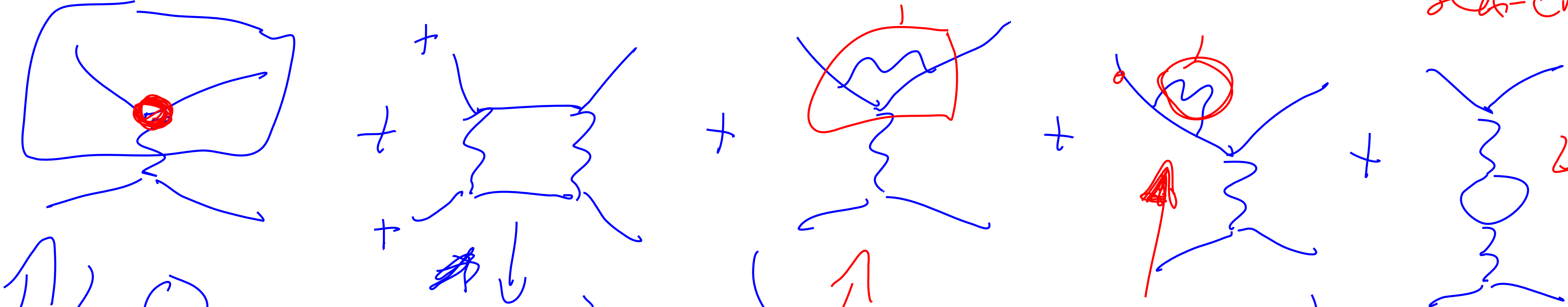
Last lectures

photon (GB) self-en.



LO

$\mathcal{M}$

Attractive $\neq$ Repulsive

Vertex correct.

???

Ferm. Self-energy

$e_0^2 \to e_R^2$

$\to q^2$-dependent

Relate diff. xpt

1) Determine $e_R^2(e_0^2)$

2) Compute other

$\sigma(e_0^2) \to \sigma(e_R^2)$

UV div. cancel sensible

$$\mathcal{L}(QED) = \bar{\Psi}\left[i(\partial_\mu + ieA_\mu)\gamma^\mu - m\right]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

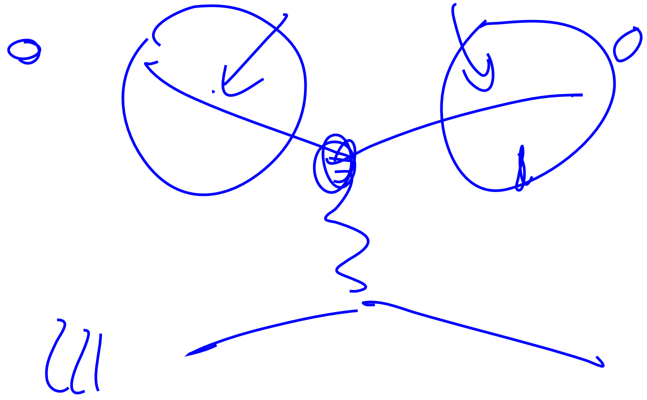
$$e^{i\int \mathcal{L}}$$

$$\frac{i}{Z(\not{p}-m)} = \frac{i(\not{p}+m)Z^{-1}}{p^2-m^2+i\epsilon}$$

$$i\cdot i\cdot ie\gamma^\mu = -ie\gamma^\mu Z$$

$$\psi_{Gry} = \frac{1}{\sqrt{Z}}\psi \qquad \psi = \sqrt{Z}\,\psi_{Gry}$$

$$\mathcal{L} = Z\,\bar{\psi}_{Gry}\left[i(\partial_\mu + ieA_\mu)\gamma^\mu - m\right]\psi_{Gry} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



$$\mathcal{M} = \langle 0|\, \bar\psi_{Gr}\; \psi_{Gr} \quad \bar\psi\psi \,|0\rangle$$

