Theoretische Physik I: Klassische Mechanik - Präsenzübung

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Übungsblatt 11

Aufgabe 11.1: About waves.

Assume that the density of air is $\rho_0(x,t) + q(x,t)$ where ρ_0 is the average density and $q(x,t)$ is some extra over- or underdensity. Such an over- or underdensity propagates according to the wave equation

$$\frac{\partial^2}{\partial t^2} q(x,t) = c_s^2 \frac{\partial^2}{\partial x^2} q(x,t) \tag{11.1.1}$$

with c_s the speed of sound.

11.1a)

Show that, for two arbitrary functions of one variable $q_1(z)$ and $q_2(z)$, that $q(x,t) = q_1(x - c_s t) + q_2(x + c_s t)$ is a solution to the wave equation.

11.1b)

Consider the "triangle function"

$$q_t(z) = \begin{cases} 0 & z < -1 \\ 1 + z & -1 < z < 0 \\ 1 - z & 0 < z < 1 \\ 0 & z > 1 \end{cases} \tag{11.1.2}$$

Draw this function. Assume that $q(x,t) = q_t(x - c_s t)$. Draw $q(x, t = 0)$ and $q(x, t = 1)$ for the case $c_s = 2$ (that is, make a plot of q vs x for each time.)

11.1c)

For the solution we just found, what is $\dot{q}(x, t = 0)$ (where the dot indicates derivative with respect to time)? Plot it.

11.1d)

Now take the same function q_t but use it for q_2, that is, $q(x,t) = q_t(x + c_s t)$. Find $\dot{q}(x, t = 0)$ and $q(x, t = 1)$. Plot them.

11.1e)

Suppose that initially $q(x, t = 0) = q_t(x)$ but $\dot{q}(x, t = 0) = 0$. Write the complete time dependent solution for $q(x,t)$ and draw it for the times $t = 0$, $t = 1$, and $t = 2$.

11.1f)

If time permits, suppose that $q(x, t = 0) = 0$ but $\dot{q}(x, t = 0) = \frac{\mathrm{d}q_t(x)}{\mathrm{d}x}$. Now what is $q(x, t)$? Draw the result at $t = 0$, $t = 1$, and $t = 2$.