Theoretische Physik I: Klassische Mechanik - Präsenzübung

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Übungsblatt 13

Aufgabe 13.1: Light from supernova.

A supernova goes off 20,000 light years from the Earth. The light from the supernova travels to Earth at the speed of light.

13.1a)

How long does the light travel?

13.1b)

How much proper time does the light itself experience on its way?

13.1c)

Radio waves are slowed down by the galactic plasma. The 700 Mhz radio signal from the supernova arrives 0.01 seconds later than the light, despite leaving at the same time. How much proper time does the radio wave experience?

Aufgabe 13.2: Force in Special Relativity.

Consider a particle of mass m, moving with velocity $\vec{v} = v\,\vec{e}_x$ in the x direction, relative to Alice, our observer. Bob moves in the same frame of reference as the particle.

Force is defined in Special Relativity in the same way that it is defined in nonrelativistic mechanics: the time rate of change of the momentum

$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}. \tag{13.2.1}$$

In this problem we will explore the relation between force and acceleration, and the relation between force and acceleration in various frames.

First consider a force acting in the $+x$ direction. It will be easiest to work from Bob's point of view, moving along with the particle (initially). Bob observes that at time $t = 0$ the velocity is $\vec{v} = 0$, and at time $\mathrm{d}t$ the velocity is $\vec{v} = \mathrm{d}v\,\vec{e}_x$. Here $\mathrm{d}t$ and $\mathrm{d}v$ are very small (infinitesimal) changes.

From Bob's (the particle's) point of view,

13.2a)

What is the acceleration?

13.2b)

What is the final value of the momentum, $\mathrm{d}p$? Work to linear order in $\mathrm{d}v$.

13.2c)

What is the force? What is $\mathrm{d}p\,/\,\mathrm{d}v$?

Find the Lorentz transformation matrix to change time and momentum to Alice's reference frame. From Alice's point of view,

13.2d)

What is the final time for Alice? It is not $\mathrm{d}t$!

13.2e)

What is the final velocity? (Use the addition of velocities and expand to linear order in $\mathrm{d}v$). From these two answers, what is the acceleration according to Alice?

13.2f)

What is the final momentum? (Use the Lorentz transform matrix on $P^\mu = (E, cp_x, cp_y, cp_z)$, and plugging in the correct values for Bob!). What is the force acting on the particle?

Your answers to e) and f) give the relation between acceleration and force when going from Bob's to Alice's frames.

Let's repeat that for a force acting in the y direction. Bob observes that at time $t = 0$ the velocity is $\vec{v} = 0$, and at time $t = \mathrm{d}t$ the velocity is $\vec{v} = \mathrm{d}v\,\vec{e}_y$. From Bob's point of view the acceleration and force are the same as in the first part, but in the y-direction.

13.2g)

What is the final momentum according to Alice? Use the Lorentz transform matrix. From this, find the force on the particle in Alice's frame of reference.

We have not learned yet how to add velocities which are not parallel. So to find the acceleration in Alice's frame, we have to first learn how to add the velocities. To see how to do that, consider a particle which moves at a constant velocity $\vec{v} = v\vec{e}_y$ in Bob's frame of reference. At time $t = 0$ it has $x^\mu = (0, 0, 0, 0)$ and at time $t = \mathrm{d}t$ it has $x^\mu = (c\,\mathrm{d}t, 0, v\,\mathrm{d}t, 0)$. Transform this into Alice's frame and use that $\vec{v} = \vec{x}/t$ to find the full velocity.

13.2h)

What is v_y in Alice's frame? What is the acceleration according to Alice?