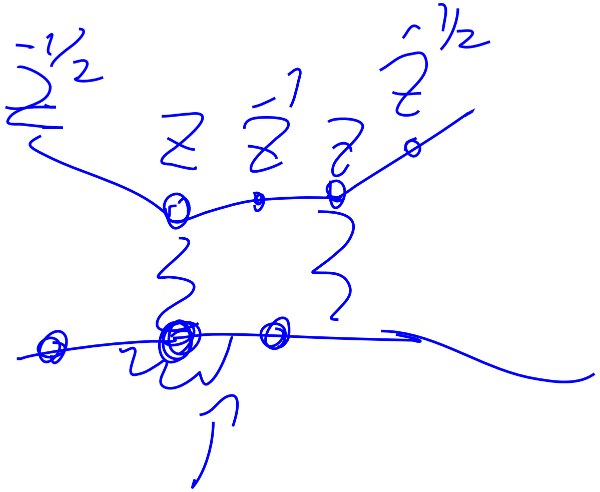
$\frac{1}{\sqrt{Z}}\psi_{Gr} \quad \frac{1}{\sqrt{Z}}\bar\psi_{Gr}$

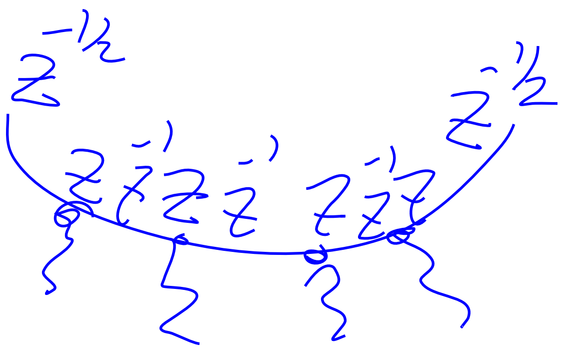
AMP

$$= \sum ie\gamma^\mu(\cdots)$$

$$Z^{-1}\frac{\not p + m}{p^2 - m^2}(\cdots)\, Z \quad Z^{-1/2}\,\frac{\not p + m}{p^2 - m^2}$$

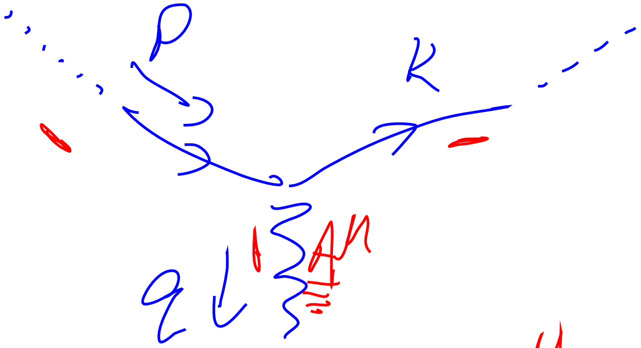
$$Z^{-1}(\not p + m) = Z\, u\bar u \quad Z^{-1/2}$$





$\longrightarrow Z^{3}$

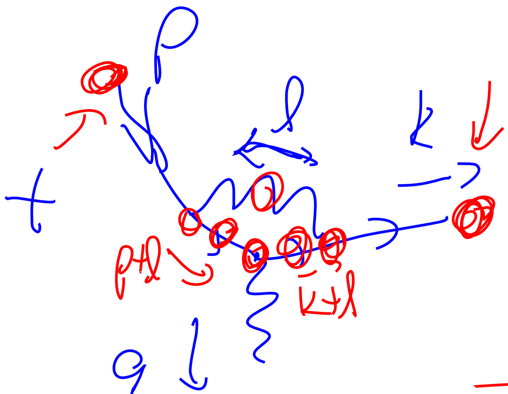
Vertex



$ie\gamma^\mu$

$q = p - k$

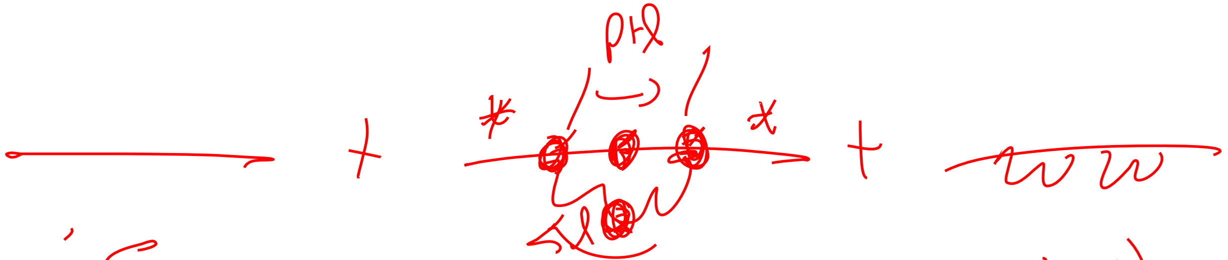
$k = p - q$



$+$ 1 loop $= ie\,\Gamma^\mu(p,q)$

$$\int \frac{d^D l}{(2\pi)^D} (ie)^3 \gamma^\alpha \frac{i(\not{k}+\not{l}+m)}{(k+l)^2-m^2+i\epsilon} \gamma^\mu \frac{i(\not{p}+\not{l}+m)}{(p+l)^2-m^2+i\epsilon} \gamma^\beta$$

