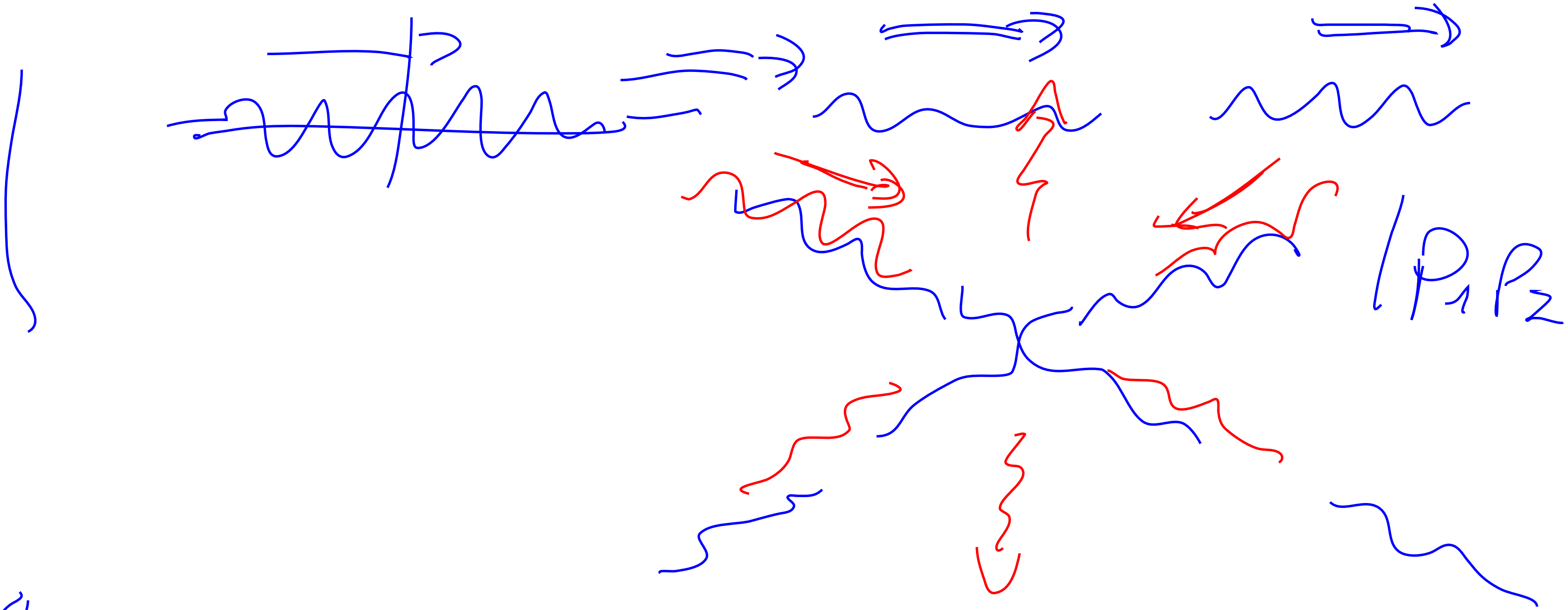
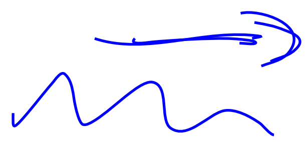
Goals & where we Are

Free thy: "particles" $|p\rangle$ $E_p^2 = \vec{p}^{\,2} + m^2$





$|p_1 p_2\rangle$ stays like that.

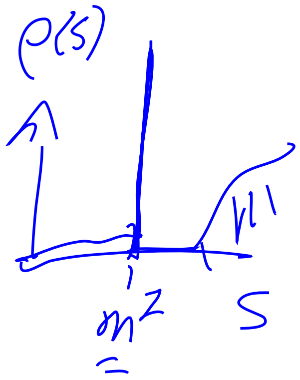
"Interactions"
Nonlinearities

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{24}\varphi^4$$

$$\rightarrow \quad \partial_\mu\partial^\mu\varphi = -m^2\varphi - \frac{\lambda}{6}\varphi^3$$

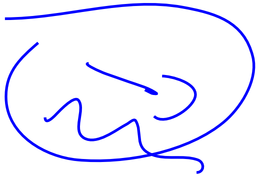
How do I figure this out?

