Theoretische Physik I: Klassische Mechanik - Präsenzübung

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Übungsblatt 2

Aufgabe 2.1: Rolling and rolling

Consider a sphere, rolling without slipping on a flat sheet. The sphere has a radius R. It may help a great deal to get spheres – for instance, balloons or balls – to perform this exercise. You may also want to draw on the spheres.

One constraint is that the center of the sphere remains a height R above the sheet. This is a holonomic constraint. Another constraint is that there is no slipping at the point of contact. This constraint is not holonomic – if we “go around in a loop” we do not come back to the same configuration we started with. That is what we want to explore.

2.1a)

Assume that the sphere starts with its south pole touching the plane. It rolls a distance a forward in the x direction. What point is now touching, and how far is that point (on the sphere) from the south pole?

2.1b)

Let us choose $a \ll R$. After rolling a distance a in the x-direction, have the sphere roll a distance a in the y direction, then $-a$ in the x direction, then $-a$ in the y direction. Show that, at linear order in a, the south pole is back on the bottom. (Hint: imagine drawing a map of the area around the south pole. Show where the contact point is moving around on the map.)

2.1c)

But now consider $a = \pi R/2$. After the ball rolls a in the x direction, what is the point of contact? Next, it rolls a distance a in the y-direction. Now where on the sphere is the point of contact? (Be careful that the sphere does not “twirl” or turn in place. It only rolls!) Next it rolls a in the $-x$ direction. Now where is the contact? Finally, it rolls back a in the $-y$ direction, coming back to where it started. What is the change in the point of contact? What is the change in the orientation of the sphere?

Aufgabe 2.2: Swinging

Consider someone sitting in a swing. The ropes are inextensible and of length L. The person in the swing has a mass of m.

The swing is moving in the $x-z-$plane. The origin of the coordinate system lies in the suspension point of the swing and the z-axis is pointing upwards. We denote the swinging angle in the $x-z-$plane with θ.

First, suppose the person is sitting at the bottom, $\theta = 0$. Show that the normal force from the ropes equals the force of gravity and that the normal force is directed upwards.

Next, suppose the person is swinging back and forth. At the moment $t = 0$, $\theta = 0$ but $\dot{\theta} \neq 0$. Our goal is to calculate the normal force at this moment.

Write the constraint for the person's coordinates. You should find $\sqrt{x^2+z^2} - L = 0$.

Next, write x and z in terms of L and θ. Next, make a small angle approximation about $\theta = 0$, that is, approximate $\cos(\theta) = 1 - \theta^2/2$ and $\sin(\theta) = \theta$. Calculate $\ddot{z}$ the acceleration in the z direction at the moment $t = 0$ (the moment when $\theta = 0$ but $\dot{\theta} \neq 0$). You should find that $\ddot{z}$ is *not* zero.

Given the acceleration you find and the force of gravity, what must be the normal force due to the swing ropes? Is it larger or smaller than the value when the swing is sitting still? Can you explain the answer you find somehow? (For instance, is there a name for the effect which is responsible?)