$$\frac{-i\, g_{\alpha\beta}}{l^2+i\epsilon}$$



$$\rightarrow \frac{i}{\not{p} - m - \Sigma(p)}$$

matrix

$$iS \qquad iS(-i\Sigma)iS \qquad iS(-i\Sigma)iS(-i\Sigma)iS$$

$$S = \frac{1}{\not{p} - m}$$

$$\Sigma(p) = i(ie)^2 \int \frac{d^D l}{(2\pi)^D}\, \gamma^\mu \frac{i(\not{p}+\not{l}+m)}{(p+l)^2 - m^2 + i\epsilon}\, \gamma^\nu \frac{-i g_{\mu\nu}}{l^2 + i\epsilon}$$

Is Σ Related to Γ? What are they?

What is Σ? Matrix $\Sigma(p)$

$$\Sigma(p) = \Sigma_m(p^2)\mathbb{1} + \Sigma_\nu(p^2)\gamma^\mu p_\mu$$

1, 4

$[\gamma^\mu, \gamma^\nu] p_\mu p_\nu$ — 6 Anti, Symm 0

$\gamma^\mu\gamma^5$ 4, γ^5 1 — Parity

$$\boxed{\Sigma(p) = \Sigma_m(p^2)\mathbb{1} + \Sigma_\nu(p^2)\not{p}}$$

$$\Sigma_m = m\Sigma_m'$$

$$\frac{1}{\not{p} - m - \Sigma} = \frac{1}{(1-\Sigma_\nu)\not{p} - (1-\Sigma_m')m}$$

$$= \frac{(1-\Sigma_\nu)^{-1}}{\not{p} - \frac{1-\Sigma_m'(p^2)}{1-\Sigma_\nu(p^2)} m}$$

$\longleftarrow$ Rescaling of ψ. $\left(Z = \frac{1}{1-\Sigma_\nu}\right)$

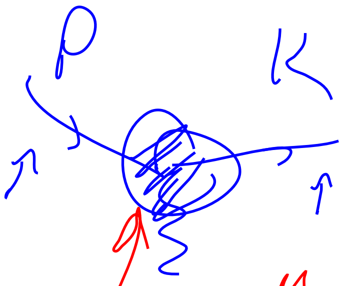
mass rescaling

Physical mass: pole

$$(\not{p} - \xi m)(\not{p} + \xi m) = p^2 - \xi^2 m^2 = 0$$

$$\rightarrow m_{phys} = \frac{1-\Sigma_m'(p^2=m^2)}{1-\Sigma_\nu(p^2=m^2)}$$

$\Gamma^\mu(p,k)$ Matrix w. 4-vector index



Special case p, k on-shell (not generic)

q $J^\mu \to \Gamma^\mu$

q_μ

$\bar u(k)\,\Gamma^\mu\, u(p)$

$$q_\mu \Gamma^\mu = A(p,p')\gamma^\mu + (p^\mu + k^\mu) B(p,p') + \cancel{q^\mu C(p,p')}$$

0 0

spinor   scalar p? p'??