Concept		Technical
Are there particles?	→	2-point func. $\langle\varphi\varphi\rangle$ Spectral func $\rho(s)$ $\varphi\|0\rangle$ makes part state
What happens when part's meet? Scattering	→	What Q should I ask QFT to determine
How to define & describe scattering	→	Scatt, Decay,
Unstable part's? How do they decay		And how do I actually calculate?



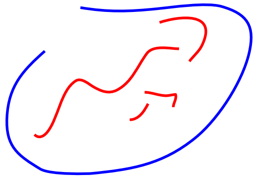
1) Are there part, how to build 1-part states

2) How build multi-part ("in" & "out") states



In before

) possible int.



out after

S-matrix $\langle f \mid i \rangle$ ← in state → formulate as QFT question.

Out state

Answer: $\int d^4x_1\, d^4x_2 \cdots d^4y_1\, d^4y_2 \cdots e^{+ix_0p_1 \cdots}\, e^{-iyk_1 \cdots} \underbrace{\langle 0|T(\varphi(x_1)\varphi(x_2)\cdots)|0\rangle}$

Tools to compute

Assume we can figure out how to compute

$$\langle 0| \, \varphi(x_1)\, \varphi(x_2)\, \varphi(x_3)\, \varphi(x_4) \dots |0\rangle$$

$$= G(x_1, x_2, x_3, x_4, \dots).$$

What do these teach us?

Physics Q $\longrightarrow$ Desired Corr. Function

Then how do you compute $G(x_1, \dots)$

Path Integral ....

Are there only scalars?

spin 0

spin 1/2

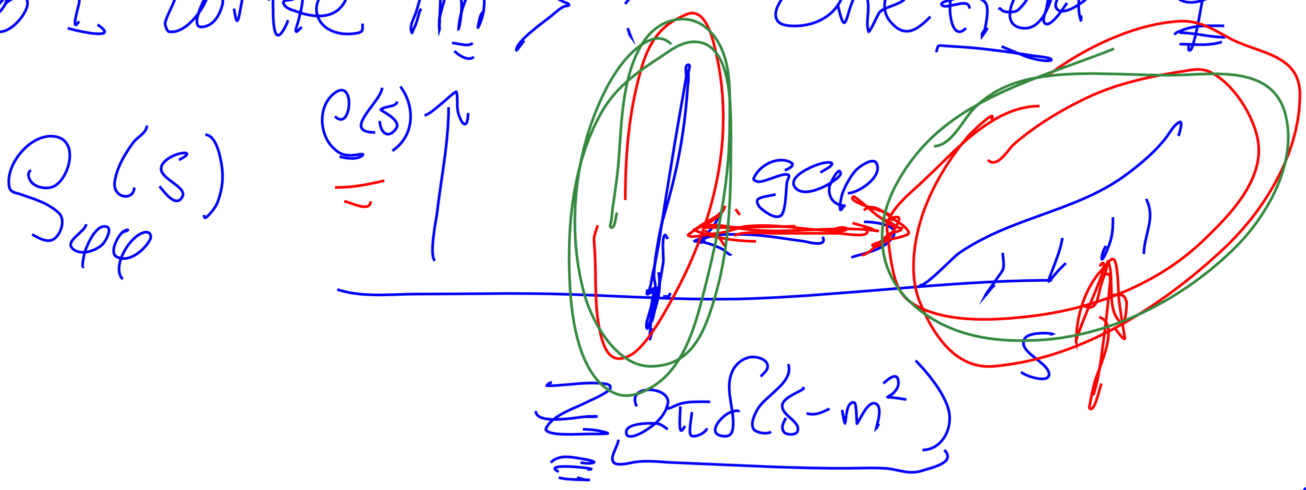
spin 1

Start w m part flying at each other

Amplitude → end w. n part. flying away??

