Last time, explained better

→ $\hat{\varphi}(x)$ creates particle — or multiparticle state — at point x.

$\int d^3x\, \psi(x)\, \hat{\varphi}(x)$ — particle w. momentum $\vec{p}$, energy $E_p = \sqrt{\vec{p}^2 + m^2}$

or mixed state with more energy.

Simple Analogy — consider 2 Harmonic Oscillators

$$H = \frac{1}{2}P_x^2 + \frac{1}{2}P_y^2 + \frac{1}{2}\omega_x^2 x^2 + \frac{1}{2}\omega_y^2 y^2 \qquad |n_1, n_2\rangle$$

I have operator $\mathcal{O} = x + y$. I want the $|1\,0\rangle$ state.

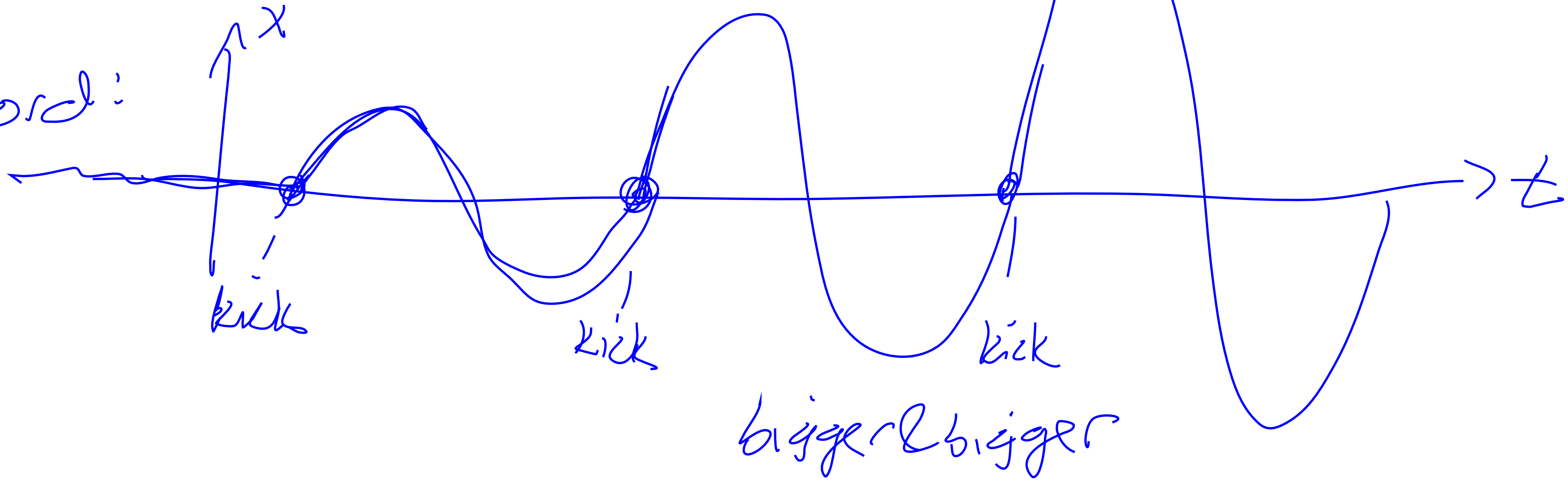
$$\mathcal{O} = \frac{a_x^\dagger + a_x}{\sqrt{2\omega_x}} + \frac{a_y^\dagger + a_y}{\sqrt{2\omega_y}}. \qquad \mathcal{O}|00\rangle = \frac{1}{\sqrt{2\omega_x}}|10\rangle + \frac{1}{\sqrt{2\omega_y}}|01\rangle \text{ oops}$$

At time t, this evolves into $\frac{e^{-i\omega_x t}}{\sqrt{2\omega_x}}|10\rangle + \frac{e^{-i\omega_y t}}{\sqrt{2\omega_y}}|01\rangle$

