Theoretische Physik I: Klassische Mechanik - Übungsblatt

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TECHNISCHE UNIVERSITÄT DARMSTADT

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Übungsblatt 13

Deadline: 15.07. 23 Uhr online

Aufgabe 13.1: Rocket. 3p.

We want to send a spaceship with one astronaut to a distant star. The spaceship, astronaut, and all supplies weighs 20 tons (20,000 kg).

13.1a)

If we use the entire yearly energy output of Germany, 1.3×10^{19} Joules, to contribute to the kinetic energy of the rocket ship, what velocity can it achieve?
Assume that we are perfectly efficiently converting energy into kinetic energy of the rocket ship.

Aufgabe 13.2: An electron on the LHC. 8p + extra 6p.

The LHC has a circular ring which is 27km in circumference. Electrons or protons can circle around in the ring. The main limitation to keeping them going in a circle is the strength of the magnetic field, needed to apply the centripetal acceleration. A proton weighs 1.6726×10^{-27} kg. I want to accelerate it to an energy of "7 TeV" which in regular people units is 1.1215×10^{-6} Joule. This is the *total* energy of the proton as it circles in the ring.

13.2a) 2p

What is the γ-factor of the proton? What is the value of $\beta = v/c$?

13.2b) 3p

What is the momentum carried by the proton?

13.2c) 3p

What is the radial force which must be exerted to keep the proton in the ring? Hint: $\mathrm{d}p/\mathrm{d}t = \omega p$ where ω is the angular frequency with which it goes around in a circle.

Extra credit questions:

13.2d) 3p

In the proton's own frame, what is the force which it feels?

13.2e) 3p

Suppose we use an electron instead of a proton. It is accelerated to achieve the same energy. Show that the answers to b) and c) are almost the same, but the answers to a) and d) are very different. [The different answer for d) turns out to be the reason we cannot accelerate electrons to the same energy as protons in the LHC.]

Aufgabe 13.3: What about the Hamiltonian? 5p.

The relation between energy and momentum is

$$E^2 = |\vec{p}|^2 c^2 + m^2 c^4 \qquad \text{or} \qquad E = \sqrt{m^2 c^4 + |\vec{p}|^2 c^2} \tag{13.3.1}$$

13.3a) 2p

Expand this relation to fourth order in the momentum p. Add a potential energy $V(x)$, and use this to write a Hamiltonian which includes the first relativistic corrections to the usual Hamiltonian. Your result should be of form:

$$H = mc^2 + \frac{|\vec{p}|^2}{2m} + A\left(|\vec{p}|^4\right) + V(\vec{x}) \tag{13.3.2}$$

with A some coefficient which you will calculate.

13.3b) 3p

Derive Hamilton's equations from this Hamiltonian.