Organizational

gmail account for homework

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Wednesday: 2 help sessions

10:00 AM

2:00 PM

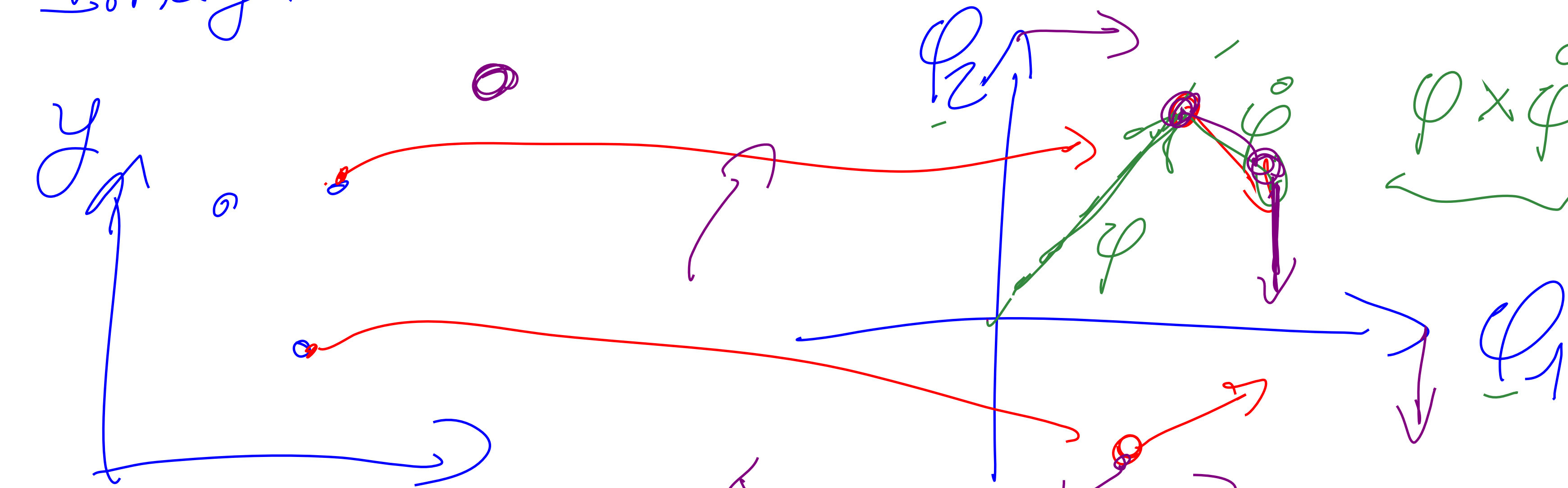
Send email to $\boxed{\text{qft1tuda@gmail.com}}$ to get on distribution list for further information.

First homework is due in 1 week. "Friday"

when? 23:59. Friday = before Saturday.

Last time: Classical mechanics

Imagine 2 scalar fields $\varphi_1, \varphi_2, \dots \varphi_N$ $\qquad \frac{N(N-1)}{2}$



$$\underbrace{\varphi \times \dot\varphi}_{} = \varphi_1 \dot\varphi_2 - \varphi_2 \dot\varphi_1$$

$$\begin{bmatrix} \varphi \times \dot\varphi(x) \\ \varphi \times \nabla\varphi(x) \end{bmatrix} = J^\mu_A(x)$$

$$Q_A = \int J^0_A \, d^3x$$

conserved

$$\partial_\mu J^\mu_A = 0$$

J^0 in a box changes only if $\vec{J}$ is "coming in" through boundaries

$N=2: A=1$ $\quad$ 12

$N=3: A=1,2,3$ $\quad$ 12 13 23

$N=4: A=1,\dots,6$ $\quad$ 12, 13, 23, 14, 24, 34

What about Spacetime symm?

QM: • Translation → $\vec{P}$

• Time-inv → E

conserved quantities.

Lorentz Transform.