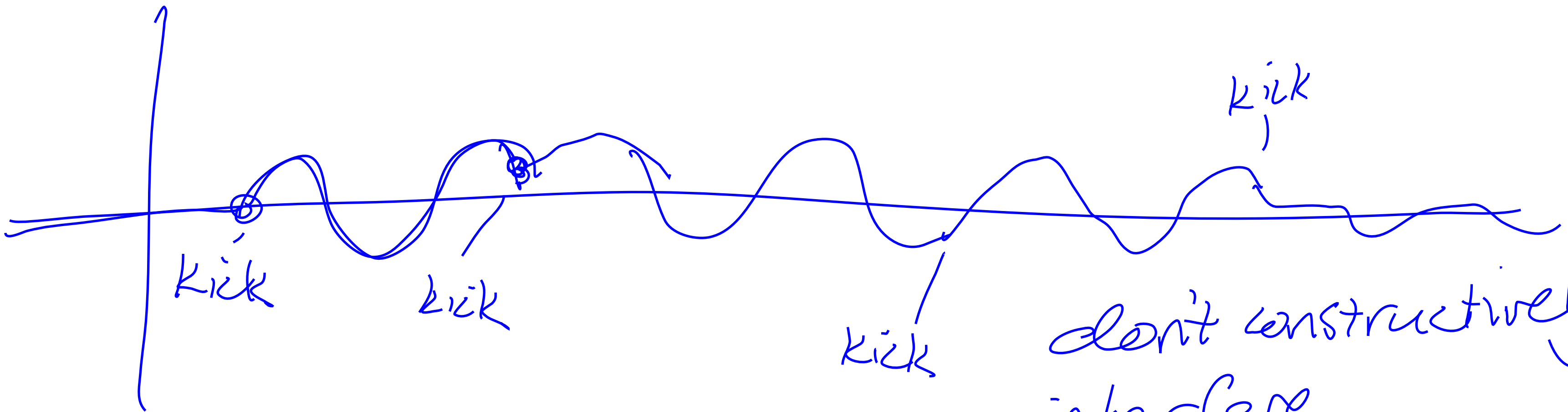
Use phase difference to avoid $|01\rangle$ component (next slide)

Imagine series of "kicks" one each $2\pi/\omega_x$ in time:

x-coord:



y-coord



don't constructively interfere.

Mathematically: $\underset{=}{\mathcal{O}}(t) = e^{iHt}\,\mathcal{O}\,e^{-iHt}$

$$= e^{i\omega_x t}\frac{a_x^+}{\sqrt{2\omega_x}} + e^{-i\omega_x t}\frac{a_x}{\sqrt{2\omega_x}} + e^{i\omega_y t}\frac{a_y^+}{\sqrt{2\omega_y}} + e^{-i\omega_y t}\frac{a_y}{\sqrt{2\omega_y}}$$

Consider some function $f(t)$.

Build "series of kicks"

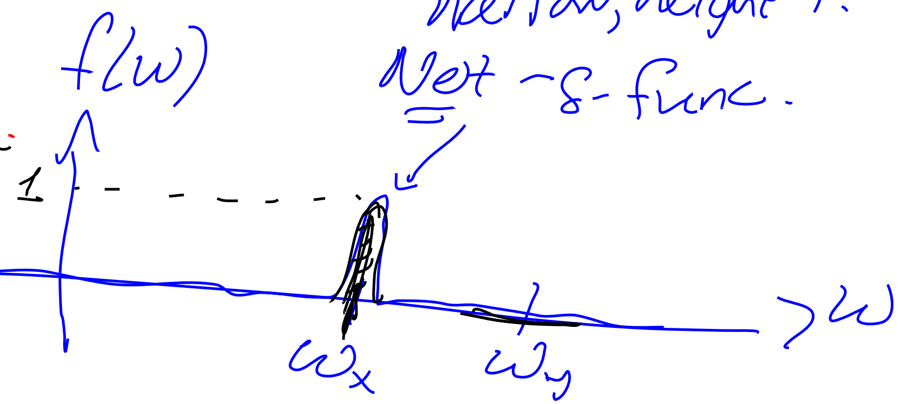
$$\sqrt{2\omega_x}\int_{-\infty}^{\infty} f(t)\,\mathcal{O}(t)\,|0\rangle\,dt$$

$$= \frac{|10\rangle}{\sqrt{2\omega_x}}\int_{-\infty}^{\infty} f(t)\,e^{i\omega_x t}\,dt + \frac{|01\rangle}{\sqrt{2\omega_y}}\int_{-\infty}^{\infty} f(t)\,e^{i\omega_y t}\,dt$$

$$= \frac{|10\rangle}{\sqrt{2\omega_x}}\,\tilde{f}(\omega_x) + \cancel{\frac{|01\rangle}{\sqrt{2\omega_y}}\,\tilde{f}(\omega_y)}$$

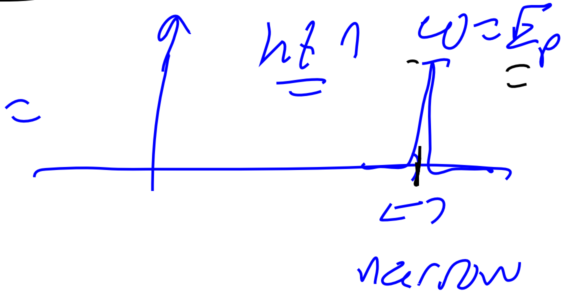
Choose $f(\omega)$:

Narrow, height 1.
Not ~δ-func.