$\langle n\text{-final} | U(-\infty,\infty) | m\text{-part} \rangle = S_{m,n}$ S-matrix

How do I write $|m\rangle$? One field φ

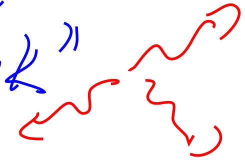


$\varphi(x)|0\rangle$

prod. $\sqrt{Z} \times |1\text{-part}\rangle$

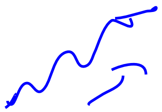
at pt. x

plus "junk"

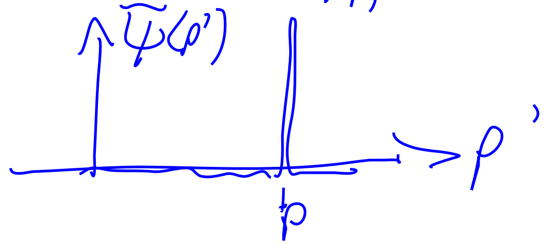


I want

wave packets $\psi_1(\vec{x}) \dots \psi_m(\vec{x})$

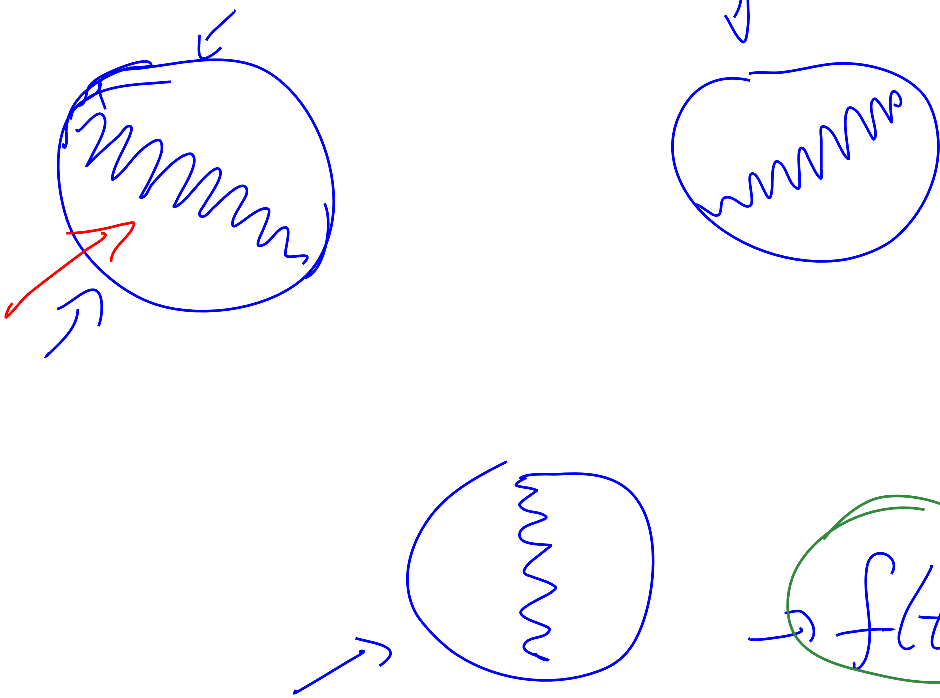


$\psi_1(\vec{x}) \longrightarrow \tilde{\psi}_1(\vec{p}')$



n wave packets each sharply (not perfectly) peaked in $\vec{p}$.