• Rotations → $\vec{J}$

• ~~Lorentz Trans~~

Boosts: mix time & space by changing velocity

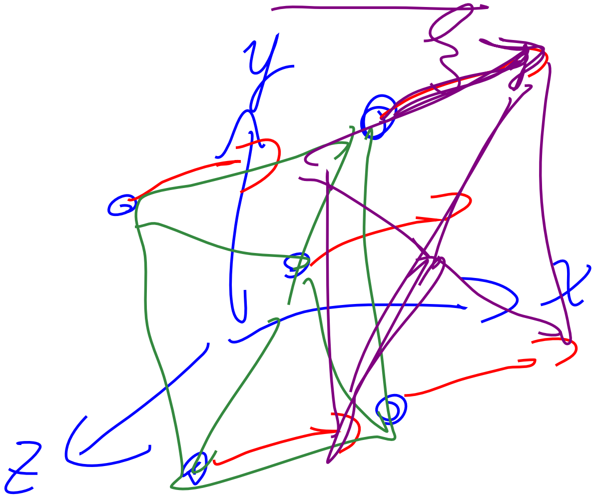
J^{μ} for each

which thing

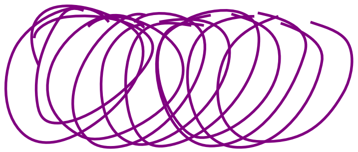
$J^{\nu\mu}$ comp's.

4-translations

$$x^\mu \longrightarrow x^\mu + \xi^\mu$$



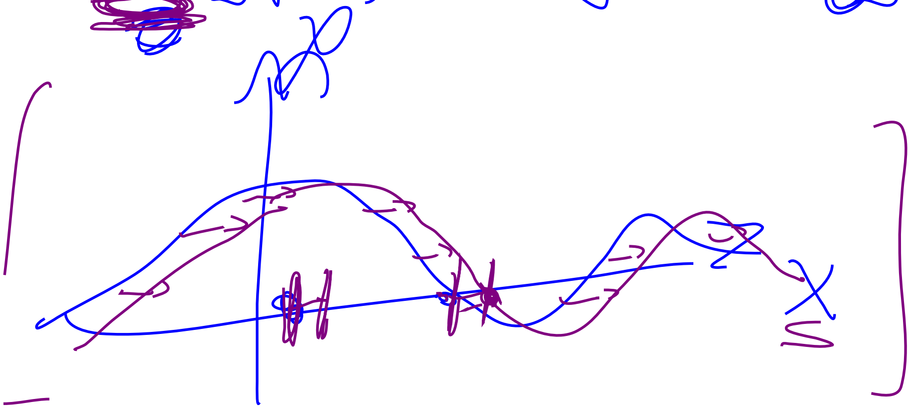
If ξ^μ is big, do N shifts of length $\frac{\xi^\mu}{N}$



Sufficient to consider ξ^μ infinitesimal.

Big shift = many little shifts.

$$\mathcal{L}[\varphi(x), \partial_\mu\varphi(x)] = \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}|\nabla\varphi|^2 - V(\varphi)$$



$$\mathcal{L}_{before}(x) = \mathcal{L}_{after}(x+\xi) \neq \mathcal{L}_{after}(x)$$

$$\varphi(x) \to \boxed{\varphi(x+\xi) = \varphi(x) + \xi^\mu \partial_\mu \varphi(x)} + \mathcal{O}(\xi^2)$$

$$\mathcal{L}(\varphi(x)) \to \mathcal{L}(\varphi(x+\xi)) = \mathcal{L}(\varphi(x)) + \boxed{\xi^\mu \partial_\mu \mathcal{L}}$$

If $\mathcal{L}$ changes, I must write it as $\boxed{\partial_\nu \mathcal{J}^\nu(x)}$

ξ^μ is a 4-comp. family of how to chg my fields

ξ^0 : time trans.

ξ^1 : x-trans.

ξ^2 : y-trans

ξ^3 : z-transl.

like the A-index before

which transform?