Last time $\psi_p(\vec{x})$ obeys $\displaystyle\int \frac{d^3p}{(2\pi)^3\, 2E_p}\, \psi^*\psi(\vec{p}) = 1$ norm.

$\psi_p(x^\mu) = f(t)\, \psi_p(\vec{x})$ $\qquad \tilde{f}(\omega) =$



$$|in\rangle = \int d^4x_1\, d^4x_2\, \psi_{p_1}(x_1)\, \psi_{p_2}(x_2)\, \varphi(x_1)\, \varphi(x_2)\, |0\rangle$$

$$\langle out| = \prod_{i=1}^{N} \int d^4y_i\, \psi^*_{k_i}(y_i)\; \langle 0|\, \varphi(y_1) \dots \varphi(y_N)$$

2 particles, approx. momentum $p_1, p_2 \to N$ part. $k_1 \dots k_N$.

$$\langle out|in\rangle = \int d^4x_1\, d^4x_2\, d^4y_1 \dots d^4y_N\, \psi_{p_1}(x_1)\, \psi_{p_2}(x_2)\, \psi^*_{k_1}(y_1) \dots \psi^*_{k_n}(y_n)$$

$$\times\, \underbrace{\langle 0|\, T\big(\varphi(y_1) \dots \varphi(y_N)\, \varphi(x_1)\, \varphi(x_2)\big)\, |0\rangle}_{\text{T-ord.}} = G(x_1\, x_2 \dots y_N)$$

N+2-Point Func.

LSZ Reduction Formula

1) Green function $G(x_1 + \xi, x_2 + \xi, y_1 + \xi, \ldots y_N + \xi)$ trans. invariant.

Fourier-space version

$$\int d^4p_a\, d^4p_b\, d^4k_a \ldots d^4k_n\, e^{+i(p_a x_1 + p_b x_2 - k_a y_1 - \ldots - k_n y_N)}\, G(x_1 \ldots)$$

$$= G(p_a, p_b ; k_a, k_b, \ldots) = (2\pi)^4 \delta^4(p_a + p_b - k_a - k_b - \ldots) \cdot G'(p_a, p_b \ldots)$$

+ sign — - sign - neg freq — must be by trans. Inv.

2) Expect $G'(p_a, p_b, \ldots)$ to be singular where $p^2 \to m^2$,

and only singularity matters

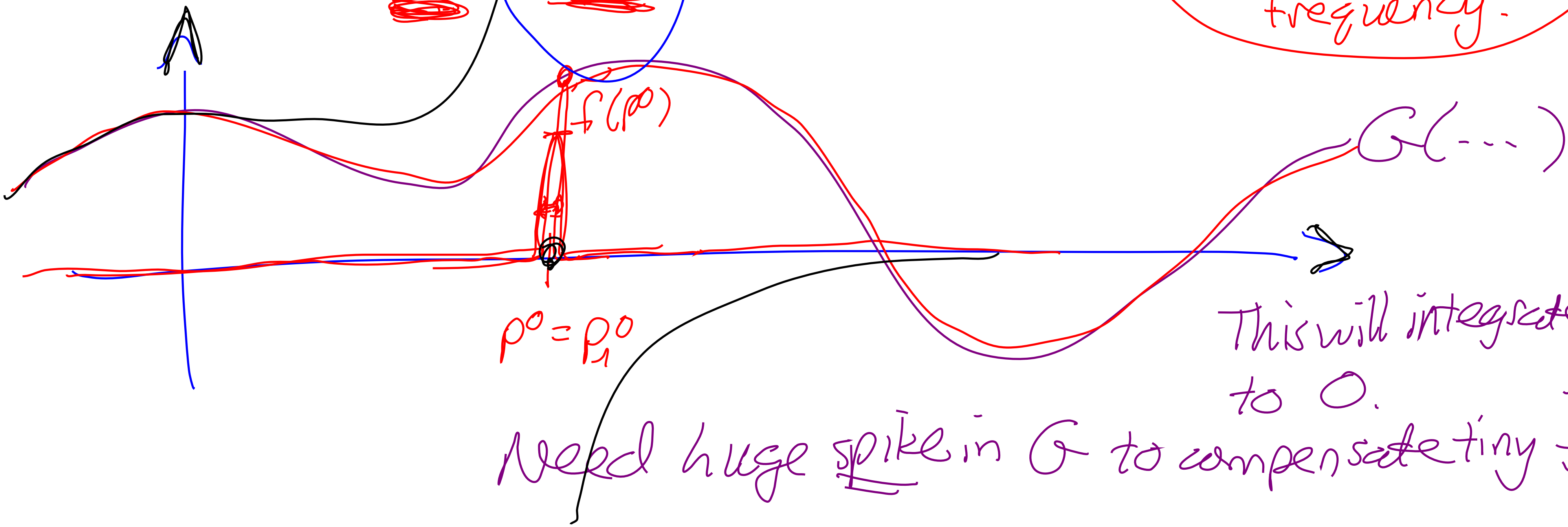
LSZ Reduction (Lehmann Symanzik Zimmermann)

Consider $G(p_a, p_b, \ldots)$ as func. only of p_a.

