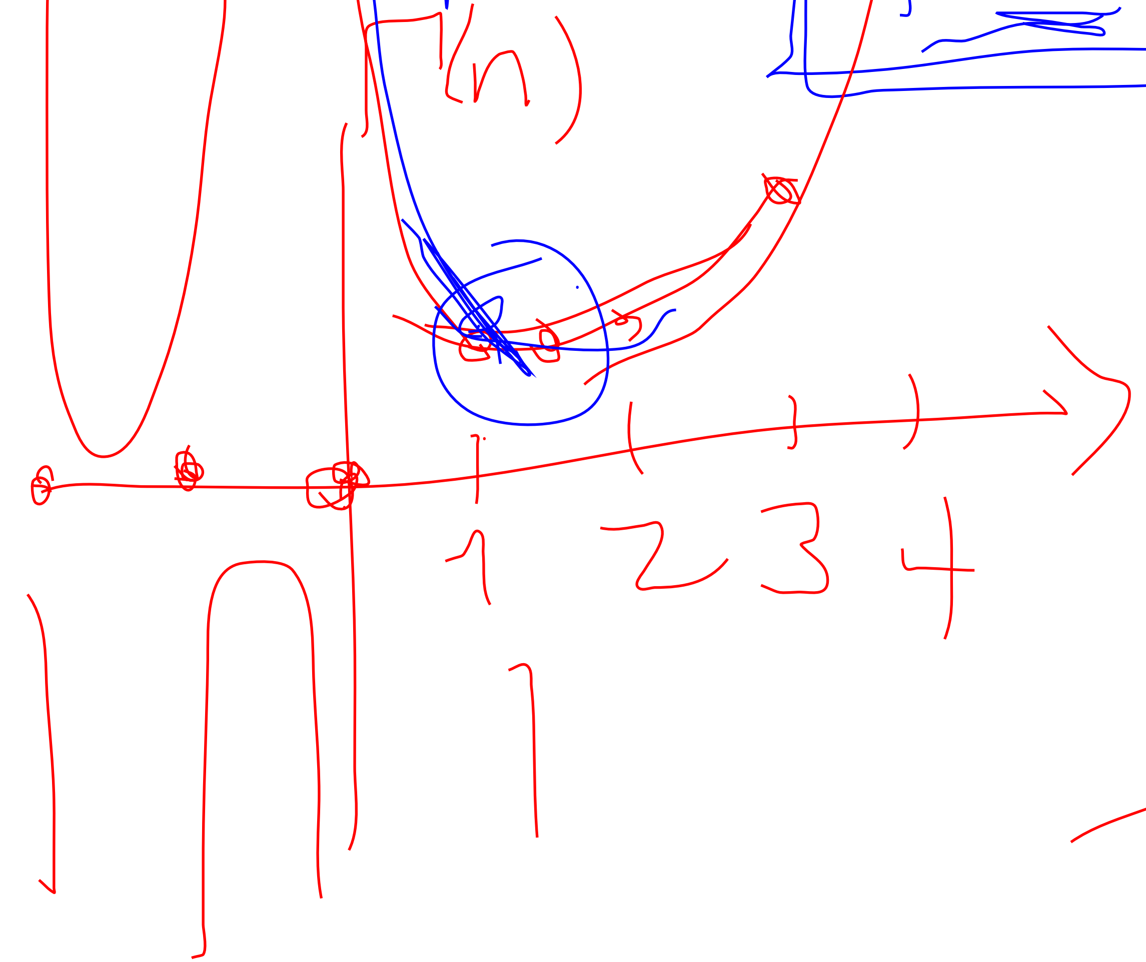
$D = 5 : \frac{8}{3}\pi^2$

$$\Gamma(n) \equiv \int_0^\infty x^{n-1} e^{-x}\, dx \qquad \text{int } (n-1)!$$

$\mathrm{Re}\, n > 0$

Int by parts

$$\Gamma(n+1) = n\,\Gamma(n)$$



$n \sim$ $\Gamma(1) = 1$

$$\varepsilon\,\Gamma(\varepsilon) = \Gamma(n+\varepsilon)$$

$$\Gamma(\varepsilon) = \frac{\Gamma(1+\varepsilon)}{\varepsilon}$$

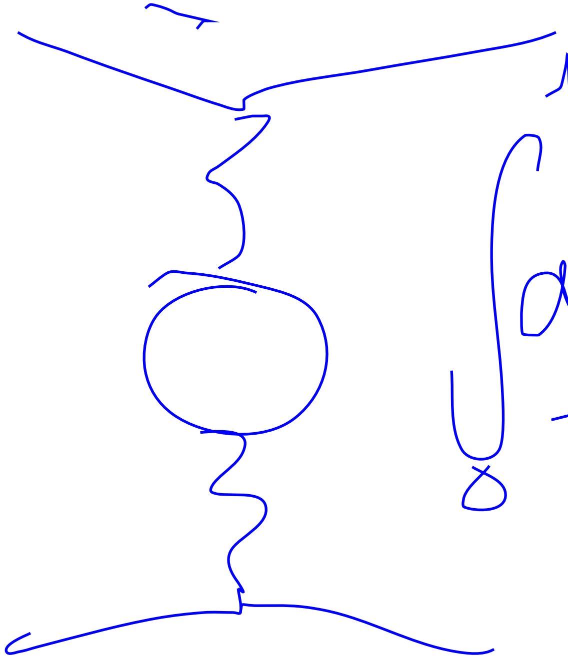
Choose $\mu^2 = m_e^2$. Then

$$\frac{1}{e_0^2} - \frac{1}{\epsilon}\frac{1}{12\pi^2} = \frac{1}{e_R^2}$$

$$\frac{1}{\left(\frac{1}{e_0^2} - \frac{1}{\epsilon\, 12\pi^2}\right)} = \frac{1}{\frac{1}{e_0^2} - \frac{1}{12\pi^2}\left(\frac{1}{\epsilon} \ldots\right)} \quad \frac{1}{e_0^2}$$

