Suppose I had some tool to compute correlation functions.

One type, say, $\langle 0 | T(\varphi_1(x_1) \ldots \varphi_n(x_n)) | 0 \rangle$, is enough - analyticity relates to all others.

What can I learn from these?

- 2-pt func. tells us if $\varphi|0\rangle$ contains particles.
- If it does, these correl. functions will tell us everything about how particles behave - how they scatter with each other.

We will see this in the next 2 lectures.

~~How~~
- What fields φ are possible?
- What Lagrangians can describe them
- Given φ's and $\mathcal{L}(\varphi, \partial_\mu \varphi)$, How do I compute the correlation functions

Remainder of 2-semester course sequence!!

Questions I want to answer:

Suppose I start with n particles flying at each other. Well separated initially. Rather well defined momenta.

Look much later. What is probability [|Amplitude|^2]

to find n particles flying apart, w. some other momenta?

"Scattering Problem"

Step 1: How do I describe initial state?

Assume I have $\hat{\varphi}_a$ a set of field operators, for today we make it 1 field $\hat{\varphi}$ so notation simpler.

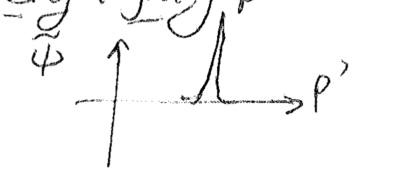
Suppose $\rho_{\varphi\varphi}(s) =$

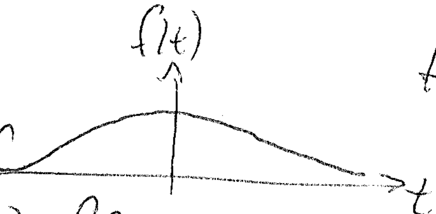
$Z\, 2\pi i\, \delta(s-m^2)$

Then $\varphi(x)|0\rangle$ produces $\sqrt{Z}$ times particles, plus some junk.

Choose your wave packet function $\psi(x)$

normalize so $\int \frac{d^3p'}{(2\pi)^3 2E_{p'}} \tilde{\psi}^*(p')\tilde{\psi}(p') = 1$ and $\tilde{\psi}(p')$ very tightly peaked at $p'=p$



now choose $f(t)$ of form 

wide t-range with (note!!) $\int f(t')dt' = 1$ not f^2!!

Choose $\psi(x^\mu) = \psi(\vec{x}) e^{-i\omega_p t} f(t)$

The idea: using $e^{-i\omega_p t} f(t)$ makes me coherently add amplitude to make particles, but incoherently cancel amplitude to make other stuff. Let's see that.

Define $$|\psi\rangle = \frac{1}{\sqrt{Z}} \int d^4x\, \psi(x^\mu)\, \varphi(x^\mu)|0\rangle \left(= \int d^4p'\, \psi(-p')\varphi(p')|0\rangle \right)$$

$$\langle\psi|\psi\rangle = \frac{1}{Z}\int d^4x\, d^4y\, \psi^*(y)\psi(x)\, \langle 0|\varphi^*(y)\varphi(x)|0\rangle$$

$$\frac{1}{Z}\int \frac{ds}{2\pi}\, \rho_{\varphi\varphi}(s) \int \frac{d^4k}{(2\pi)^4}\, 2\pi\delta(k^2-s)\,\Theta(k^0)\, e^{-ik\cdot(y-x)}$$

But $\int d^3y\, \psi^*(y) e^{+i\vec{k}\cdot\vec{y}} = \left(\tilde{\psi}^*(\vec{k})\right)^*$, $\int d^3x\, \psi(x) e^{-i\vec{k}\cdot\vec{x}} = \tilde{\psi}(\vec{k})$

$$\cancel{\langle\psi|\psi\rangle = \int \frac{ds}{2\pi}\, \rho_{\varphi\varphi}(s) \underbrace{\int \frac{d^4k}{(2\pi)^4}\, \Theta(k^0)\, 2\pi\delta(k^2-s)}_{\int \frac{dk^0}{2\pi}\Theta(k^0)\, 2\pi\delta(k_0^2-p^2-s)}\, \tilde{\psi}^*\tilde{\psi}(k)}$$