Other Lorentz indices are Lorentz...

$$\xi^\mu g^\nu_\mu \partial_\nu \mathcal{L}$$

ν - 4-vect comp

μ - which symm.

$$\xi^\mu \partial_\mu \mathcal{L} = \xi^\mu \partial_\nu \mathcal{J}^\nu = \xi^\mu \partial_\nu \left[g^\nu_\mu \mathcal{L} \right]$$

$x^\mu \to x^\mu + \xi^\mu$ does change $\mathcal{L}$! but only by tot deriv

$$\partial_\nu J^\nu \quad \text{with} \quad J^\nu \equiv [g^\nu_\mu \mathcal{L}]\, \xi^\mu$$

4-vect

$$\underset{\text{"A"}}{J_\mu^{\ \nu}} \equiv T_\mu^{\ \nu} = \underset{\text{Nöther Thy}}{\underline{\pi^\nu_a}\, \underline{\partial_\mu \varphi_a}} - g^\nu_\mu \mathcal{L}$$

many fields: $\underline{\pi^\nu_a} \cdot \underline{\delta \varphi_a}$

$$g^\mu_\nu = \delta^\mu_\nu$$

Example: $\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi)$

$$T_\mu^{\ \nu} = \partial^\nu \varphi \partial_\mu \varphi - g^\nu_\mu \left[\tfrac{1}{2} \partial_\alpha \varphi \partial^\alpha \varphi - V(\varphi)\right]$$

$$T^{\mu\nu} = \partial^\mu \varphi \partial^\nu \varphi - g^{\mu\nu} [\quad " \quad]$$

<u>symmetric</u>

$T^{\nu}{}_{\mu}$

ν ← dir. I change space

μ ← comp. of current.

Stress Mom. Energy

$T^{1}{}_{0}$ chg. assoc. w. x-trans.

Charges: $\mu = 0$

$\int d^3x \, T^{1}{}_{0} = p^{1}$

$\int d^3x \, T^{\mu}{}_{0} = p^{\mu} = \begin{bmatrix} E \\ \vec{p} \end{bmatrix}$

E flux

endens.

T^{00} T^{0x} T^{0y} T^{0z}

T^{x0} T^{xx}

T^{y0} T^{yy}

T^{z0} T^{zz}

Mom dens.

Stress

$T^{\boxed{0}}{}_{x}$ ← timetrans - Energy

x ← x comp. current

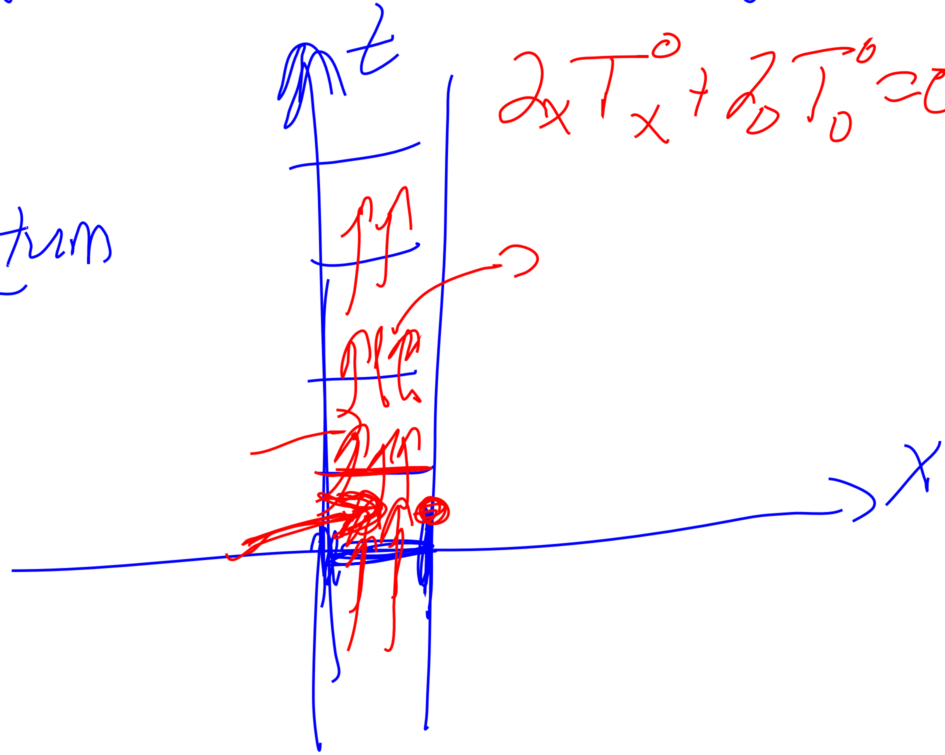
x-dir. flux of energy

$T^{y}{}_{x}$ x-dir. flux of y-momentum

energy in cell $T^{0}{}_{0}$

energy entering cell $T^{0}{}_{x}$

$\partial_x T^{0}{}_{x} + \partial_0 T^{0}{}_{0} = 0$



Lorentz Trans. // Rotation $\underbrace{X_i \to R_{ij} X_j}$ lin. transform.