Throw out all $\frac{1}{\epsilon}$ $e^2 \to e_R^2$

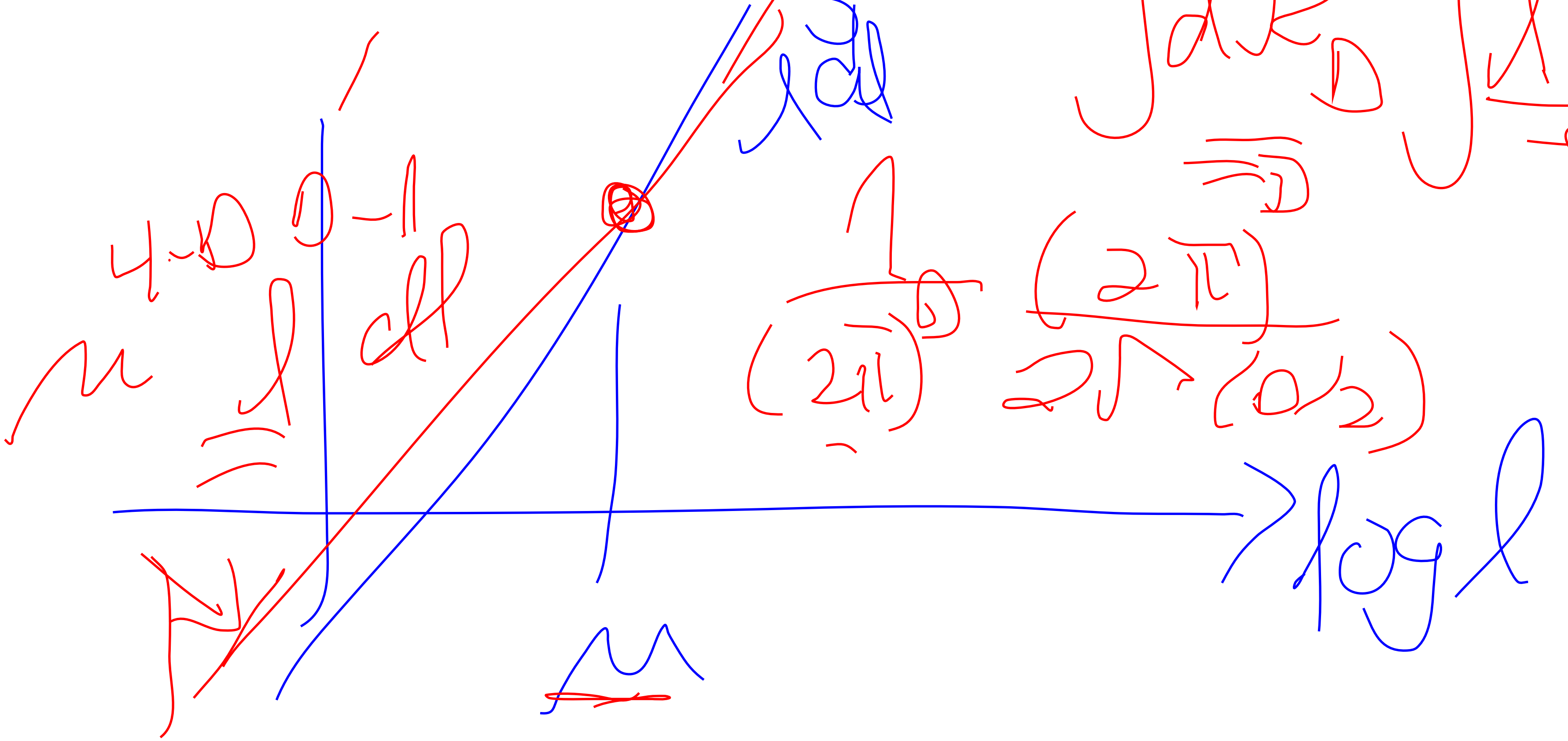
That turns all



$$\int_0^1 dx \int \frac{d^D l}{(2\pi)^D} \frac{1}{(l^2+m^2)^2}\, \mu^{4-D}$$

$$\int d\Omega_D \int \frac{l^{D-1}\, dl}{\quad}$$

$$\frac{1}{(2\pi)^D} \frac{2\pi^{D/2}}{\Gamma(D/2)}$$



$$\int_0^\infty \frac{x^{2x-1}\,dx}{(1+x^{D})^{y}} = \frac{1}{\lambda}\,\lambda^{x-y}\,\frac{\Gamma(x)\,\Gamma(y-x)}{2\,\Gamma(y)}$$

$$\frac{1}{\epsilon} \qquad \Gamma(\epsilon)$$

$$x = D/2$$

$$y = 2$$

$$y - x = 2 - D/2 = \epsilon$$

$$D = 4 - 2\epsilon$$

$$\Pi(q) = \frac{-e^2}{12\pi^2}\,\Gamma(\epsilon)\int_0^1 dx\; 6x(1-x) \times$$

$$(2\pi)^4 \to (2\pi)^D \quad \left[\frac{4\pi\mu^2}{m^2 - x(1-x)q^2}\right]^{\epsilon} \qquad \frac{1}{\epsilon} - \gamma_E$$

$$A^{\epsilon} = e^{\epsilon \log A} = 1 + \epsilon \log A + \epsilon^2$$

$$\left(\frac{1}{\epsilon} - \gamma_E\right)\int_0^1 dx\; 6x(1-x)\left(1 + \epsilon \log\left[\;\right]\right)$$