Also space-separated



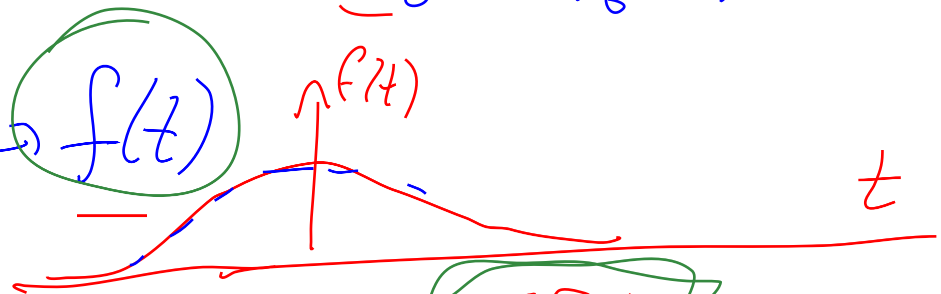
Expect energies

p_1, p_2, p_3 central values

$$E_1 = \sqrt{p_1^2 + m^2}$$

$$E_2 = \sqrt{p_2^2 + m^2}$$

$$E_3 = \sqrt{p_3^2 + m^2}$$



$$\int f(t)\,dt = 1$$

$$\psi(x_1^\mu) = \psi(\vec{x}_1)\, f(t)\, \boxed{e^{-iE_1 t}}$$

avoids wrong energies

Let's build 1 particle

$$|\psi\rangle = \frac{1}{\sqrt{Z}} \int d^4x \, \psi(x^\mu)\, \hat{\varphi}(x^\mu)|0\rangle = \int \frac{d^4p}{(2\pi)^4} \psi(-p)\, \varphi(p)|0\rangle$$

$$\langle \psi|\psi\rangle = \frac{1}{Z} \int d^4x\, d^4y \, \langle 0| \psi^*(y)\, \hat{\varphi}(y^\mu)\, \psi(x)\, \hat{\varphi}(x^\mu)\, |0\rangle$$

$$= \frac{1}{Z} \int d^4x\, d^4y \, \psi^*(y)\, \psi(x) \, \langle 0| \varphi(y^\mu)\, \varphi(x^\mu) |0\rangle$$

$d^3x \quad d^3y$

$$\int ds\, \rho(s) \int \frac{d^4k}{(2\pi)^4}\, 2\pi\, \delta(k^2 - s)\, \Theta(k^0)\, e^{-ik_\mu(y^\mu - x^\mu)}$$

2π

$$e^{i\vec{k}\cdot\vec{y}} \quad e^{-i\vec{k}\cdot\vec{x}} \quad e^{ik^0(x^0-y^0)}$$

$$\int d^3y\, e^{i\vec{k}\cdot\vec{y}}\, \psi^*(y)$$

$$= \psi^*(\vec{k})$$

$$\langle\psi|\psi\rangle = \frac{1}{Z}\int dx^0 dy^0 \int \frac{d^3\vec{k}\, dk^0}{(2\pi)^3} \int ds\, \rho(s)\, 2\pi\,\delta(s-k^2)\, \boxed{\psi^*(\vec{k})\,\psi(\vec{k})}$$

$$s - k_0^2 + \vec{p}^{\,2} \qquad f^*(y^0)\, f(x^0)$$

$$\Theta(k^0)\, e^{-i(E)(x^0-y^0)}$$

$$e^{+ik_0(y^0-x^0)}$$

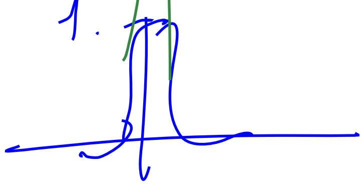
$$\int \frac{d^3\vec{k}}{(2\pi)^3}\, \psi^*\psi(\vec{k}) = 2E_k$$

$$k_0 = E \qquad E^2 - \vec{p}^{\,2} = s$$

$$\langle\psi|\psi\rangle = \int \frac{ds}{2\pi}\, \frac{\rho(s)}{Z} \int \frac{dk^0}{2\pi}\, 2\pi\,\delta(k_0^2 - \vec{p}^{\,2} - s)\, \Theta(k^0)$$

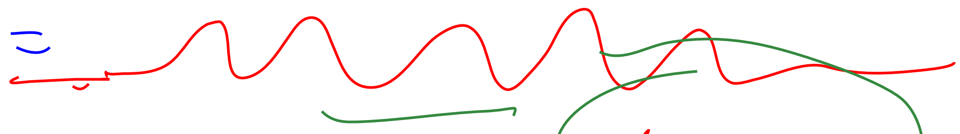
$$\int dx^0 dy^0\, e^{ix^0(E-k^0)}\, e^{iy^0(k^0-E)}\, f(x^0) f^*(y^0)$$

1.