<u>We want</u> $\int d^3\vec{x}\, dt \; \psi_{p_1}(\vec{x})\, f_1(t)\, G(x_1^\mu, \ldots)$

$$= \int \frac{d^3\vec{p}_a}{(2\pi)^3} \frac{dp^0}{2\pi} \tilde{\psi}_{p_1}(\vec{p}_a)\, \tilde{f}(p^0)\, G(p_a^\mu, \ldots)$$

Just look at frequency.



This will integrate to 0.

Need huge spike in G to compensate tiny $\tilde{f}$.

LSZ

If $G(p_a, \ldots) \propto \boxed{\dfrac{+i}{p_a^2 - m^2 + i\epsilon}}$ then $\displaystyle\int \frac{dp^0}{2\pi} f(p^0) \frac{i}{p^2 - m^2 + i\epsilon} \to 1/2p^0$

principle part $+ 2\pi i\,\delta(p^2 - m^2)$

Otherwise if G is regular, get 0.

Define

$$G_{Amp}(p_a, p_b, k_a, k_b, \ldots k_n) = \left(\frac{p_a^2 - m^2 + i\epsilon}{i}\right)\left(\frac{p_b^2 - m^2 + i\epsilon}{i}\right)\left(\frac{k_a^2 - m^2 - i\epsilon}{i}\right) \cdots \times G(p_a, p_b, \ldots)$$

or

$$G = \frac{i}{p_a^2 - m^2 + i\epsilon} \; \frac{i}{p_b^2 - m^2 + i\epsilon} \cdots \times G_{Amp}(p_a, p_b, \ldots)$$

Amputated Green function.

what I really need.

Also generates all the $\frac{1}{2p^0}$ I expect...

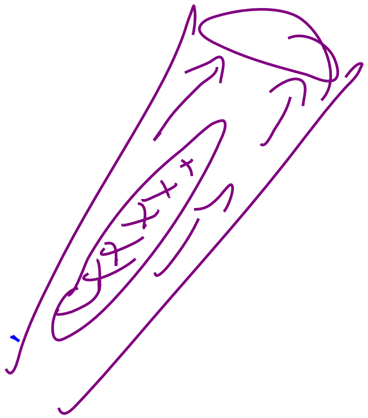
Matrix Element

$$G(p_a, p_b; k_a, k_b, \ldots k_n) = (2\pi)^4 \delta^4(p_a + p_b - k_a \ldots - k_n) \frac{iZ}{p_a^2 - m^2 + i\epsilon} \frac{iZ}{p_b^2 - m^2 + i\epsilon} \frac{iZ}{k_a^2 - m^2 + i\epsilon} \cdots \times i\mathcal{M}(p, k)$$

What to do with it?

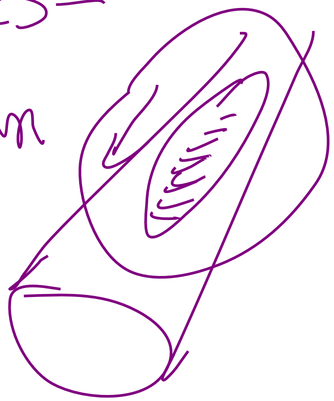
A) Final States

B) Initial States



$$\langle out | in \rangle = 0$$

Final states — use momentum basis

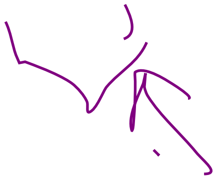


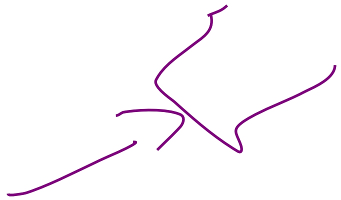
But $\int d(\text{out-states}) \langle in | out \rangle \langle out | in \rangle$

