Theoretische Physik I: Klassische Mechanik - Übungsblatt

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TECHNISCHE UNIVERSITÄT DARMSTADT

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Übungsblatt 10

Deadline: 24.06. 23 Uhr online

Aufgabe 10.1: A particle in a cylinder. 7p. + 3p. Bonus

A small mass m rests on a cylinder, laid on its side. That is, the mass's motion in the (x, y) plane is constrained to obey $x^2 + y^2 \geq R^2$ with R the radius of the cylinder. Here y is the vertical direction. The mass begins almost at rest on the top of the cylinder, $(x, y) = (0, R)$, and slides down without friction. (You can completely ignore motion in the z direction.)

10.1a)

(1p) Write the problem in cylindrical coordinates. Assuming that the mass must stay on the cylinder, what is the Lagrangian using the Lagrange II formulation?

10.1b)

(1p) Find the equation of motion and the velocity as a function of the angle.

10.1c)

(2p) Write the problem (still in cylinder coordinates) in the Lagrange I formalism. Show that the equation of motion for the angular coordinate is the same.

10.1d)

(3p) Calculate the normal force on the mass. At what angle does the normal force become zero? At this point, the mass will leave the surface of the cylinder.

10.1e)

(Bonus 3p) Suppose that we replace the mass with a small sphere. The radius of the sphere r_b is much smaller than R. But the sphere rolls without slipping, meaning that it spins as it rolls down. Calculate the moment of inertia of the sphere, and recompute the Lagrange function, taking into account the kinetic energy associated with the sphere rotating. At what angle does the sphere leave the surface – is it the same, higher up, or lower down than the mass which slides without friction?

Aufgabe 10.2: A swing. 3p +3p Bonus

A girl swings on a very long swing, with ropes of length L. The girl's mass is M and she has a negligible moment of inertia. The strength of gravity is g.

10.2a)

(3p) Write the Lagrangian for this system in the Lagrange I formalism, using the angle θ and the length of the rope r as coordinates. If the girl swings back and forth with a maximum angle θ_0, calculate the normal force (tension) in the ropes as a function of the angle θ.

10.2b)

(Extra 3p) Usually one swings higher by leaning forward and backward. But the girl has another way. She stands up on the swing. At the top of each swing, she squats down, and at the bottom, she stands up. This effectively changes the length of the rope by ΔL (how far her center of mass changes). Remembering that work is $W = F\Delta L$, how much more work does she do, standing up, than she absorbs, squatting down? How much does the angle increase each time she does this? You may approximate $\Delta L \ll L$.