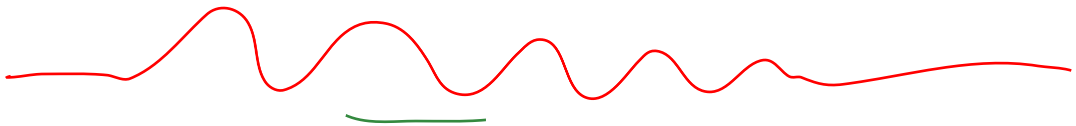
Fourier comp. of f: $\tilde{f}(E-k^0)\, \tilde{f}^*(E-k^0)$

$$\int d^3x \, \psi(\vec{x}) \, \varphi(\vec{x}) \, |0\rangle =$$



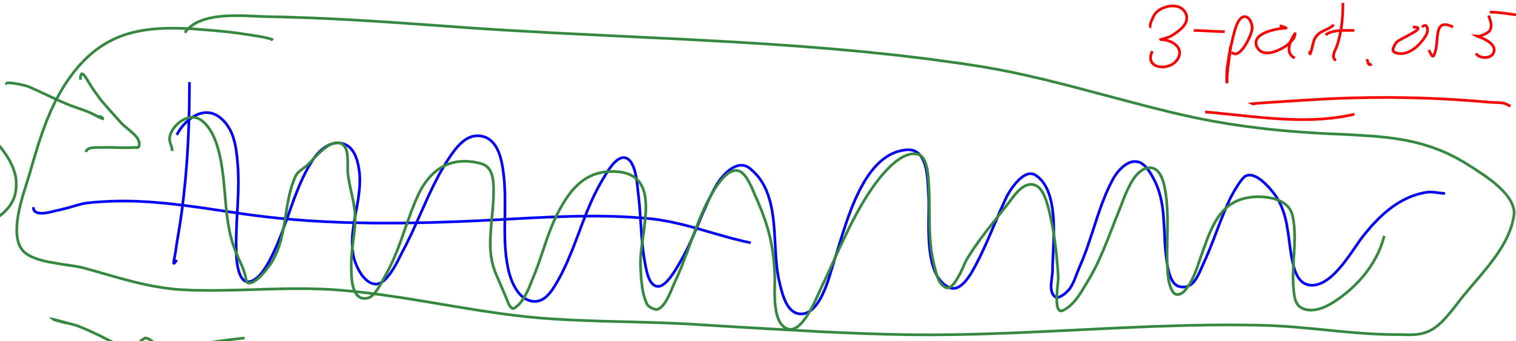
1-part.