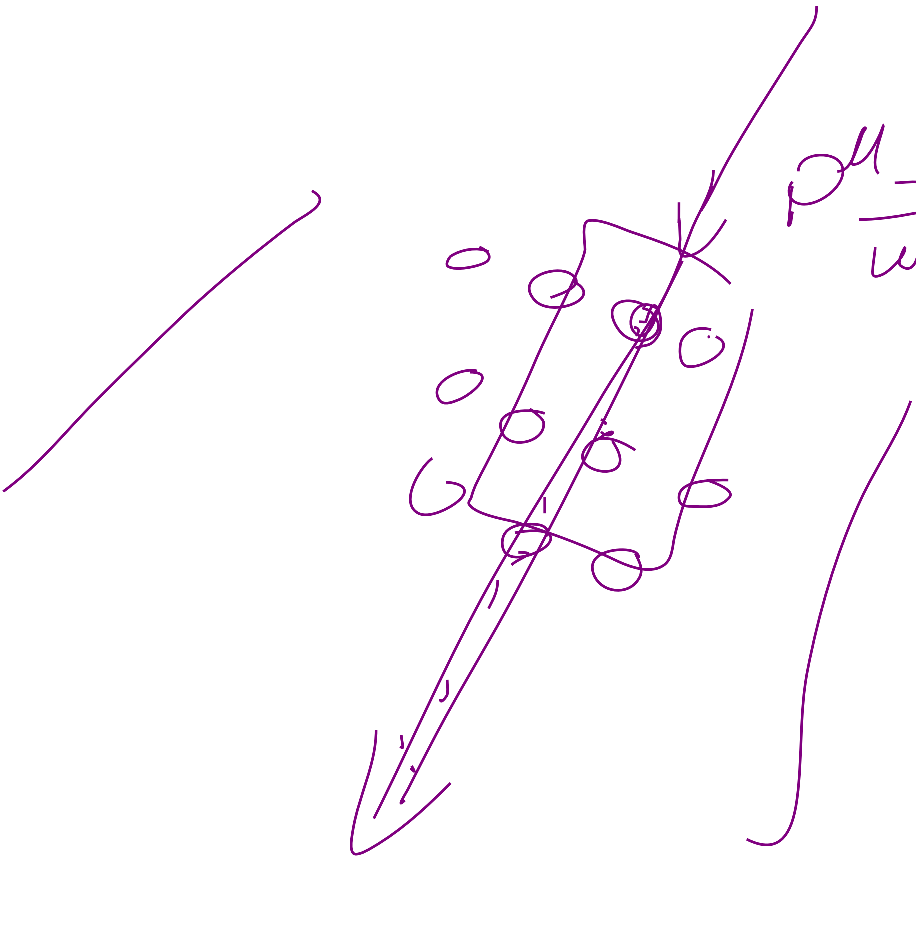
$$\int \frac{d^3k_1\, d^3k_2 \cdots d^3k_n}{(2\pi)^{3n}\, 2k_1^0\, 2k_2^0 \cdots 2k_n^0} \underbrace{\langle in | k_1 k_2 \cdots k_n \rangle \langle k_1 k_2 \cdots k_n | in \rangle}_{\left| G_{Amp}(p_1, p_2 ; k_1, k_2 \cdots k_n) \right|^2}$$

Range

$$\int dp_1\, dp_2 \cdots$$







p^μ-info well determined

$$\int \frac{d^3k}{(2\pi)^3 2k^0} |k\rangle\langle k| \quad \text{is well-def. projection operator}$$

Value 1 if part in range

0 if not

$$\langle in| \int |out\rangle\langle out| \; |in\rangle \to \text{Prob. of scatt.}$$

Range

$$|in\rangle = \underbrace{\int d^4x_1\, d^4x_2\; \psi_{p_1}(x_1)\,\psi_{p_2}(x_2)}$$

$$\varphi\varphi|0\rangle$$

$$\int dx_1^0\, dx_2^0 \;\; f \;\; f$$

kill $\frac{1}{p^2-m^2}\frac{1}{p^2-m^2}$ from G_{Amp}.

