Fermions & Vectors & ..

$$\mathcal{L} = \bar{\Psi}(i\cancel{\partial} - m)\Psi + \phi^*\big(\underset{-m^2}{-\partial_\mu \overset{\text{or } D^\mu}{\partial^\mu}}\big)\varphi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\langle\Psi\bar{\Psi}\rangle = \frac{1}{i\cancel{\partial}-m} = \frac{1}{\cancel{p}-m} \qquad \langle\phi\phi^*\rangle\ \frac{1}{-\partial_\mu\partial^\mu - m^2} = \frac{1}{p^2-m^2}$$

what as $\langle A^\mu A^\nu\rangle$?

$$-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) = -\frac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

$\int$ by parts $\quad +\frac{1}{2}A_\nu(\partial^\mu\partial_\mu A^\nu - \partial^\mu\partial^\nu A_\mu)$

$$= \frac{1}{2}A_\nu(\partial_\alpha\partial^\alpha g^{\nu\mu} - \partial^\nu\partial^\mu)A_\mu$$

so I want the inverse of $g^{\mu\nu}\partial_\alpha\partial^\alpha - \partial^\mu\partial^\nu$, right?

p-space: inverse of $-g^{\mu\nu}p^2 + p^\mu p^\nu$, as in

$$(p^\mu p^\nu - g^{\mu\nu}p^2)G_{\nu\alpha}(p) = g^\mu_\alpha.$$ Except there <u>is</u> no such thing!

$(p^\mu p^\nu - g^{\mu\nu}p^2)$ is singular: $(p^\mu p^\nu - g^{\mu\nu}p^2)p_\nu = p^\mu p^2 - p^\mu p^2 = 0$ <u>oops</u>

what does this mean? $(p^\mu p^\nu - g^{\mu\nu}p^2)\langle A_\nu A_\alpha\rangle$ has problem if $A_\nu \propto p_\nu$

And that means when... "$A_\mu \propto \partial_\mu$ as in $\partial_\mu\theta$"

Gauge change $A_\mu \to A_\mu + \partial_\mu\theta \to A_\mu - ip_\mu\theta$ in p-space

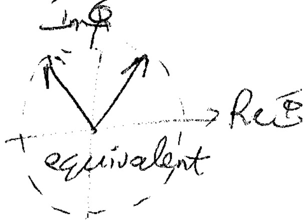
$$(p^\mu p^\nu - g^{\mu\nu}p^2)\langle A_\mu - ip_\mu\theta\ A_\alpha\rangle \text{ should } = (\,..\,)\langle A_\mu A_\alpha\rangle$$

This zero mode makes sure nothing changes when $A_\mu \to A_\mu + \partial_\mu\theta$.

Geometric Interpolation: $A_\mu \to A_\mu + \partial_\mu \theta$ is a change of some coordinates. Not physical.

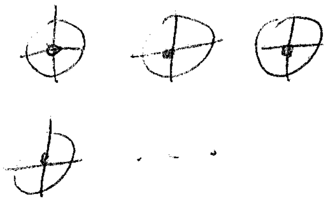
Picture: lattice

At each pt. we have Φ. Now $\Phi^\dagger \Phi = |\Phi|^2$ invariant. But Arg Φ not.



To concentrate on this angle info, just look at direction of Φ at each pt.
Lives on a circle $U(1)$ "S^1" space of θ's

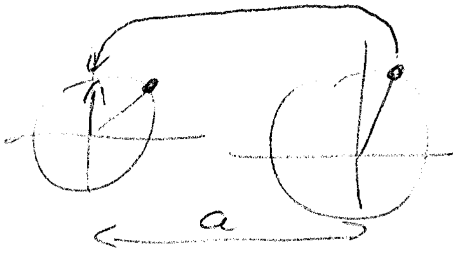
Gauge change:
To freely rotate each space $e^{i\theta(x)}$



Same as choosing diff't coord's for each circle. (Start pt. on circle)

Call each circle a "fiber" at pt x
Coms. of 1 fiber at each x: fiber bundle ~~space~~. Almost $\mathbb{R}^4 \times S^1$ ~~$U(1)$~~
(topology: same
geometry: not!)

To define deriv., we need comparison instruct. between neighbors.



need to know "in moving back, also rotate by angle ϕ."
Name that angle $-aA_\mu$. $+aA_\mu$ moves you forward.