$$\Pi(q^2) = -\frac{e^2}{12\pi^2}\left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi\right) + F\left(\frac{\mu^2}{m^2}, \frac{\mu^2}{q^2}\right)$$

Always $\frac{1}{\epsilon}$

$$\frac{1}{\epsilon} = \log\frac{\Lambda^2}{\mu^2}$$

$m^2 \gg q^2$

$$\ln\frac{\mu^2}{m^2}$$

$$F = \int_0^1 dx\, 6x(1-x)\ln\frac{\mu^2}{m^2 - x(1-x)q^2}$$

$$\ln\frac{\mu^2}{q^2} - \frac{5}{3}$$

$m^2 \ll q^2$

$$\frac{1}{e^2(\mu)} - \frac{1}{12\pi^2}\log\frac{\mu^2}{q^2}$$

How do I measure e^2?

$$\ominus \longleftarrow\longrightarrow \ominus \quad \left(F = \frac{e^2}{4\pi r^2} \right. \qquad q^2 \to 0$$

$$F = \log \frac{\mu^2}{m^2} \quad \text{choose } \mu^2 = m^2$$

$$V = -\frac{e_0^2}{12\pi^2}\left(\frac{1}{\epsilon}\right)$$

$$\frac{e_0^2}{r^2\left(1 - \frac{e_0^2}{12\pi^2}\frac{1}{\epsilon}\right)}$$

$$q^2\left(\frac{1}{e_0^2} - \frac{1}{12\pi^2}\epsilon\right) = \frac{1}{e_R^2}$$