$$\int d^3x_1\, d^3x_2\;\; \psi_{p_1}(\vec{x}_1)\,\psi_{p_2}(\vec{x}_2)$$

$$\int d^3x_1'\, d^3x_2'\;\; \psi^*_{p_1}(\vec{x}_1')\,\psi^*_{p_2}(x_2')$$

$$|G_{Amp}|^2$$

Goal (I Think): Prob that in-state $|in\rangle$ $|\langle out|in\rangle|^2$

scatt into range of out-states $\int d^3k\ |k\rangle\langle k|\ldots$

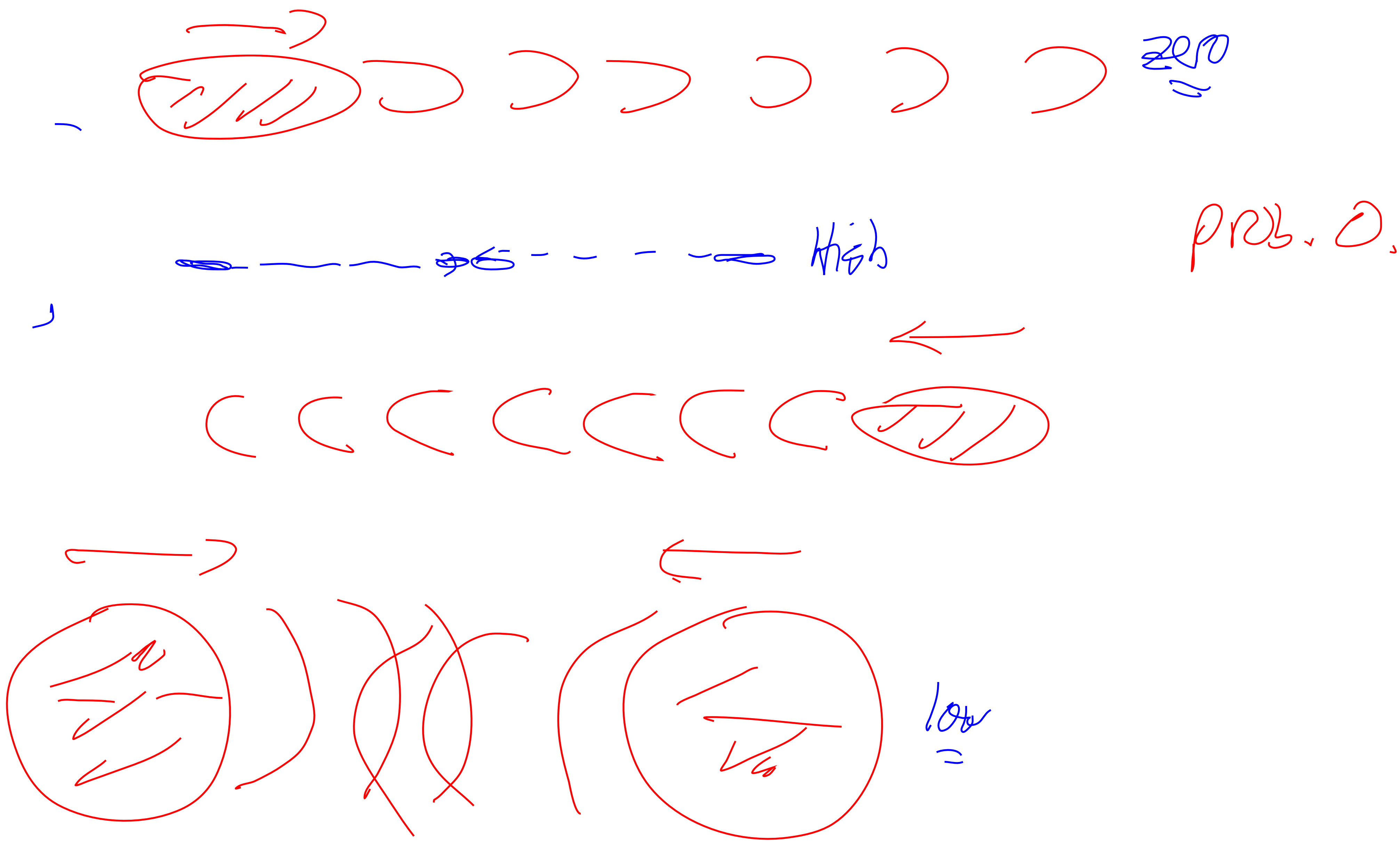
$$\int d^3\vec{x}_1\, d^3\vec{x}_2\, d^3\vec{x}_1{}'\, d^3\vec{x}_2{}'\ \psi_{p_1}(x_1)\,\psi^*_{p_1}(x_1')\,\psi_{p_2}(x_2)\,\psi^*_{p_2}(x_2')$$

$$\int_{\text{Range}} \frac{d^3k_1 \cdots d^3k_n}{(2\pi)^{3n}\, 2k_1^0 \cdots 2k_n^0}\ \left(G_{Amp}(x_1 x_2; k_1 \ldots k_n)\right)\left(G^*_{Amp}(x_1', x_2'; k_1 \ldots k_n)\right)$$

This depends on wave Packets

1) Info - did experimental colleagues do good job?