$$= |p_1\rangle + |p_3\rangle + |p_5\rangle + \dots$$



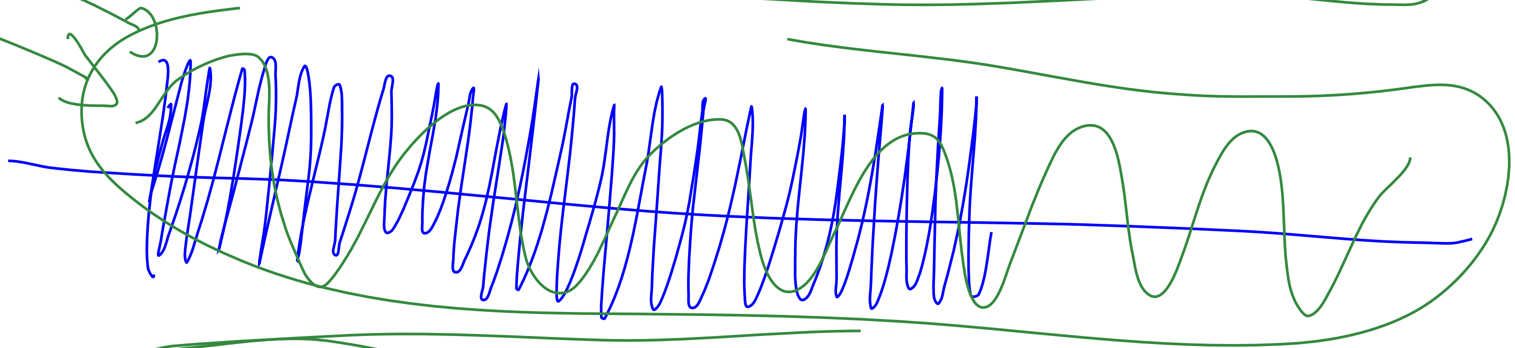
$$E = \sqrt{p^2+m^2}$$

$$\langle p_1(t) \, | \, p_1(0)\rangle$$

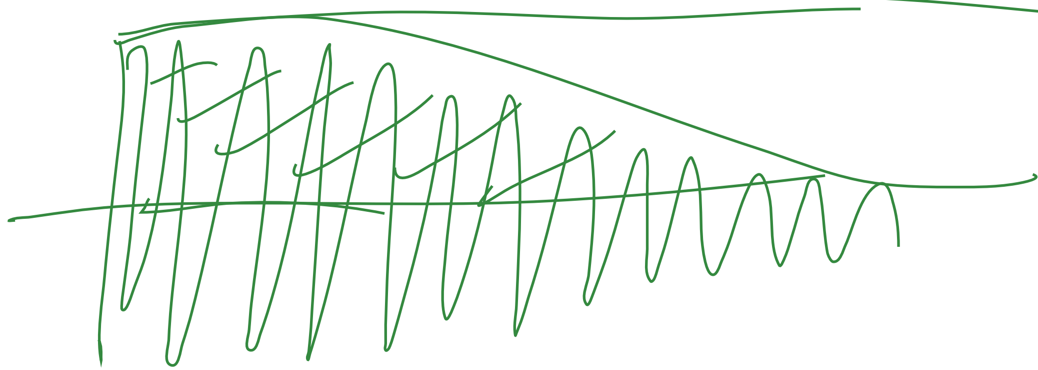


3-part. or 5-part.

$$\langle p_3(t) \, | \, p_3(0)\rangle$$



$$\int e^{+iEt} f(t) \, dt$$



Procedure to make 1 part:

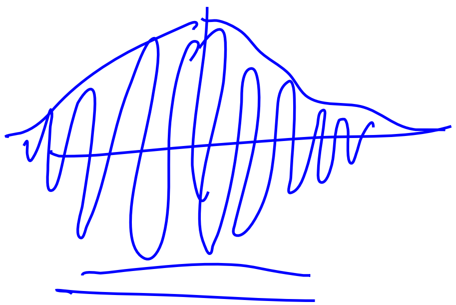
$\varphi(x)|0\rangle$

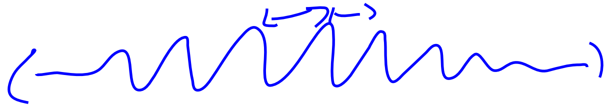
mix of stuff at pt x

→ many $\vec{p}$

→ 1, 3, 5 ... part's. Yuck

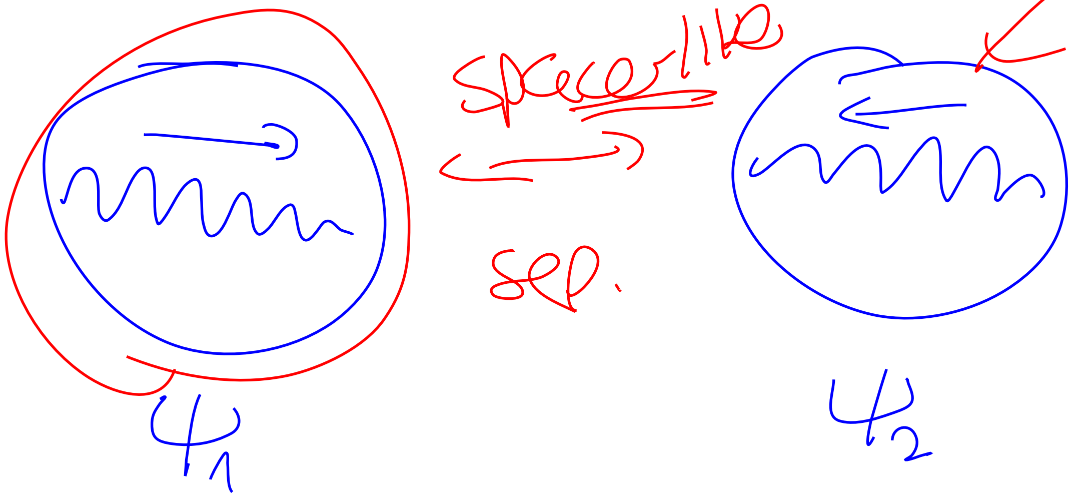
$\int dt\, e^{iEt} f(t) \int d^3x\; \psi(x)\; \varphi(x)|0\rangle$ → almost definite mom. local in space





~~mix w 3 + part.~~

Multiple particles?



All $(x_1-x_2)^2<0$ space-sep.