$\bar u(k)\, \not k\, \not p\, u(p)$

$\bar u(k) m$

$(\not p - m)u(p) = 0$ $\not p\, u(p) \to m u(p)$

I can treat A, B as scalar fncs of $(p^2, p'^2, p\cdot p', m)$

Gordon Identity: if $\bar{u}(k)\,\Gamma^\mu\, u(p)$

then

$$\bar{u}(k)\,\gamma^\mu\, u(p) = \bar{u}(k)\left[\frac{p^\mu + k^\mu}{2m} + \frac{i\,\sigma^{\mu\nu}}{2m}\, q_\nu\right] u(p)$$

q

$$q_\nu\, \sigma^{\mu\nu} = \frac{i}{2}\,[\gamma^\mu, \gamma^\nu]\,(p_\nu - k_\nu)$$

$$\frac{i}{2}\left(-2\gamma^\mu \not{p} - 2\not{k}\gamma^\mu + \{\gamma^\mu, \not{p}\} + \{\gamma^\mu, \not{k}\}\right)$$

$\not{p}u \to mu$ $\quad \bar{u}\not{k} \to \bar{u}m$ $\quad 2p^\mu$ $\quad 2k^\mu$

$$\bar{u}\gamma^\mu u \longrightarrow \bar{u}\left(\underbrace{\frac{p^\mu + k^\mu}{2m}}_{\text{Scalar}} + \underbrace{\frac{i\,\sigma^{\mu\nu}}{2m}\, q_\nu}_{\text{spin-B int.}}\right) u$$

$$\sigma^{\mu\nu} A_\mu q_\nu \equiv \sigma^{\mu\nu} F_{\mu\nu}$$

If I compute Γ^{μ} $\longrightarrow$ $C_1 \dfrac{p^\mu + k^\mu}{2m}$ $\leftarrow$

$$\frac{C_2}{C_1} = \frac{g}{2} \text{ if } \neq 1 \qquad + C_2 \frac{\sigma^{\mu\nu} q_\nu}{2m} \leftarrow$$

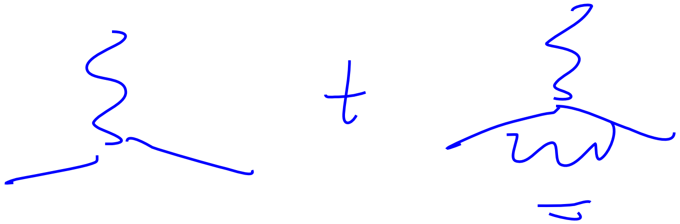
Anom Mag Moment

$$\frac{g}{2} - 1 = \frac{\alpha}{2\pi}$$

Went, but don't have time, to show you

Relation between Σ and Γ

$$\frac{1}{\not p - m} \quad \overset{\Sigma}{\rule{0pt}{0pt}} \longrightarrow \frac{1}{(1-\Sigma_v)\not p - m(1-\Sigma_m)} \simeq \frac{(1-\Sigma_v)^{-1}}{\not p - m_{phys}(p^2\text{-dep})}$$



$$= F_1(q^2,m)\,\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m}\, q_\nu\, F_2(q^2,m)$$

$g-2$ AnomMag

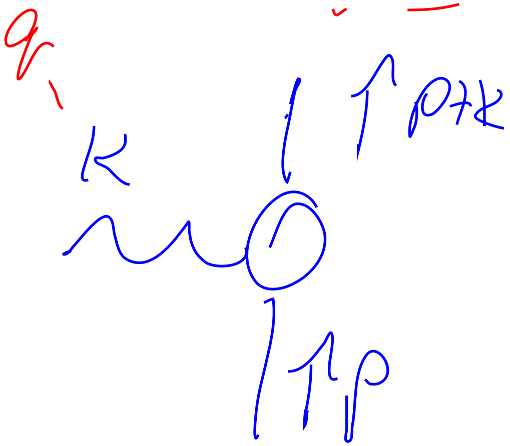
$q^2 \to 0$ On-shell Ext

$\epsilon_\mu \sigma^{\mu\nu} q_\nu \to 0$

Ward: Totalscatt

$$\sim (1-\Sigma_v)^{-1}(1+e^2 F_1) = 1$$

Ward-Takahashi Id.



$$-ik_\mu \Gamma^\mu(p+k,k) = S^{-1}(p+k) - S^{-1}(p) \quad \text{exactly}$$

Lowest order $\Gamma^\mu = \gamma^\mu$

$$-ik_\mu \gamma^\mu = -i(\not p + \not k - \not p) = -i(\not p + \not k - m - (\not p - m))$$

$$= S^{-1}(p+k) - S^{-1}(p)$$

$$S(p) = \frac{i}{\not p - m}$$

$$\frac{1}{S(p)} = -i(\not p - m)$$

Next order

$$-i(\not p' - \not p)e^2\Gamma_1 = -i(1-\Sigma_1)(\not p' - m) + i(1-\Sigma_1)(\not p - m)$$