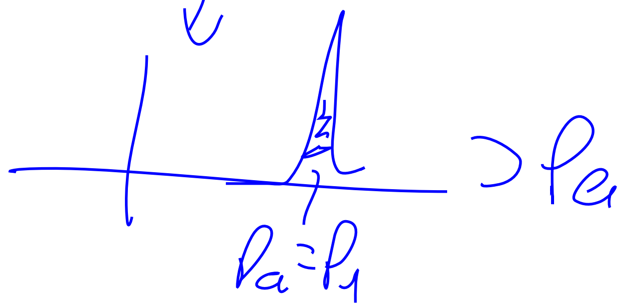
2) what's phys of my thy? cross-sections, not probabilities


zero
High
PROB. 0.
low

Because $G_{Amp}(p_1 p_2 ; k_1 k_2 \ldots) = (2\pi)^4 \delta^4(p_1+p_2-k_1-\ldots-k_n) M$

Fourier transform

$$\int d^3x_1 \, \psi_{p_1}(x_1) \, G(x_1 \ldots) = \int d^3 p_a \, \psi_{p_1}(p_a) \, G(p_a, \ldots)$$



Frame where $\vec{p}_1, \vec{p}_2$ are head-to-head.

z axis

$$\psi_{p_1}(x_1) = \psi_{z_1}(z_1) \, \psi_{\perp}(x_1, y_1)$$

$$\int \frac{dp_z}{(2\pi) \, 2p^0} \, \psi_z^* \psi_z = 1 \qquad \int \frac{d^2 p_\perp}{(2\pi)^2} \, \psi_\perp^* \psi_\perp = 1$$

$$\text{Prob} = \int d^2p_a\, d^2p_b\, d^2p_a'\, d^2p_b' \; \psi_1(p_a)\, \psi_1^*(p_a')\, \psi_2(p_b)\, \psi_2^*(p_b')$$

$$\int dp_{az}\, dp_{az}'\, dp_{bz}\, dp_{bz}' \; \psi_z^*(p_a')\psi_z(p_a)\; \psi^*(p_b')\, \psi(p_b)$$

$$\int \frac{d^3k_1}{2\ldots} \ldots (2\pi)^4\, \delta(p_a+p_b-p_a'-p_b')\; (2\pi)^4\, \delta(p_a'+p_b'-\Sigma k)\; |\mathcal{M}|^2$$

p_1+p_2 $\qquad \mathcal{M}(p_1,p_2,k,\ldots)$

weak p-dep.

ψ's peaked about $p_a = p_1$
$p_b = p_2$

$$\int \frac{d^3k}{(2\pi)^3\ldots} \ldots (2\pi)^4\delta^4(p_1+p_2-\Sigma k)\, |\mathcal{M}(p_1,p_2;k_1,\ldots)|^2 \times (\psi\text{-integrals})$$

What is

$$\int \frac{dp_{az}\, dp_{az}'\, dp_{bz}\, dp_{bz}'}{(2\pi)^4 (2E_a)^2 (2E_b)^2} (2\pi)^2 \delta\left(p_{az} + p_{bz} - p_{az}' - p_{bz}'\right) \delta\left(E_{az} + E_{bz} - E_{az}' - E_{bz}'\right)$$

$$E_a = \sqrt{p_a^2 + m^2}$$

$$\psi^*_{z1}(p_a')\, \psi_{z1}(p_a)\, \psi^*_{z2}(p_b')\, \psi_{z2}(p_b)$$

$$\boxed{p_{bz}' = p_{bz} + \left(p_{az} - p_{az}'\right)}$$

$$-\frac{d}{dp_{az}'}\left(E_{az} + E_{bz} - E_{az}' - E_{bz}'\right) = \frac{d}{dp_z'}\left(-\sqrt{p_z'^2 + m^2} - \sqrt{\left(p_{bz} + p_{az} - p_{az}'\right)^2 + m^2}\right)$$

$$= \frac{p_{az}'}{E_{az}'} - \frac{p_{bz}'}{E_{bz}'} = v_a - v_b$$

$$\frac{1}{v_a - v_b} \int \frac{dp_a\, dp_b}{(2\pi)^2 (2E_a)^2 (2E_b)^2} \left(\psi^*_z \psi_z(p_a)\right) \left(\psi^*_z \psi_z(p_b)\right) = \frac{1}{2E_1\, 2E_2\, |v_a - v_b|}$$