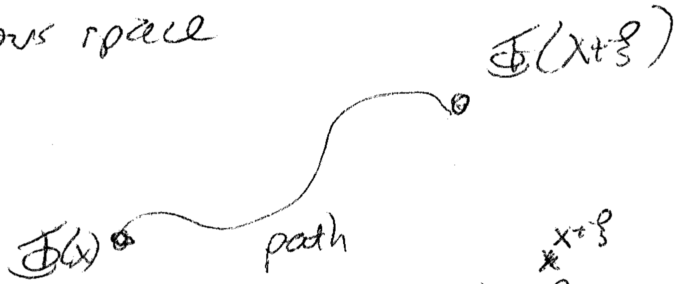
Difference between pullback of $\Phi(x+a\hat{\mu})$ and $\Phi(x)$ is

$$e^{-iaA_\mu}\Phi(x+a\hat{\mu}) - \Phi(x). \quad \frac{(\text{Diff})}{a} \simeq \text{"}D_\mu \Phi\text{"} = \frac{-iaA_\mu \Phi(x+\hat{\mu})}{a} + \frac{\Phi(x+a\hat{\mu}) - \Phi(x)}{a}$$

$D_\mu \Phi \simeq \partial_\mu \Phi - iA_\mu \Phi$. Ah.

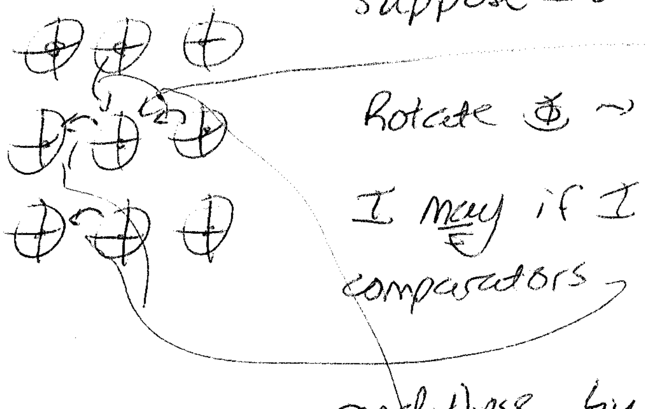
So A_μ is a comparator to tell how to rotate Φ when comparing in diff'nt places.

Continuous space



compare $\Phi(x)$ to $\underbrace{\exp\left(-i\int_{x}^{x+\xi} A_\mu dx^\mu\right)}_{\text{comparator - Wilson line}} \Phi(x+\xi)$

Back to latt. picture



Suppose I want new coord's here,

Rotate $\Phi \to e^{i\theta}\Phi$.

I may if I promise to rotate these comparators by $e^{-i\theta}$

and these by $e^{+i\theta}$

So neighbors brought to x also have the $e^{i\theta}$

Φ moved away from x loses the $e^{i\theta}$.

$A_\mu(\text{in front}) \to A_\mu - \frac{\theta}{a}$

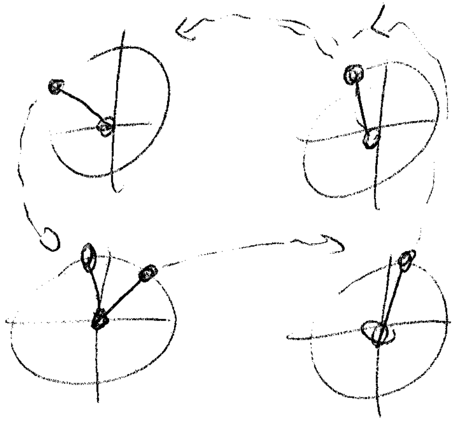
$A_\mu(\text{behind}) \to A_\mu + \theta/a$

change θ at both ends of a link: $A_\mu \to A_\mu + \frac{\theta(\text{front}) - \theta(\text{back})}{a}$

$\simeq A_\mu + \partial_\mu \theta$

So what's coord. and what's physical? (L22P4)

Suppose, if I take $\Phi(x)$ and I move it around



comes back somewhere new!

$$\Phi(x) \to e^{iaA_1} e^{iaA_2} e^{-iaA_3} e^{-iaA_4} \Phi$$

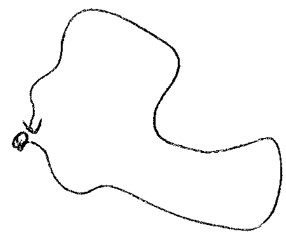
$$= e^{ia\left(A_x(x) + A_y(x+a\hat{x}) - A_x(x+a\hat{y}) - A_y(x)\right)} \Phi$$

$$\simeq e^{ia^2\left(\frac{-A_x(x+a\hat{y}) + A_x(x)}{a} + \frac{A_y(x+a\hat{x}) - A_y(x)}{a}\right)} \Phi$$

$$\simeq e^{ia^2(\partial_x A_y - \partial_y A_x)} \Phi \quad \text{aha!}$$

Amount you fail to return to original point by is $a^2 F_{xy}$

or generally



$$\iint F_{\alpha\beta}\, dx^\alpha \wedge dx^\beta$$

surface bounded by path

$F_{\mu\nu}$: physical invariant.