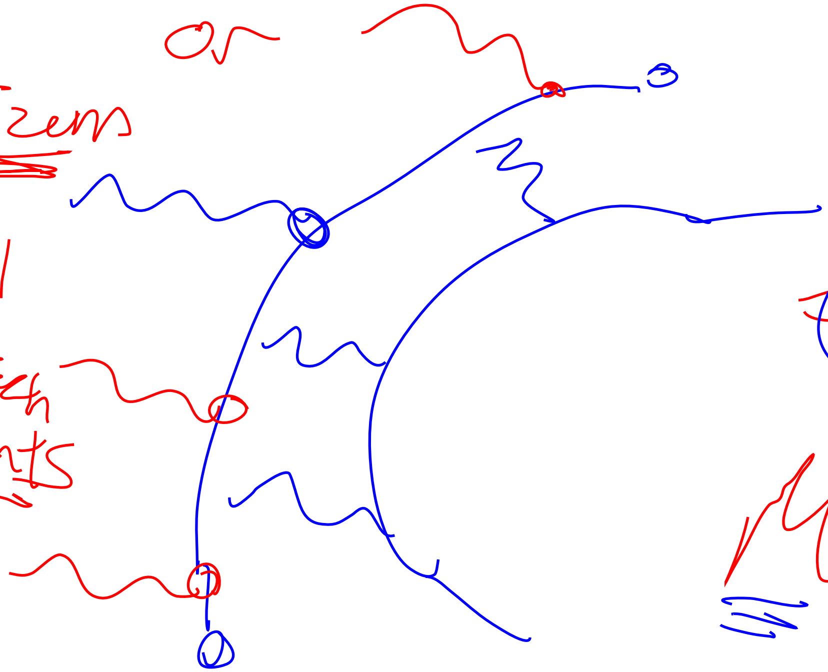
$$e^2\Gamma_1 = 1 - \Sigma_1$$

$$\langle 0 | T[A^\mu(x)\ \bar\psi(y_1) \cdots \psi(y_2)] | 0 \rangle$$

$n\ \bar\psi \qquad n\ \psi$

or

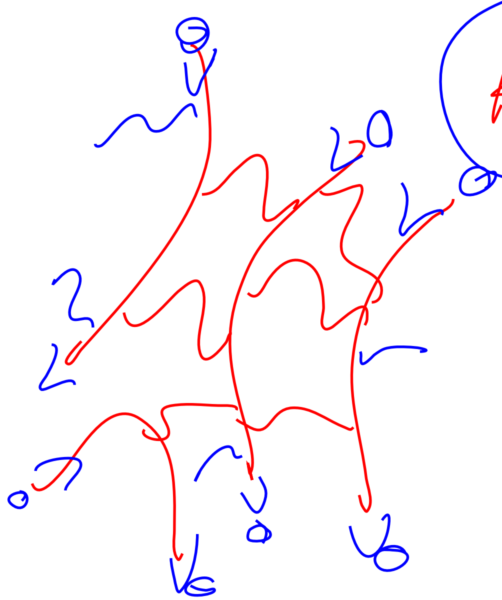
sum all attachments



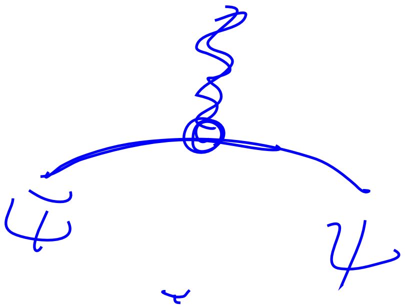
$$\to M_\mu(k; \overset{\bar\psi}{p_1 \ldots p_n}; \overset{\psi}{q_1 \ldots q_n})$$

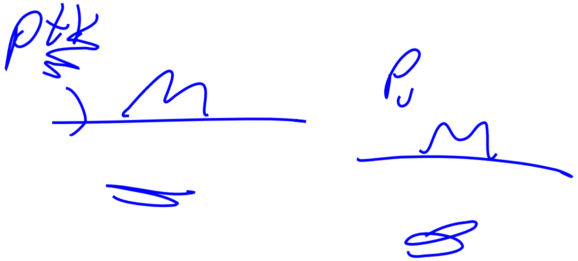
$$= A^\mu [M_\mu]$$

M_0 diagram w/o A^μ attach



$$k_\mu M^\mu(k; p_1 \ldots p_n; q_1 \ldots q_n) = e \sum_{i=1}^{n} \Big[M_0(p_1 \ldots p_n; q_1 \ldots (q_i - k) \ldots q_n) - M_0(p_1 \ldots (p_i + k) \ldots p_n; q_1 \ldots q_n) \Big]$$





