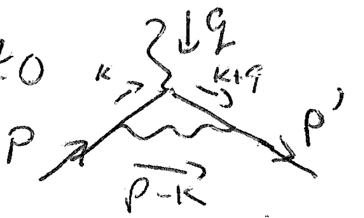
And $g-2$ of electron?

Return to  IR behavior: consider $q \ll m$ or eventually $\lim_{q\to 0}(\dots)$.

$$\bar u(p')\delta\Gamma^\mu u(p) = \int \frac{d^Dk}{(2\pi)^D} \frac{-ig_{\nu\rho}}{(k-p)^2+i\epsilon}\, \bar u(p')(-ie\gamma^\nu) \frac{i(\not k+\not q+m)}{(k+q)^2-m^2+i\epsilon}\gamma^\mu \frac{i(\not k+m)}{k^2-m^2+i\epsilon}(-ie\gamma^\rho) u(p)$$

Combining & simplifying,

$$= ie^2 \int \frac{d^Dk}{(2\pi)^D}\, \bar u(p') \frac{\left[\overset{2}{\cancel{(D-2)}}\, \not k\gamma^\mu(\not k+\not q) + (D-2)m^2\gamma^\mu + (4-D)(\not k+\not q)\gamma^\mu\not k - 4m(k+k')^\mu + (4-D)\big((\not k+\not q)\gamma^\mu+\gamma^\mu(\not k)\big)\right]}{((k-p)^2+i\epsilon)(k^2-m^2+i\epsilon)((k+q)^2-m^2+i\epsilon)}$$

3 Denominators: $\displaystyle \frac{1}{ABC} = \int dx\,dy\,dz\, \frac{\delta(x+y+z-1)}{(Ax+By+Cz)^3}$

Yikes! Focus, $k^2+2k\cdot(yq-zp)+yq^2+zp^2-(x+y)m^2+i\epsilon$

Shift to $\ell = k+yq-zp$ (some average of k, $k+q$, $k-p$.)

Note, $p^2 = m^2 = (p+q)^2$

Denom $= \ell^2 + xyq^2 - (1-z)^2m^2 + i\epsilon$

Numerator: $\displaystyle \int d^D\ell\, \frac{\ell^\mu\ell^\nu}{(\dots)}$ must be $\frac{1}{D}g^{\mu\nu}\ell^2$.

$\displaystyle \int d^D\ell\, \frac{\ell^\mu}{(\dots)}$ must $\to 0$

Boring algebra $\to$ numerator $= \bar u(p')\Big[\gamma^\mu\big(-\tfrac{1}{2}\ell^2 + (1-x)(1-y)q^2 + (1-2z-z^2)m^2\big)$

$+ (p^\mu+p'^\mu)\,mz(z-1)$

$+ q^\mu m(z-2)(x-y)\Big] u(p)$

Note: $\frac{1}{2}$ really has $\mathcal{O}(\epsilon)$ correction. To get γ^μ term right, I need that, as $k^2/(k^2\dots)^3 \to$ log div in 4D or $\frac{1}{\epsilon}$ in D-dim.

Other terms finite

~~Remark: mutor how much do large l con~~

The fun part is of course $(p^\mu + p'^\mu) m z(z-1)$ <0 note

Confusing part is $q^\mu m (z-2)(x-y)$. I thought $q_\mu \Gamma^\mu = 0$??

$$\text{Ah-but } \underbrace{\int dx\,dy\,dz\, \delta(1-x-y-z)}_{x\text{-}y \text{ symm}} \frac{q^\mu m (z-2)\overbrace{(x-y)}^{x\text{-}y \text{ antisymm}}}{\underbrace{(l^2 + xy q^2 - (1-z)^2 m^2)^3}_{x\text{-}y \text{ symm}}}$$

Vanishes on $\int dx\,dy$. Good...

Gordon identity: $p^\mu + p'^\mu \longrightarrow -i\sigma^{\mu\nu} q_\nu + 2m\gamma^\mu$

(Alternatively, $p^\mu + p'^\mu$ is coupling without $S_{\mu\nu} F^{\mu\nu}$ spin-magnetic. And it's negative. So total spin-mag. is extra...)

Let's just go after the $\sigma^{\mu\nu} q_\nu$ term.

$$\bar{u}\,\delta\Gamma^\mu u = 2ie^2 \int \frac{d^D l}{(2\pi)^D} \int dx\,dy\,dz\, \delta(1-x-y-z) \times$$

$$\left(\frac{i\sigma^{\mu\nu} q_\nu}{2m}\right) \bar{u}(p') \frac{4m^2 z(1-z)}{(l^2 - (1-z)^2 m^2 + xyq^2 + i\epsilon)^3} u(p)$$

Hey - this is finite $\int \frac{d^D l}{(l^2)^3}$. Converges by 2 powers of l. I can do it in 4D if I want.