$$\frac{1}{(\sqrt{2})^2}\int d^4x_1\, d^4x_2\; \psi_1(x_1)\,\psi_2(x_2)$$

$$\times\ \varphi(x_1)\,\varphi(x_2)|0\rangle$$

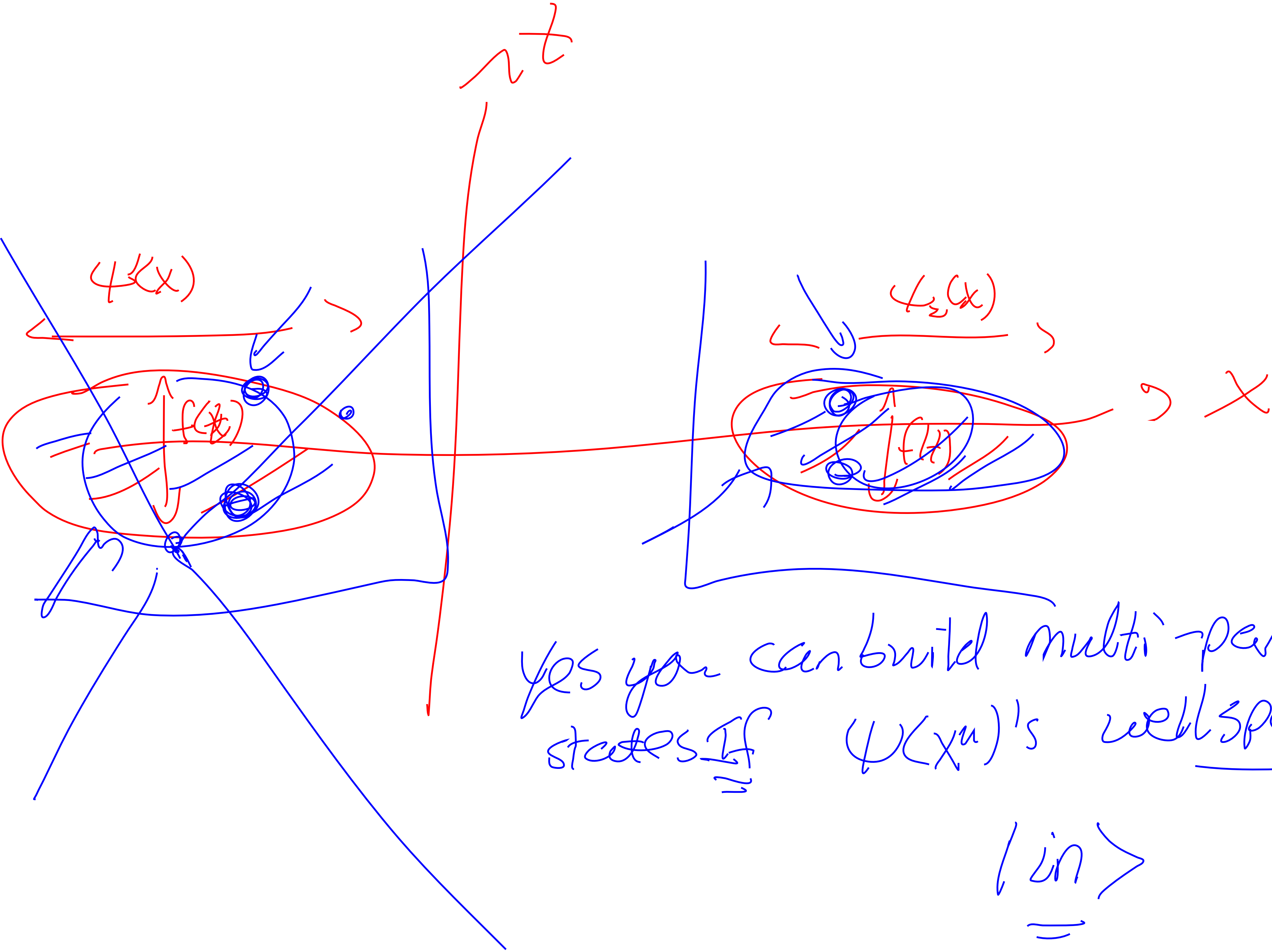
$$= |\psi_1\psi_2\rangle$$

Are you sure? $\varphi(x_2)\varphi(x_1)|0\rangle$??

Does making part 1 interfere w. part 2?

$\varphi(x)\varphi(y)=\varphi(y)\varphi(x)$ if $(x-y)^2<0$ spacelike sep.

Locality! It's OK!



Yes you can build multi-part states if $\psi(x^\mu)$'s well space-sep.

$|in\rangle$

in-state – $\vec{x}, \vec{p}$ of packets chosen so never had chance to cross.

out state – $\vec{x}, \vec{p}$ such that they're done crossing (not in future)

S-matrix

start at t_f — complete at t_i

$$\langle \Psi_{out} | \Psi_{in} \rangle$$

or

$$\langle \Psi_{out} | \, \mathcal{U}(t_f, t_i) \, | \Psi_{in} \rangle$$

$$\prod \int d^4x_{1i}\, d^4x_{2i} \cdots \times \prod \int d^4y_{1out}\, d^4y_{2out} \cdots \psi_{1in}(x_{1in})\, \psi_{2in}(x_{2in}) \cdots$$

$\psi_{1i}\, \psi_{2i} \cdots$

$$\times\ \psi^*_{1out}(y_{1out})\, \psi^*_{2out}(y_{2out}) \cdots \langle 0 | \varphi(y_{1f})\, \varphi(y_{2f}) \cdots \varphi(x_{1i})\, \varphi(x_{2i}) | 0 \rangle$$

bra — out on left

ket — in on right

Question you want to ask.

INFO provided by Quantum Field Thy

m+n-point correl. funz's "know"

whether m-part. scatter into n-part.

$$\langle 0 | T(\varphi(y_1)\varphi(y_2) \cdots \varphi(x_1)\varphi(x_2)\cdots) | 0 \rangle$$

T-ordering symbol - shorthand for saying what operator order op's are in.

$$T(\varphi(t_1)\,\varphi(t_2)) = \varphi(t_1)\varphi(t_2)\,\Theta(t_1-t_2) + \varphi(t_2)\varphi(t_1)\,\Theta(t_2-t_1)$$

$$T(a(t_1)\,b(t_2)\,c(t_3)) = abc\,\Theta(t_1-t_2)\Theta(t_2-t_3) + bac\,\Theta(t_2-t_1)\Theta(t_1-t_3) + cab \; \vdots$$

6 terms.

for any $(t_1 t_2 t_3)$ exactly 1 term is present

$$\int d^4x_1\, d^4x_2 \quad \varphi_1(x_1)\, \varphi_2(x_2) \cdots$$

$$\langle 0 | T \Big(\varphi(y \cdots) \quad \varphi(x_1)\varphi(x_2) \Big) | 0 \rangle$$

spacelike - so they commute