Proof

$$\langle \psi\psi\psi(y_1 \dots y_n)\, \bar\psi\bar\psi(y_{n+1} \dots y_{2n}) \rangle$$

$$= \int \mathcal{D}\bar\psi\psi A_\mu \, e^{iS} \, \psi(y_1) \dots \psi(y_n)\, \bar\psi(y_{n+1}) \dots \bar\psi(y_{2n})$$

$$\psi'(x) = e^{i\alpha(x)} \psi(x)$$

$$\bar\psi'(x) = e^{-i\alpha(x)} \bar\psi(x)$$

$$\text{Lin-in } \alpha \text{ change} = 0 = \int \mathcal{D}\bar\psi\psi A \, \frac{\delta}{\delta\alpha(x)} \left[\psi \dots \psi \, \bar\psi \dots \bar\psi \, e^{iS(\bar\psi(\dots)\psi)} \right]$$

$$\frac{\delta}{\delta\alpha(z)}\psi(y_m) = i\delta^4(z-y_m)\,\psi \qquad \left(e^{i\alpha}\right)$$

$$\frac{\delta}{\delta\alpha(z)}\bar\psi(y_{n+m}) = -i\delta^4(z-y_{n+m})\,\bar\psi \left(e^{-i\alpha}\right)$$

$$\frac{\delta}{\delta\alpha(z)} e^{iS} = e^{iS}\frac{\delta}{\delta\alpha(z)}\int d^4x\ \bar\psi e^{-i\alpha}(i\partial\!\!\!/-m)\psi e^{i\alpha} \qquad J^\mu$$

$$\left(\partial_\mu\right) \qquad (i\partial_\mu\alpha)\,\bar\psi\gamma^\mu\psi$$

$$= -i\,\partial_\mu J^\mu e^{iS}$$

$$0 = \langle \partial_\mu J^\mu(z)\,\psi\cdots\bar\psi\cdots\rangle - \sum\langle\psi\cdots \delta^4(y-z)\,\psi\,\bar\psi\cdots\bar\psi\ \delta(y_{n+m}-z)\rangle$$

$$\frac{\delta}{\delta\alpha(z)} \to \int d^4z\, e^{-ik_\mu z^\mu} \quad \text{now what?}$$