preserving length $\vec{x}^{\,2} = \delta_{ij} x_i x_j$ stay same

Length →
Proper
Length

$(x_0)^2 - (\vec{x})^2$

$g_{\mu\nu} x^\mu x^\nu$

$$\begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$$x_i \delta_{ij} x_j \to R_{ik} x_k \delta_{ij} R_{jl} x_l$$

$$= x_k R^T_{ki} \delta_{ij} R_{jl} x_l$$

$$\underbrace{x_i \delta_{ij} x_j} = \underbrace{x_i R^T_{ik} \delta_{kl} R_{lj} x_j}$$

$$\underbrace{\mathbb{1} = R^T \mathbb{1} R}$$

$R \in O(3)$
orthogonal
3x3 matrices

$$R_{ij} = \delta_{ij} + \Theta_{ij}$$

$$\Theta_{ij} + \Theta_{ji} = 0$$

Θ antisymm

$$\begin{bmatrix} 1 & \theta \\ -\theta & 1 \end{bmatrix} \quad \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$x^\mu g_{\mu\nu} x^\nu$ unchanged when $x^\mu \to \Lambda^\mu{}_\nu x^\nu$

$$x^\mu g_{\mu\nu} x^\nu = \Lambda^\mu{}_\alpha x^\alpha g_{\mu\nu} \Lambda^\nu{}_\beta x^\beta$$

$$= x^\alpha \Lambda^\mu{}_\alpha g_{\mu\nu} \Lambda^\nu{}_\beta x^\beta$$

$$x^\mu g_{\mu\nu} x^\nu = x^\mu \Lambda^\alpha{}_\mu g_{\alpha\beta} \Lambda^\beta{}_\nu x^\nu$$

if these are same, $\Lambda^\mu{}_\nu$ is valid Lorentz trans.

$\Lambda^\mu{}_\nu = g^\mu{}_\nu + \omega^\mu{}_\nu$ infinitesimal

$$g_{\mu\nu} = (g^\alpha{}_\mu + \omega^\alpha{}_\mu) g_{\alpha\beta} (g^\beta{}_\nu + \omega^\beta{}_\nu)$$ to lin. order in ω.

$$g_{\mu\nu} = g_{\mu\nu} + \omega_{\nu\mu} + \omega_{\mu\nu}$$

$$\boxed{\omega_{\nu\mu} + \omega_{\mu\nu} = 0}$$

$$\underline{\underline{\omega_{\mu\nu}}} = \begin{bmatrix} 0 & b_x & b_y & b_z \\ -b_x & 0 & r_z & -r_y \\ -b_y & -r_z & 0 & r_x \\ -b_z & r_y & -r_x & 0 \end{bmatrix}$$

$r_x\, r_y\, r_z$: angles to rot. about x, y, z dir.

$b_x\, b_y\, b_z$: velocities to boost in x, y, z.

But $x^\mu \to \Lambda^\mu{}_\nu x^\nu = (g^\mu{}_\nu + \omega^\mu{}_\nu) x^\nu = x^\mu + \underline{\omega^\mu{}_\nu x^\nu}$

$$\omega^\mu{}_\nu = g^{\mu\alpha}\omega_{\alpha\nu} = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

same

flipped

$$\begin{bmatrix} 0 & b_x & b_y & b_z \\ b_x & 0 & -r_z & r_y \\ b_y & r_z & 0 & -r_x \\ b_z & -r_y & r_x & 0 \end{bmatrix}$$

