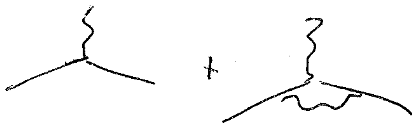
Ward Identity

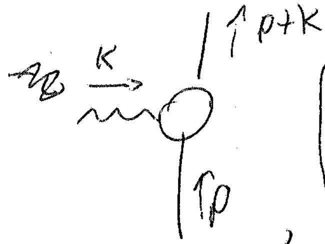
 $= \dfrac{1}{(1-\Sigma_V)\not{p} - m(1-\Sigma_m')} \simeq (1\mp\Sigma_V)^{-1}\dfrac{1}{\not{p}-m_{phys}}$

 $= F_1(q^2; m^2)\,\gamma^\mu + \dfrac{i\sigma^{\mu\nu}}{2m} q_\nu F_2$

Scattering rate  , ... $\underbrace{(1\mp\Sigma_V)^{-1}(1+e^2F_1^{(1)})}$

This is 1 for $q^2\to 0$, I claim. Why? How?

Ward-Takahashi Identity:



$$\boxed{-ik_\mu \Gamma^\mu(p+k,p) = S^{-1}(p+k) - S^{-1}(p)}$$

Lowest order: $-i(\not{p}'-\not{p}) = -i(\not{p}'-m) + i(\not{p}-m)$ yup.

Next order: $-i(\not{p}'-\not{p})F_1 = -i(1-\Sigma_V)(\not{p}'-m) + i(1-\Sigma_V)(\not{p}-m)$

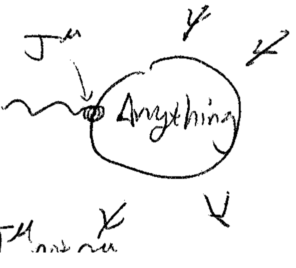
$F_1 = (1-\Sigma_V)$ cool!! If it's true!

Prove it! Prove something much more general

Ward-Takahashi: Consider

$$\int d^4x\, d^4y_1 \dots d^4y_{2n}\, e^{-ik\cdot x}\, e^{-i\Sigma(y_m p_n - y_{m+n} q_n)} \langle 0|T A^\mu(x)\,\psi(y_1)\dots\psi(y_{2n})|0\rangle$$

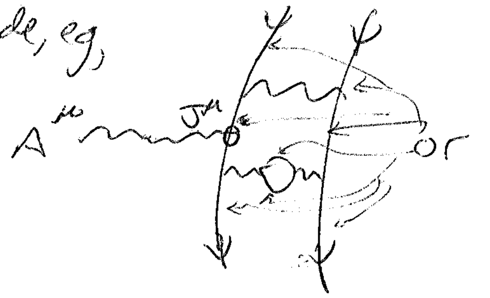
p-space correlator of A^μ with n ψ's, n $\bar\psi$'s



Amputate photon line.

Call it $\mathcal{M}^\mu(k; p_1 \dots p_n; q_1 \dots q_n)$. Really J^μ ...

Individual diagrams include, eg,



many places where A^μ can attach ...

Define $\mathcal{M}_0(p_1 .. p_n ; q_1 .. q_n)$ = diagram w.o. photon. Then

$$k_\mu \mathcal{M}^\mu(k; p_1 .. p_n ; q_1 .. q_n) = e \sum_{i=1}^{n} \Big[\mathcal{M}_0(p_1 .. p_n ; q_1 .. (q_i - k) .. q_n) - \mathcal{M}_0(p_1 .. (p_i + k) .. p_n ; q_1 .. q_n) \Big]$$

Case of just $\bar\psi\psi$, one p and one k, is identity we want.

$$k_\mu \Gamma^\mu(k; p, p' = p - k) = S^{-1}(p) - S^{-1}(p-k) \ ...$$

Note: does NOT say ~~Not~~ $\gamma N\bar N$ func. determined by $N\bar N$ func. Only one component.

Proof: Consider $\int \mathcal{D}(\bar\psi \psi A^\mu)\, e^{i\int [\bar\psi(i\not\partial - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}]}$

$$\psi_1(y_1) ... \psi_n(y_n)\, \bar\psi_1(y_{n+1}) ... \bar\psi_n(y_{2n})$$

Change variables $\psi'(x) = e^{i\alpha(x)}\psi(x)$
$\bar\psi'(x) = e^{-i\alpha(x)}\bar\psi(x)$

Change of var. inside integral - nothing should change.

Linear-in-α change should $= 0 = \int \mathcal{D}(\bar\psi\psi A) \frac{\delta}{\delta\alpha(z)} \Big[\psi_1 .. \psi_n \bar\psi_1 .. \bar\psi_n e^{i\int \bar\psi i\not\partial\psi ...} \Big]$

$$\frac{\delta}{\delta\alpha(z)} \psi(y_m) = i\delta^4(z - y_m) \quad \text{due to } e^{i\alpha}$$

$$\frac{\delta}{\delta\alpha(z)} \bar{\psi}(y_{n+m}) = -i\delta^4(z - y_{n+m}) \quad " \quad e^{-i\alpha}$$

And $\cancel{\text{ef}}$ $\frac{\delta}{\delta\alpha(z)} e^{iS} = i\left(\frac{\delta S}{\delta\alpha(z)}\right) e^{iS} = -\frac{\delta}{\delta\alpha}\left[\bar{\psi} e^{-i\alpha} \gamma^\mu \underbrace{\partial_\mu e^{+i\alpha}}_{i\partial_\mu\alpha} \psi\right] e^{iS}$

$$= -i\,\partial_\mu J^\mu(z) e^{iS}.$$

$$0 = \langle\ \partial_\mu J^\mu(z)\, \psi \cdots \bar{\psi} \cdots - \sum_i \delta^4(y_i - z)\, \psi \cdots \bar{\psi} \cdots + \sum_i \delta^4(y_{mi} - z)\, \psi \cdots \bar{\psi} \cdots\ \rangle$$

Now $\int e^{-ik_\mu z^\mu} d^4z \ldots$

$$\partial_\mu J^\mu \to k_\mu J^\mu. \quad \underbrace{\int d^4z \int d^4y\ e^{-ip\cdot y} e^{-ik\cdot z} \delta(y - z)}_{e^{-ik\cdot y} \text{ is } p \to p+k \ldots}$$