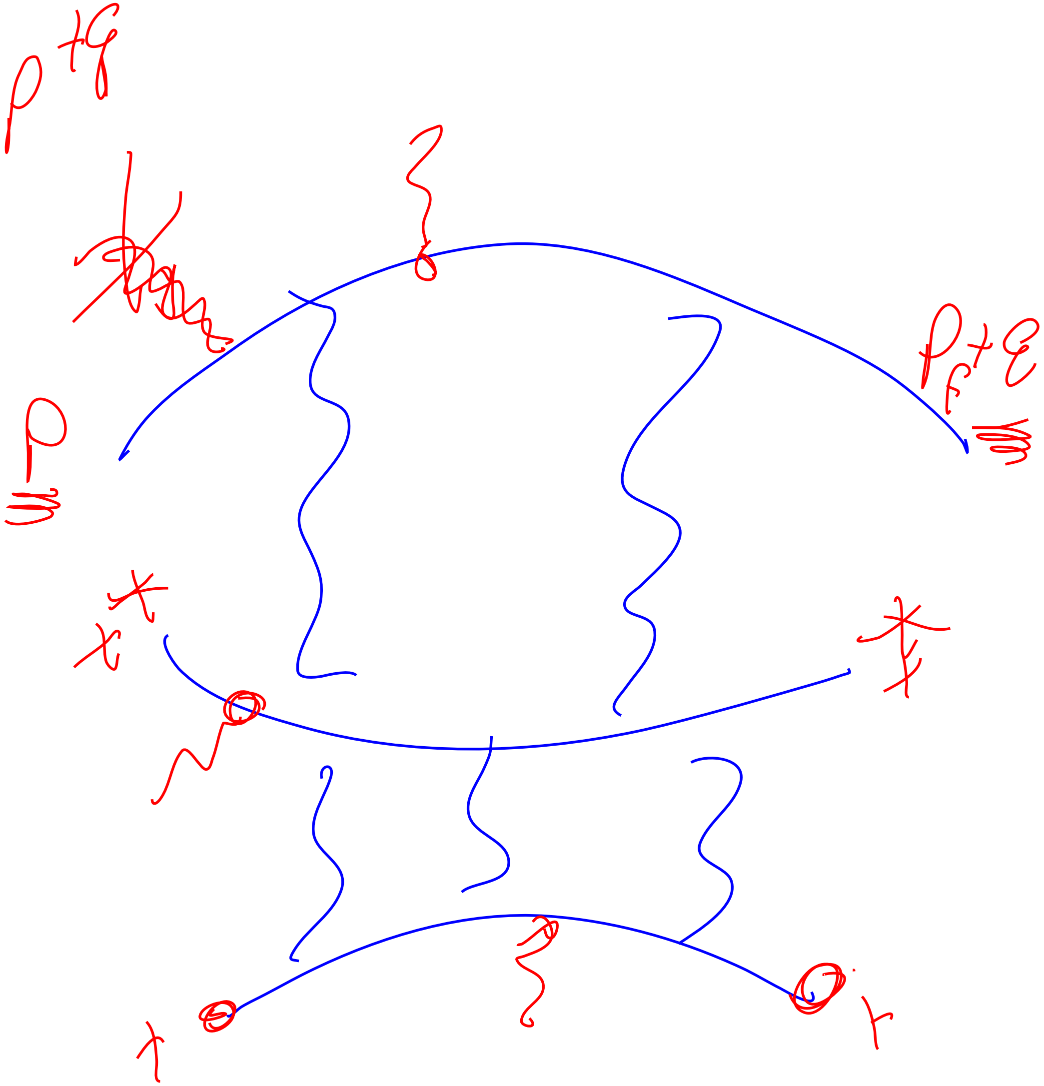
$$\partial_\mu J^\mu(z) \to k_\mu J^\mu(k) \to k_\mu M^\mu$$

$$\int d^4z\, e^{-ik_\mu z^\mu}\, \delta(z - y_m) \to \text{shift in } p_m \text{ momentum}$$

Symmetry! $\psi \to e^{i\alpha}\psi$ is gauge symm.

$\bar\psi \to e^{-i\alpha}\bar\psi$ $\quad \& \quad M$ $\quad p+q \quad p$

one $\bar\psi$ and one ψ $\to$ $k_\mu \cdot$ (p, $p+q$) $= M - M$

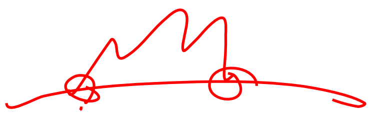


$p+q$

$$-\begin{pmatrix} p+q \text{ vert} \\ \text{no } \gamma \end{pmatrix} - \begin{pmatrix} p\text{-vert} \\ \text{no } \gamma \end{pmatrix}$$

$$= q_\mu \left(\Gamma^\mu \text{ with } \gamma \right)$$

Diverg. in Γ^μ 

equal Diverg. in 

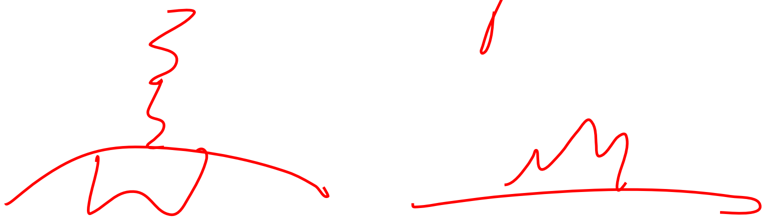
Nontrivial Relation

$$\mathcal{L} = \bar{\psi}_0 \left(i\partial^\mu (\partial_\mu - ie_0 A_\mu) - m_0 \right) \psi_0$$

$$\bar{\psi}_m, \ \psi_m, \ e_m, \ m_m$$

$\bar{\psi}, \psi$ create part. with 1-norm.

$e_m, m_m \to$ meas. chg. mass



Next Semester

Finish Ren.-story

Introduce QCD