Theoretische Physik I: Klassische Mechanik - Präsenzübung

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Übungsblatt 1

Aufgabe 1.1: The ball is moving

Consider a football, with a mass $m = 1$ kg, which is spherical. For this problem, approximate it as a point-like object.

Imagine that it starts at the point $\vec{r}_i = (0, 0, 0)$ and it is kicked with an initial velocity $\vec{v} = (10, 0, 10) = 10\hat{e}_x + 10\hat{e}_z$, that is, 10 m/s in the forward (x) direction and 10 m/s in the vertical (z) direction. It flies under the influence of gravity, which exerts a downward force, giving rise to an acceleration $g = -10\hat{e}_z\ \mathrm{m/s}^2$.

1.1a)

Show that the ball flies for 2 seconds and lands at the point $\vec{r}_f = (20, 0, 0)$. This should be easy.

1.1b)

Write an expression for the potential energy $V(\vec{r})$ and for the kinetic energy $T(\vec{v})$. This should also be easy.

1.1c)

Consider three hypothetical paths the ball could take:
A) the true path, $\vec{r}(t) = (10t, 0, 10t - 5t^2)$
B) the straight-line path $\vec{r}(t) = (10t, 0, 0)$
C) the circular path $\vec{r}(t) = \left(10 - 10\cos\left(\frac{\pi t}{2}\right), 0, 10\sin\left(\frac{\pi t}{2}\right)\right)$.

Each path is chosen to start at the same point and end at the same point as the true path, and to take the same amount of time. But in between, they differ from the true path.

For each path, calculate the time-integrated value of the potential energy

$$\int_0^{t_f=2} V(\vec{r}(t))\, dt$$

and the time-integrated value of the kinetic energy

$$\int_0^{t_f=2} T(\dot{\vec{r}}(t))\, dt.$$

Which has the largest/middle/smallest average potential energy?
Which has the largest/middle/smallest average kinetic energy?

1.1d)

Consider the special combination of kinetic and potential energy:

$$S \equiv \int_{t=0}^{t=t_f=2} (T - V)\, dt\,.$$

(The combination $\mathcal{L} = T - V$ is called the *Lagrangian*, and its time integral S is called the *action*. We will have much more to say about each of these in the future.)

Show that the action is smaller for the true path than for either of the two hypothetical paths, even though one of the paths has a smaller integral of T and another has a smaller integral of $-V$.

1.1e)

Time permitting:
Build your own hypothetical path, and test whether you can find one which has an action S smaller than that for the true path.

1.1f)

Time permitting:
Explain/prove that there are paths where $\int -V dt$ is as small as you want. Show why these tend to have large values for $\int T dt$, and therefore they do not lead to small S values.
Similarly, show that there is a path with a minimum $\int T dt$. What path has this property? Why does it have a larger value of S than the true path?