If $\omega_x \neq 0$

$$\omega^{\mu}{}_{\nu} = \begin{bmatrix} 0 & v_x & 0 & 0 \\ v_x & & & \\ 0 & & 0 & \\ 0 & & & \end{bmatrix} \qquad \Lambda^{\mu}{}_{\nu} = \begin{bmatrix} 1 & v_x & & \\ v_x & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

or $\exp \omega^{\mu}{}_{\nu} = \begin{bmatrix} \cosh v_x & \sinh v_x & 0 & 0 \\ \sinh & \cosh & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$ $\quad \cosh = \gamma$, $\sinh = \gamma v$

$$\omega^{\mu}{}_{\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\theta & 0 \\ 0 & \theta & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & & & \\ & 1 & -\theta & 0 \\ & \theta & 1 & 0 \\ & 0 & 0 & 1 \end{bmatrix} \text{ or } = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Symmetries:

$$x^\mu \to x^\mu + \omega^\mu{}_\nu x^\nu$$

2-index set of symm. ${}^\mu{}_\nu \to A$ in Nöther Theorem

$$\varphi(x^\mu) \to \varphi(x^\mu + \omega^\mu{}_\nu x^\nu) \quad \text{``like'' } \xi^\mu$$

$$= \varphi(x^\mu) + \left[\omega^\mu{}_\nu x^\nu\right] \partial_\mu \varphi$$

"like" ξ^μ

$$= \varphi(x^\mu) + \omega^\mu{}_\nu \left[g^\mu{}_\alpha g^\beta{}_\nu x^\alpha \partial_\beta \varphi \right]$$

$$\mathcal{L}(\varphi) \to \mathcal{L}(\varphi) + \omega^\mu{}_\nu x^\nu \partial_\alpha \left[g^\alpha{}_\mu \mathcal{L} \right]$$

$$= \mathcal{L}(\varphi) + \omega^\mu{}_\nu \partial_\alpha \left[x^\nu g^\alpha{}_\mu \mathcal{L} \right]$$

Differ by $\omega^\mu{}_\nu (\partial_\alpha x^\nu) g^\alpha{}_\mu \mathcal{L}$

$$g^\nu_\alpha = \omega^\alpha{}_\alpha \mathcal{L} = 0$$

$$\partial_\alpha J^\mu{}_\nu{}^\alpha$$

$$J^{\mu\nu\alpha} = (\delta\varphi)^{\mu\nu} \Pi^\alpha + J^\alpha$$

$$J^{\mu\nu\alpha} = x^\nu \partial^\mu \varphi \Pi^\alpha - x^\nu g^{\mu\alpha} \mathcal{L}$$

$J^{\mu\nu\alpha} \to M^{\mu\nu\alpha}$ antisymm μ,ν

$$M^{\mu\nu\alpha} = (x^\nu \partial^\mu \varphi - x^\mu \partial^\nu \varphi)\pi^\alpha - (x^\nu g^{\mu\alpha} - x^\mu g^{\nu\alpha})\mathcal{L}$$

$\partial_\alpha M^{\mu\nu\alpha} = 0$ conserved current

Antisymm tensor-worth of conserved currents

$\int d^3x\, M^{\mu\nu 0} = M^{\mu\nu}$ Charge - constant in time $\partial_t M^{\mu\nu} = 0$

$$M^{ij} = \int x^j \underbrace{\partial^i \varphi \pi^0}_{T^{i0}} - x^i \underbrace{\partial^j \varphi \pi^0}_{T^{j0}} = \int_x \underline{x^j p^i - x^i p^j} = J^{ij}$$

Ang momentum

$$M^{0i} = \int d^3x\, (x^0 T^{0i} - x^i T^{00})$$

[time x Momentum - Energy · CM coord.]

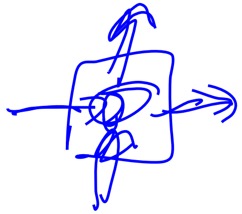
$\frac{d}{dt}(\text{CM coord}) = \text{Mom}/\text{En.}$

Class Mech.

Symm $\rightarrow$ Cons. laws

Trans $\rightarrow$ $T^{\mu\nu}$ $\qquad \int T^{\mu 0} d^3x = p^\mu$ 4-momentum

$$\boxed{\partial_\mu T^{\mu\nu} = 0} \longrightarrow$$



Lorentz: $\omega_{\mu\nu} = -\omega_{\nu\mu}$ $\qquad \partial_\alpha M^{\mu\nu\alpha} = 0$

$\mu\nu = ij \rightarrow$ Ang Mom

$0i \rightarrow$ Something