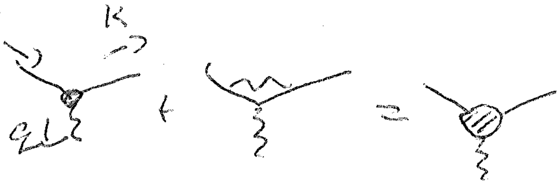
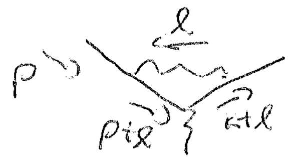
Now let's look at

Call



$ie\gamma^\mu$ + 1 loop $= ie\Gamma^\mu(p,q)$

explicitly 1-loop $= \int \frac{d^D l}{(2\pi)^D} (ie)^3 \gamma^\alpha \frac{i(\not k+\not l+m)}{(k+l)^2-m^2+i\epsilon} \gamma^\mu \frac{i(\not p+\not l+m)}{(p+l)^2-m^2+i\epsilon} \gamma^\beta \frac{-ig_{\alpha\beta}}{l^2+i\epsilon}$



which is a matrix and does not have a trace

Also

iS $\quad$ $iSi\Sigma iS$ $\quad$ $iSi\Sigma iSi\Sigma iS$ $\quad = \frac{i}{\not p - m - \Sigma(p)}$

Σ is a matrix. No trace

$S = \frac{1}{\not p - m}$ $\qquad \Sigma = (i)(ie)^2 \int \frac{d^D l}{(2\pi)^D} \gamma^\mu \frac{i(\not p+\not l+m)}{(p+l)^2-m^2+i\epsilon} \gamma^\nu \frac{-ig_{\mu\nu}}{l^2+i\epsilon}$

from def.

Before I compute these are they related somehow?
And what must they look like?

What must they look like? $\quad$ L 26 P2

Σ is a matrix depending on p^μ alone.

$$\Sigma(p) = \Sigma_v \not{p} + \Sigma_m \mathbb{1} \qquad \Sigma_v, \Sigma_m \text{ funcs of } p^2 \text{ only}$$

why? only 16 4x4 matrices: $\{ \mathbb{1}, \gamma^\mu, [\gamma^\mu, \gamma^\nu], \underbrace{\gamma^\mu \gamma^5, \gamma^5}_{\text{parity odd}} \}$

So what else is there? $\qquad p^\mu p^\nu [\gamma_\mu, \gamma_\nu] = 0$

$g^{\mu\nu} [\gamma_\mu, \gamma_\nu] = 0$

Interpretation

$$\frac{1}{\not{p}(1-\Sigma_v) - (m + \Sigma_m)} \qquad \text{Claim: } \Sigma_m = m \Sigma_m' \text{ linear in } m.$$

where is this zero? eg. $((1-\Sigma_v)\not{p} - (1-\Sigma_m')m)u = 0$

$$[(1-\Sigma_v)\not{p} + (1-\Sigma_m')m][(1-\Sigma_v)\not{p} - (1-\Sigma_m')m] = 0$$

$$(1-\Sigma_v)^2 p^2 = (1-\Sigma_m')^2 m^2 \qquad p^2 = \underbrace{\left(\frac{1-\Sigma_m'}{1-\Sigma_v}\right)^2 m^2}_{m^2_{phys} \text{ physical mass}}$$

Does $\langle \bar{\psi} | 0 \rangle$ give particle with normalization $u(p)$? Not anymore.

$$\frac{\not{p}(1-\Sigma_v) + (m)(1-\Sigma_m')}{(p^2 - m^2_{phys})(1-\Sigma_v)^2} = \underset{\uparrow}{(1-\Sigma_v)^{-1}} \; \frac{\not{p} + m_{phys}}{p^2 - m^2_{phys}} \ldots$$

I want $(1-\Sigma_v)^{-\frac{1}{2} \# \text{ferm. lines}} \times$ usual Amp GF. $\qquad$ produces part. w. norm. $u(1-\Sigma_v)^{-1/2}$.

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and Γ^μ? $\Gamma^\mu(p', p)$ is
1) matrix
2) 4-vector index
3) dep. on 2 ext. momenta p, p' or p, q (equiv.)

Consider $\bar{u}(p') \Gamma^\mu u(p)$

$$\Gamma^\mu = \gamma^\mu A(p,p') + (p^\mu + p'^\mu) B(p,p') + q^\mu C(p,p')$$

A, B, C funcs of p, p', m, e may include $\not{p}, \not{p}'$, etc.

But $\not{p} u(p) = m u(p)$, $\bar{u}(p') \not{p}' = \bar{u}(p') m$ so forget $\not{p}, \not{p}'$'s

A, B, C scalar funcs of m, $q^2 = (p-p')^2$ only. $\quad (p^2 = p'^2 = m^2)$

And $q_\mu \Gamma^\mu = 0$ (see in a moment) $\to C = 0$.

A - spinor-like coupling.
B - scalar-like coupling.

L26P3

Slippery Trick (Gordon identity)

$$\bar{u}(p')\,\gamma^\mu\, u(p) = \bar{u}(p')\left[\frac{p^\mu + p'^\mu}{2m} + i\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

proof: $\sigma^{\mu\nu}q_\nu = \frac{i}{2}[\gamma^\mu,\gamma^\nu](p_\nu - p'_\nu)$

$$= \frac{i}{2}\Big(\underset{=4m\gamma^\mu \text{ by Dirac}}{-2\gamma^\mu \not{p} - 2\not{p}'\gamma^\mu} + \underbrace{\{\gamma^\mu, \not{p}'\} + \{\gamma^\mu, \not{p}\}}_{2p'^\mu + 2p^\mu}\Big)$$ there you go

So 1 lin. dependence between γ^μ, $(p^\mu + p'^\mu)$, and $\frac{i\sigma^{\mu\nu}}{2m}q_\nu$.
So I can write

$$\Gamma^\mu = F_1(q^2 m^2)\,\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m}q_\nu\,F_2(q^2 m^2)$$

Lowest order $F_1 = 1$, $F_2 = 0$. Recall γ^μ contracts w. A_μ

$\sigma^{\mu\nu}A_\mu q_\nu = S^{\mu\nu}F_{\mu\nu}$ is spin-magnetic coupling.

Aha: F_2 is $\frac{g-2}{2}$ of electron. And it's UV, IR finite!

F_1 related to renormalization of e-charge. But so is Σ_V

Relation between Γ^μ and Σ?? Yes: e-chg does not renormalize except for Π.

(Would be bad: different for each ψ, ϕ field??)

What relation? Ward (Ward-Takahashi) Identity

Consequence of gauge symm, best understood w Path $\int$.