Theoretische Physik I: Klassische Mechanik - Übungsblatt

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Übungsblatt 11

Deadline: 01.07 23 Uhr online

Aufgabe 11.1: Double pendulum, again. 9p.

Consider the double pendulum: the top pendulum has a string of length ℓ_1 and bob of mass m_1, the pendulum hanging from it has a string of length ℓ_2 and bob of mass m_2. They hang under gravity, $V = mgz$ for each mass.

11.1a) 1p

Write the Lagrangian for the double pendulum.

11.1b) 2p

Simplify the Lagrangian: in the potential energy, approximate $\cos(\theta) = 1 - \theta^2/2$ and in the kinetic energy approximate $\cos(\theta_1 - \theta_2) \simeq 1$. Show that, under these approximations, the problem now looks like coupled harmonic oscillators.

11.1c) 2p

Write the Lagrange function in the standard form

$$L = \sum_{i,j} \frac{1}{2}\dot{\theta}_i T_{ij} \dot{\theta}_j - \frac{1}{2}\theta_i V_{ij} \theta_j. \qquad (11.1.1)$$

What are the matrices T_{ij} and V_{ij}?

11.1d) 2p

Solve the eigenvalue problem for $T^{-1}V$.

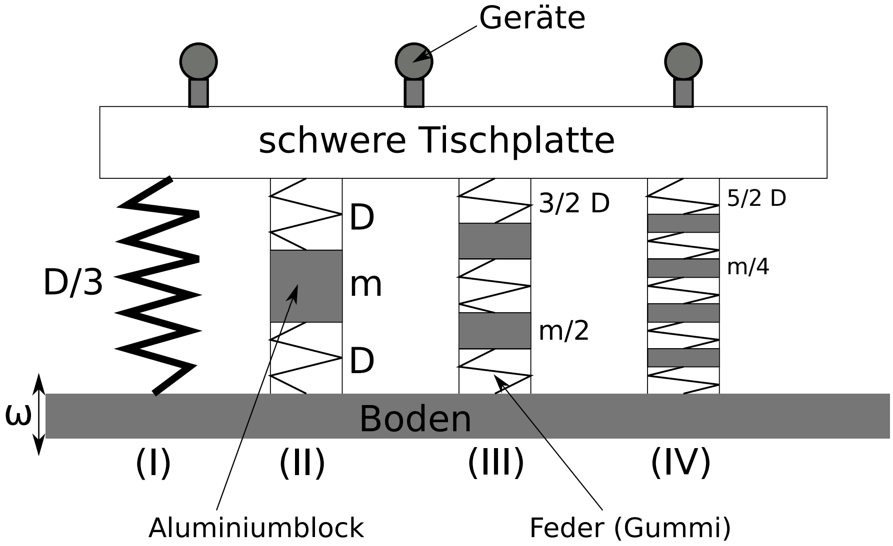
11.1e) 2p

If $\ell_1 = \ell_2 = \ell$ and $m_1 = m_2 = m$, what are the eigenvalues and eigenfrequencies of the pendulum? Sketch each.

Aufgabe 11.2: Optical table. 11p.

An optical table is a table for building sensitive experimental apparatus. The key property of a good table is that it prevents vibrations from the floor from shaking the top of the table.

Suppose the floor is vibrating at frequency ω. The tabletop has a mass of M, and rather than stiff legs, we use a spring of spring constant $D/3$ (we will see why $D/3$ in a moment) connecting the table to the floor. Consider the spring to be massless.

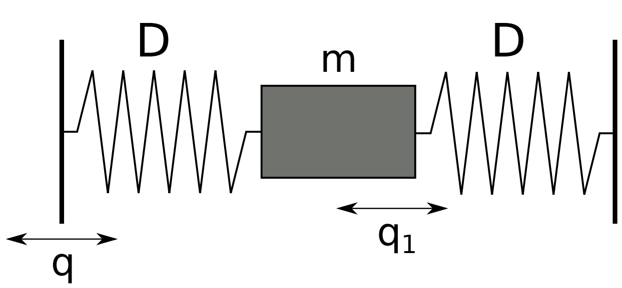


11.2a) 2p

What is the force on the table, if the floor oscillates with frequency ω and amplitude q? Assume here and throughout that the table is so heavy that it does not move. We are just trying to determine the force on the table via the spring, if the floor rises and falls with the given frequency and amplitude. This should be relatively easy and should not require a calculation.

11.2b) 3p

To improve the isolation, we replace the middle third of the spring with an aluminium block of mass m. This takes up 1/3 of the length of the table leg; so now the table leg is a spring of constant D, a block of mass m, and another spring of constant D. A simplified version is shown in the figure. Approximating the table to be motionless and the floor to oscillate with amplitude q and frequency ω (you may assume that $q(t) = q\cos(\omega t)$ describes the oscillation of the floor), find the amplitude of the mass's motion q_1. You may as well just study the steady-state solution (i.e. assume that $q_1(t)$ looks like $q(t)$). From this, determine the force on the table. If $\omega \gg \omega_0 \equiv \sqrt{D/m}$, is this smaller than in part a?



11.2c) 3p

That worked so well that we now use 3 springs of constant $3D/2$ and two masses of mass $m/2$. Recalculate the force on the table in this configuration.

11.2d) 3p

What if we use four masses of mass $m/4$ and five springs of spring constant $5D/2$? If $\omega = 20\omega_0$, compute the force for each system we have described (as a function of D and q).