$$\langle\psi|\psi\rangle = \int \frac{ds}{2\pi}\, \frac{\rho_{\varphi\varphi}(s)}{Z} \underbrace{\int \frac{d^3k}{(2\pi)^3}\, \tilde{\psi}^*\tilde{\psi}(\vec{k})}_{\text{forces } \vec{k}=\vec{p} \text{ and gives } 2\omega_p} \wedge \int dt_1 dt_2\, e^{i\omega_p t_1} e^{-i\omega_p t_2} e^{-ik^0(t_1-t_2)} f(t_1)f(t_2)$$

$$\int \frac{dk^0}{2\pi}\, 2\pi\delta(k_0^2 - \vec{k}^2 - s)\,\Theta(k^0)$$

$$\langle\psi|\psi\rangle = \int_0^\infty ds\, \frac{\rho(s)}{Z}\, 2\omega_p \overbrace{\int dk^0\, \Theta(k^0)\, \delta(k_0^2 - \vec{p}^2 - s)}^{k^0 \to \sqrt{\vec{p}^2+s}}$$

$$\times \int dt_1 dt_2\, e^{it_1(k^0-\omega_p)}\, e^{it_2(\omega_p - k^0)}\, f(t_1) f(t_2)$$

$$= \int_0^\infty ds\, \frac{\rho(s)}{Z} \left| \int dt\, f(t)\, e^{it(\sqrt{\vec{p}^2+s} - \sqrt{\vec{p}^2+m^2})} \right|^2$$

If $s = m^2$ this is 1.

~~If $\sqrt{s} - m$ is $>$ width of^{-1}~~

If $e^{it(..)}$ varies significantly over range of $f(t)$, this phase-cancels.

For instance, if $f(t) = \frac{e^{-t^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$ ($\int f\, dt = 1$ ✓)

then we get $e^{-\sigma^2(\sqrt{p^2+s} - \sqrt{p^2+m^2})^2/2}$ from integral

Kills everything but δ-function

$\int ds \frac{\rho(s)}{Z} = 1$ and $s \to m^2$ where δ-func. occurs.

My wave packet gives me particle only, with momentum $\simeq p$.

Make particle at location x_1: $\frac{1}{\sqrt{Z}} \int d^4x\, \psi(x - x_1)\, \varphi(x)\, |0\rangle$

Make more particles:

$$\frac{1}{Z^{n/2}} \int d^4y_1\, d^4y_2 \ldots d^4y_n\, \psi_{p_1}(y_1 - x_1) \ldots \psi_{p_n}(y_n - x_n)$$
$$\varphi(y_1) \ldots \varphi(y_n)\, |0\rangle$$

works if $x_1, \ldots x_n$ spacelike separated by more than width of wave packets.

Physically

$$\int d^4y \;\; \psi_p(y - x_1)\, \varphi(y)\, |0\rangle$$

builds

wavepacket: phase structure makes it a traveling wave w. momentum p (approx) at loc. x_1

$$\int d^4y_1\, d^4y_2 \;\psi_{p_1}(y_1 - x_1)\, \psi_{p_2}(y_2 - x_2)\; \varphi(y_1)\, \varphi(y_2)\, |0\rangle$$

packets of well separated particles.

evolution: they will fly at each other until packets overlap. And then ...... Good Question!!

Call a state built this way an "in" state $|\psi_{in}\rangle$

I can do same thing - wide separated, deep-future outward-moving particles - to build "out" state. $\langle\psi_{out}|$

Difference: locations, momenta (x_i, p_i) of $|\psi_{in}\rangle$ chosen to be wider - separated in past. Overlap after state built.

For (x_f, p_f) of $|\psi_{out}\rangle$, choose such that future evol. will take ever farther apart.

$$\text{"S-matrix"} = \langle \psi_{out} | \psi_{in} \rangle = \langle \psi_{out} |\, \mathcal{U}(t_f - t_i)\, | \psi_{in} \rangle$$

if you wanna be explicit about time evolution.