Theoretische Physik I: Klassische Mechanik - Präsenzübung

Prof. Dr. Guy Moore

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Übungsblatt 10

Aufgabe 10.1: Hockey on a hill

Let's walk through the very simplest example of constrained motion, an object sliding without friction down an inclined plane.

Consider a hockey puck on a tilted ice rink. There are two coordinates, x and z (horizontal distance and height). The height depends on x through the constraint

$$z = x\tan(\alpha).$$

The puck moves under the influence of gravity with strength $F_g = -mg\hat{e}_z$ and the normal force caused by the constraint.

10.1a)

Write the Lagrange-II Lagrangian for x.

10.1b)

Use it to find the equation of motion. This should look like motion under gravity, but with the wrong strength of gravity: $g \to gf(\alpha)$. What is the function of α?

10.1c)

Write the Lagrange-I Lagrangian, with Lagrange multiplier λ.

10.1d)

Write the separate equations of motion for $\ddot{x}$ and for $\ddot{z}$, in terms of the (not-yet-determined) λ value.

10.1e)

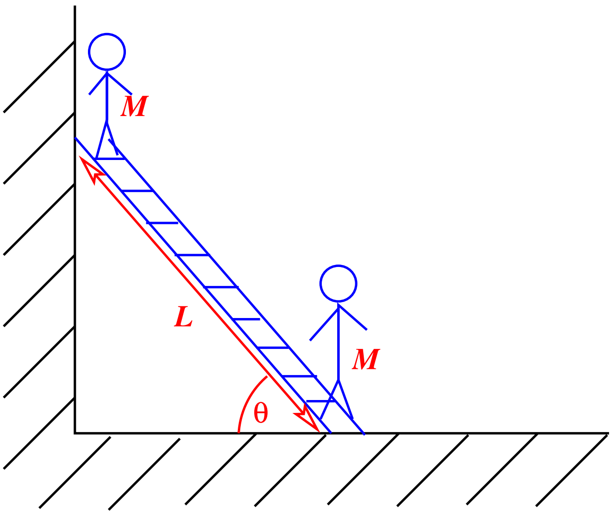
Take the second time derivative of the constraint equation $z = x\tan(\alpha)$, remembering that α is a constant. Use the result to replace $\ddot{z}$ with $\ddot{x}$ in one of your equations of motion, and use the resulting equations to determine λ.

10.1f)

Recalling that $\ddot{x} = F_x/m$ and $\ddot{z} = F_z/m$, determine the normal force components F_x and F_z. Is the normal force at right angles to the surface?

Aufgabe 10.2: Physics on a ladder

Consider the ladder leaning on a wall, discussed in the lecture.



The Lagrange-II Lagrangian is:

$$L(\theta) = \frac{ML^2\dot{\theta}^2}{2} - MgL\sin(\theta)$$

We found in lecture that the normal force on the wall is

$$-\lambda_x = -\frac{d}{dt}\left(mL\sin(\theta)\dot{\theta}\right)$$

but we did not evaluate this or find when the ladder leaves the wall. Let's do so now.

10.2a)

Find the Euler-Lagrange equation for θ.

10.2b)

Rather than solve this, we will use energy methods. Write an expression for the system energy $T + V$. If the ladder starts at an angle θ_0 with zero kinetic energy, what is the starting energy?

Use this result to find an expression for $\dot{\theta}$ as a function of θ and θ_0.

10.2c)

Consider the case $\theta_0 = \pi/2$ so the ladder starts out up against the wall. Use your expression for $\dot{\theta}$ to rewrite $(mL\sin(\theta)\dot{\theta})$ as a function of θ alone.

10.2d)

What θ value maximizes this function? Show that this is the θ value at which the ladder leaves the wall.