Actual information : $\{A_\mu(x)\}$ minus that part which is just coord. change, that is "modulo $\partial_\mu \Theta$"

$\{$ equivalence classes of $A_\mu(x)$, $A_\mu(x)$ $A'_\mu(x)$ equivalent iff $\forall \Theta : A'_\mu = A_\mu + \partial_\mu \Theta$ $\}$

Set of all fiber bundles.

Don't $\int \mathcal{D}A_\mu$. Only $\int \mathcal{D}$(equiv. classes, or distinct fiber bundles)

Great but how do you *do* that?

Find some condition which <u>does</u> tell one ~~fiber~~ A_μ apart from equivalent A_μ, and pick its value.

For instance consider $\partial_\mu A^\mu$. $\quad \partial_\mu A^\mu \to \partial_\mu A^\mu + \partial_\mu \partial^\mu \theta$

And I can choose $\partial_\mu \partial^\mu \theta$ = anything I want, invert to find θ
$\forall \theta$ to make $\partial_\mu A^\mu$ anything you want.

(Almost uses all freedom to choose θ, too. Only "harmonic gauge" θ: $\partial_\mu \partial^\mu \theta = 0$, that is, wave eq sol's, still free. Correspond to small subset, eg, fix at $t=-\infty$ surface ...)

If I insert $\delta(\partial_\mu A^\mu)$ into path $\int$, it ~~replaces~~ picks out one copy of each set of gauge-equiv. config's.
But changes no physics.

$$\int \mathcal{D}A_\mu \, e^{i\int d^4x \frac{1}{2} A_\mu (\partial_\alpha \partial^\alpha g^{\mu\nu} - \partial^\mu \partial^\nu) A_\nu} \, \delta(\partial_\mu A^\mu)$$

Claim: propagator $= \dfrac{-i(g^{\mu\nu} - p^\mu p^\nu / p^2)}{p^2 + i\varepsilon}$

Slightly better: force $\partial_\mu A^\mu = \lambda(x)$ any λ. which? Avg w. Gauß weight

$$\int \mathcal{D}A_\mu \, \mathcal{D}\lambda(x) \, e^{i\int d^4x \left(\frac{1}{2} A_\mu (\partial_\alpha \partial^\alpha g^{\mu\nu} - \partial^\mu \partial^\nu) A_\nu + \frac{1}{2\alpha}\lambda^2\right)} \, \delta(\partial_\mu A^\mu - \lambda)$$

Do $\mathcal{D}\lambda$ int. using δ-function: $\frac{1}{2} A_\mu (\partial_\alpha \partial^\alpha g^{\mu\nu} + (\alpha^{-1} - 1)\partial^\mu \partial^\nu) A_\nu$

has an inverse of

$$\frac{-i(g^{\mu\nu} + (\alpha - 1) p^\mu p^\nu / p^2)}{p^2 + i\varepsilon}$$

check: $\dfrac{-g^{\mu\nu} + (1-\alpha)\frac{p^\mu p^\nu}{p^2}}{p^2} \left(-p^2 g_{\nu\alpha} + (1 - \alpha^{-1}) p_\nu p_\alpha\right) = g^\mu_\alpha$ ✓ previous is just $\alpha \to 0$ limit.

But surely it matters, what α I use when I write

$$\langle A^\mu_{(\alpha)} A^\nu_{(\alpha)} \rangle = \int d^4x \, e^{ip\cdot x} \quad \frac{-ig^{\mu\nu} + i(1-\alpha) p^\mu p^\nu / p^2}{p^2 + i\epsilon} \quad ??$$

well, no it doesn't. A_μ always contracts against J^μ

$A_\mu J^\mu$. Only $A_\mu \propto p_\mu$ cares about $\frac{p^\mu p^\nu}{p^2}$ term.

But $p_\mu J^\mu = 0$ (p-space version of $\partial_\mu J^\mu = 0$)

Essential that A_μ couples to J^μ w. $\partial_\mu J^\mu = 0$.

Otherwise, you are in <u>trouble</u>.