~~Fasto~~

L28P3

Wick rotate

$$\int\frac{d\ell^0}{(\ell^{0\,2}-E)^3}\to i\int\frac{d\ell^0_E}{(-\ell^{0\,2}_E-E)^3}=-i\int\frac{d\ell^0_E}{(\ell_E^2+\ldots)^3}$$

oops

$$2e^2\left(\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right)\bar u(p')4m^2u(p)\int dx\,dy\,dz\,\delta(1-x-y-z)\,z(1-z)\int\frac{d^4\ell_E}{(2\pi)^4}\frac{1}{(\ell_E^2+(1-z)^2m^2)^3}$$

Here I dropped $q^2 \ll m^2$...

$$\frac{1}{(1-z)^2m^2}\int\frac{d^4\ell}{(2\pi)^4(\ell^2+1)^3}=\underbrace{\frac{1}{16\pi^2}\int\frac{z\,dz}{(z+1)^3}}_{\frac{1}{32\pi^2}}\qquad =1/2$$

$$\Gamma^\mu=\gamma^\mu F_1(q^2)+\frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2),$$

$$F_2(q^2\to 0)=\frac{\alpha}{2\pi}\int dx\,dy\,dz\,\delta(1-x-y-z)\,\frac{2m^2z\cancel{(1-z)}}{m^2(1-z)^{\cancel{2}}}$$

$$=\frac{\alpha}{2\pi}\quad \text{wow.}$$

$$\frac{g-2}{2}=\frac{\alpha}{2\pi}=\frac{1}{2\pi\cdot 137.0599\ldots}=0.0011614$$

Experiment: 0.0011597. Close.

In fact, off by α^2...

1-loop Schwinger 1948 $\frac{\alpha}{2\pi}$ 1 diagram

2-loop Petermann Sommerfeld 1957 $\frac{\alpha^2}{\pi^2}\left(\frac{197}{144}+\frac{\pi^2}{12}+\frac{3}{4}\zeta(3)-\frac{1}{2}\pi^2\ln(2)\right)=-.328\ldots\frac{\alpha^2}{\pi^2}$ 5 diagrams

3 loop also analytic (1996) $\frac{\alpha^3}{\pi^3}(\ldots\pi^2\zeta(3)\cdots\zeta(5)\ldots\ln^4 2\ldots)=\left(\frac{\alpha}{\pi}\right)^3(1.18124\ldots)$ Asymptotic series!

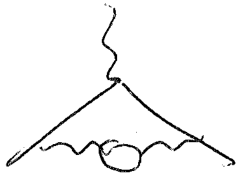
4-loop numerical ~~-1.728~~ $\left(\frac{\alpha}{\pi}\right)^4 \ast -1.9106$ 891 diagrams

5-loop numerical $\left(\frac{\alpha}{\pi}\right)^5 \ast 9.16$ 12672 diagrams

Comparison: $\frac{g-2}{2}\Big|_{thy} = 1.159\,652\,181\,78(77)\times 10^{-3}$ ← error dom. by error in α.

$\phantom{\frac{g-2}{2}}\Big|_{xpt}$ $1.159\,652\,180\,73(28)\times 10^{-3}$

Note: at 2-loops



μ, quark, ...

We saw $\int \frac{d^D k}{(k^2)^3} \sim \int \frac{dk}{k^3}$

Modified where $|k^2| > m_\mu^2$.

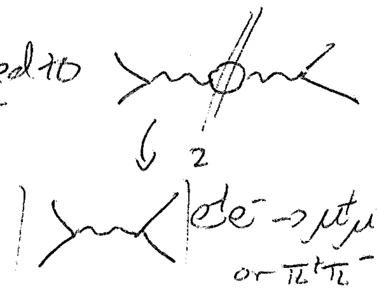
Effect suppressed by $\left(\frac{\alpha}{\pi}\right)^2$ and $\left(\frac{m_e}{m_\mu}\right)^2$.

Not negligible but not so bad.

quark: $\underbrace{\left(\frac{\alpha}{\pi}\right)^2}_{\sim 10^{-6}} \underbrace{\left(\frac{m_e}{m_\pi}\right)^2}_{\sim 10^{-5}}$ suppressed → $1{,}87\times 10^{-12}$. New problem - how do you compute?

Cleverness: $\int \frac{d^2 k_E}{(\dots)}\, \Pi(k_E)$

↳ related to

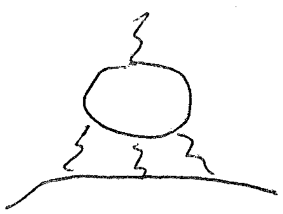


Experimental input!

$e^+e^- \to \mu^+\mu^-$ or $\pi^+\pi^-$ or ...

Dispersion relation

And at 3-loops



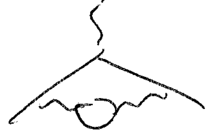
(why not  ??)

for π - no good way to compute (lattice? In development)

But $\frac{\alpha}{\pi}$ smaller.

Muon

now



suppressed by $\left(\frac{m_\mu}{m_\pi}\right)^2$ not $\left(\frac{m_e}{m_\pi}\right)^2$

so, $\left(\frac{m_\mu}{m_e}\right)^2 = 40{,}000\times$ more sensitive.

electron $g-2$: tool to test QED

muon $g-2$: tool to probe (new?) physics.