Quantum Field theory I

Remarks: QFT is hard. { Lots of Time
Book and Lectures
Homework and Final
New math things
new concept/intuition things

QFT is important. { Language of Part & Nucl. Thy
Often applicable in CMT etc
Highly predicted & well tested
New intuition in other fields (GR)

You know what Quantum is
What is field theory?

Theory with 1+ Degree of Freedom at each point in space

Example 1: Class. mech. is: particle at location $\vec{r}$

$$L(\vec{r}, \dot{\vec{r}}) = \frac{m}{2}\dot{r}^2 + V(r)$$

1 or 3 DOF (gen. coord.): $r_x r_y r_z$

Class field is like E&M, $\vec{E}(x)$, $\vec{B}(x)$

$$L(E, B) = \int d^3x \left[\frac{B^2}{2} - \frac{E^2}{2} \right]$$

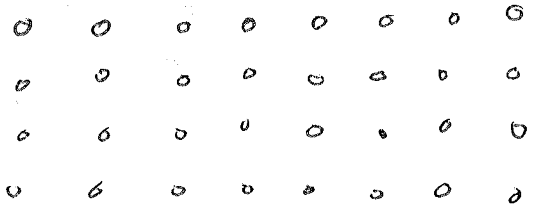
(in some unit convention for E, B)

$B_x(r)\ B_y(r)\ B_z(r)\ E_x \ldots E_z(r)$ 6 DOF for each r value.

Infinite DOF. { Continuous ∞ if B, E arbitrary
Countable ∞ if enforce that they are continuous and if we put thy in (large) box.

Field Thy is also the limit of other things when the scale of interest is much longer than the intrinsic scale.

For instance, consider solid (crystaline)



regular array of atoms

Each atom has 3 DOF: x, y, z

Interactions betw. atoms - sort of like springs linking them.

Realworld: $3 \times \underset{(\#\,\text{atoms})}{10^{27\pm3}}$ DOF is alot! Cannot follow everything

Useful notation: rather than having 10^{27} labels, use that each atom has equilibrium ~~prefer~~ location.

Define displacement from usual as

$\varphi_x(\vec{r})$
$\varphi_y(\vec{r})$
$\varphi_z(\vec{r})$

x, y, z displacement of atom normally at location $\vec{r}$.
Defined for each $\vec{r}$ of latt.

If φ_i vary smoothly - or if I Don't Care about short-scale variation - can treat $\varphi_i(\vec{r})$ as continuous variable.

$\varphi_x(\vec{r})$ = displacement of atoms near $\vec{r}$.

What is L? $L = K_{in} - V_{(pot)}$

$$K_{in} = \sum_{\text{atoms}} \frac{m}{2}\left(\partial_t \varphi_x \partial_t \varphi_x + \partial_t \varphi_y \partial_t \varphi_y + \partial_t \varphi_z \partial_t \varphi_z\right)$$

$$\Rightarrow \int d^3x \, \frac{\rho}{2} \, \partial_t \varphi_i \partial_t \varphi_i$$

$i = x, y, z$ sum on i implied
$\rho = m \times$ (#atoms/volume)

What is V_{pot}? Hard question. In atom-pair approx, it would be..

$$V = \sum_{r_1, r_2} \frac{C_{ijlm}}{2} \left(\varphi_i(r_1) - \varphi_j(r_2)\right)\left(\varphi_l(r_1) - \varphi_m(r_2)\right)$$

who knows what??

But only r_1 very close to r_2 could possibly matter: atomic forces are atomic range

And φ is smooth - or I only care about smooth part.

$$\varphi_i(r_1) - \varphi_i(r_2) = (r_1 - r_2)_j \partial_j \varphi_i + \frac{(r_1-r_2)_j (r_1-r_2)_k}{2} \partial_j \partial_k \varphi_i + \dots$$

Taylor series

$$V \to \int d^3x \; K_{ijlm} \partial_i \varphi_j \partial_l \varphi_m + K_{ijlmno} \partial_i \partial_j \varphi_l \partial_m \partial_n \varphi_o$$

$$+ \text{ terms of order } \partial^4 \varphi^4, \partial^6 \varphi^2, \dots$$

~~get rapidly~~ le

Size of K_{ijlm}? Unit analysis, we have

V: energy. Natural energy: eV

d^3x: length^3 natural length: Å or nm

φ: length

$$K_{ijlm} \sim \frac{[V]}{[d^3x][\partial^2][\varphi^2]} = \frac{[eV]}{[Å^3][Å^{-2}][Å^2]} \sim \frac{eV}{Å^3}$$

$$K_{ijlmno} : \frac{eV}{Å^5}$$

If the scales of separation I care about are $\gg Å$, $\partial^4 \varphi^2$ term tiny

$\partial^4 \varphi^4$ term tiny if I care about scale l and $|\varphi| \ll l$

I don't need specifics of atomic potential.

Just need a few long-wave info's : ρ density

K_{ijlm} elastic modulus

for rotationally symmetric systems, $K_{ijlm} = K_1 \delta_{il} \delta_{jm} + K_2 \delta_{ij} \delta_{lm}$ 2 param's

generally 21 param's as crystaline axes can play role

General lessons

Energy & action depend on fields : things which take value at every point in space, $\varphi(\vec{r})$

$$L = \int d^3x \cdot \mathcal{L}(\varphi, \partial_i \varphi, \partial_t \varphi) \quad \text{Lagrangian Density}$$

$$S = \int dt\, L = \int \underbrace{d^3x\, dt}_{d^4x \text{ all spacetime}}\ \mathcal{L}(\varphi, \partial_\mu \varphi)$$

all deriv's

~~Look~~ look, spacetime picture & sp. & time on same footing emerge naturally if I work in terms of S.

All relativistic consistent QP theories are field thy's
And easiest to study them (spacetime, Lorentz manifest) Sp. Relat.,
in terms of S and $\mathcal{L}$

$$H = \int d^3x\, \mathcal{H}(\varphi, \partial_i \varphi, \partial_0 \varphi)$$ also plays a role, but not covariant. We will use it early on to make contact with QM you know.

Also : use premise/assumption that fundamental scale is very short (our excuse to use FT) to pare down what terms we need to write in $\mathcal{L}$ using dimensional arguments.

And don't worry that # of DOF's appears to be ∞. That may just be that you have not probed short enough scales where DOF become "grainy" & number finite but really really large.

Back of our minds, always assume that's true: at some super short scale a (super high $\Lambda = \hbar c/a$) our DOF become grainy & density ~~#~~ of DOF is finite (but large)