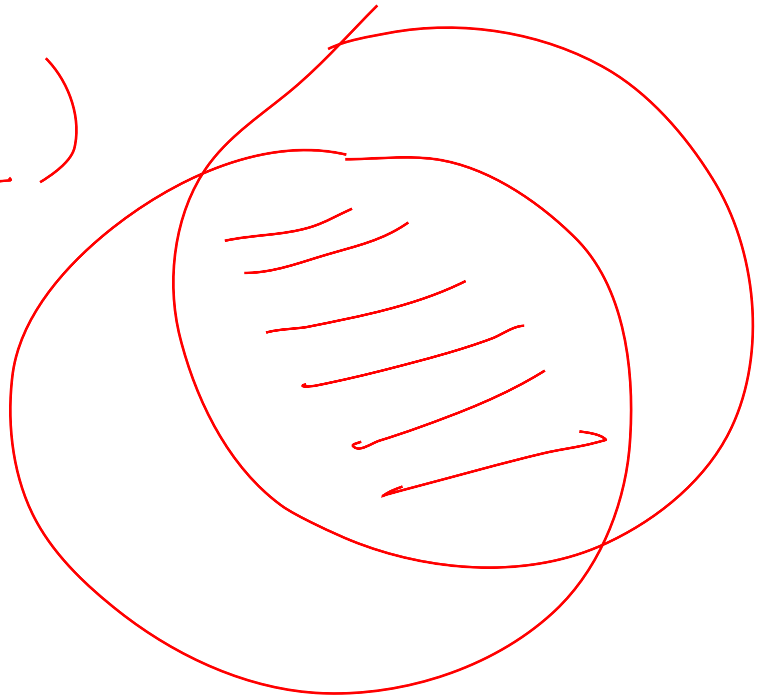
Transverse Stuff.

$$\int d^2 p_{a\perp}\, d^2 p'_{a\perp}\, d^2 p_{b\perp}\, d^2 p'_{b\perp}\; \psi^*_{1\perp}(p'_{a\perp})\, \psi_{1\perp}(p_{a\perp})\, \psi^*_{\perp 2}(p'_{b\perp})\, \psi_{\perp 2}(p_{b\perp})$$

$$(2\pi)^2 \delta^2(p_{a\perp} + p_{b\perp} - p'_{a\perp} - p'_{b\perp})$$

$$\int d^2 x_\perp\, e^{i x_\perp \cdot (p_{a\perp} + p_{b\perp} - p'_{a\perp} - p'_{b\perp})}$$

$$\int d^2 x_\perp\; \psi^{+}{}_1 \psi_1 (x_\perp)\; \psi^{+}{}_2 \psi_2 (x_\perp)$$



How well will parts hit?

Prob. to scatter = $\left(\frac{1}{2E_1 2E_2 |v_1 - v_2|} \int \frac{d^3k_1 \dots d^3k_n}{(2\pi)^{3n} 2k_1^0 \dots 2k_n^0} (2\pi)^4 \delta^4(p_1+p_2-\Sigma k) |M|^2 \right)$

× (how well are part's focused?)

Prob to Scatt / how-well-focused = $\equiv \sigma$ cross-section

how-well-focused = 1/A

$\psi_1^* \psi_1 = \frac{1}{A}$ uniformly over area A

Fraction hit = $\frac{\sigma}{A}$ = Prob to scatt.

$\int d^2x \, \psi^* \psi \, \psi_1^* \psi = \frac{1}{A}$

σ = A · Prob to scatt

= Prob / how well focused.

Take Home Lesson:

We want Cross-section = $\dfrac{\text{Prob to scatt}}{\text{Quality of beam focus.}}$

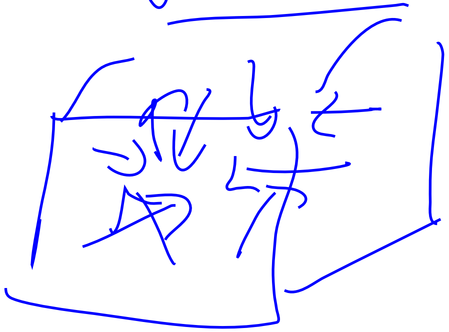
[expts: Luminosity – Quality × flux of particles

$$\frac{\#\,\text{scatt}}{\text{sec}} = \sigma \cdot \text{Luminosity}$$

σ Barn : $10^{-28}\,m^2$ $= (10^{-12}\,cm)^2$]

$$\sigma = \frac{1}{2E_1\, 2E_2\, |v_1 - v_2|} \int \frac{d^3k_1 \ldots d^3k_n}{(2\pi)^{3n}\, 2k_1^0 \ldots 2k_n^0} (2\pi)^4 \delta^4(p_1 + p_2 - \Sigma k_i)\, |M|^2$$

Last generaliz. occupancy $f(\vec{x},\vec{p})$: $\frac{\text{number}}{d^3x\, d^3p}$ of part. in box



$$n = \int d^3x \int \frac{d^3p}{(2\pi)^3} f(x,p)$$

$$\int \frac{d^3p_1\, d^3p_2}{(2\pi)^3 (2\pi)^3} f\, f \, \frac{1}{2E_1 2E_2}$$

$$\frac{\#\text{scatt}}{\text{space-time vol}} = \prod_{p_f} \int \frac{d^3p_f}{(2\pi)^3 2E_f} \prod_i \int \frac{d^3p_i}{(2\pi)^3 2E_i} \prod f(p_i, x_i)$$

$$\times (2\pi)^4 \delta^4\left(\sum p_i - \sum p_f\right) |M|^2$$