

Theoretische Physik I: Klassische Mechanik - Präsenzübung

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Übungsblatt 3

Aufgabe 3.1: The Earth moves!

Consider motion in the x, y plane, representing the equatorial plane of the Earth. Because the Earth is round, it makes sense to change to ρ, φ as variables. But because the Earth turns, it makes sense to take a different definition of φ than usual:

$$\rho = \sqrt{x^2 + y^2} \quad (3.1.1)$$

and

$$\varphi = \text{atan}\left(\frac{y}{x}\right) - \Omega t \quad (3.1.2)$$

where Ω is the angular velocity of the Earth's rotation. Call the two original coordinates $(x, y) = (r_1, r_2)$ and the two new coordinates $(q_1, q_2) = (\rho, \varphi)$.

3.1a)

Express (r_1, r_2) as functions of (q_1, q_2, t) .

3.1b)

Compute $\partial r_i / \partial q_j$ for i, j each 1, 2 (4 terms in all)

3.1c)

Express \dot{r}_1 and \dot{r}_2 in terms of $q_1, q_2, \dot{q}_1, \dot{q}_2, t$.

3.1d)

Compute $\partial \dot{r}_i / \partial \dot{q}_j$ and show that it equals your result in part b.

3.1e)

The kinetic energy is

$$T = \frac{m}{2} (\dot{x}^2 + \dot{y}^2). \quad (3.1.3)$$

Use your results above to rewrite this in terms of the q and \dot{q} variables.