Theoretische Physik I: Klassische Mechanik - Übungsblatt

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Sommersemester 2022
Übungsblatt 1

Deadline: 22.04 11:30 Uhr

Aufgabe 1.1: A simple spring

This problem should be easy. If you find it difficult, you are making it harder than it is.

Consider a small ball with mass $M = 1\,\mathrm{kg}$, suspended from a spring. The energy depends on the vertical position of the ball, z, through $V = \frac{K}{2}(z - z_0)^2$, where $K = 1\ \mathrm{J/m^2}$ is the spring constant. The ball only moves vertically (in the z direction) in this problem.

1.1a)

Starting with Newton's second law, and using $F = -dV/dz$, find a second-order differential equation for the motion of the ball.

1.1b)

If the ball has an initial position z and velocity v, what is its position as a function of time?

Aufgabe 1.2: A goalie kicking a goalie kick

This problem should also be easy, but the later parts may take a moment's thought.

The goalie kicks a goalie kick from a point 11 meters in front of the center of the goal. He kicks it upwards and straight forward, with velocity components v_z (vertical component) and v_h (horizontal component). The ball is affected by gravity, with strength $g = 10\,\mathrm{m/s^2}$, and you should ignore air resistance. Ignore the physical size of the ball for this problem (take it to be a point). Assume that, after kicking the ball, the goalie stands in place / stays put.

Calculate the ball's trajectory $\vec{r}(t)$...

1.2a)

In the coordinate system where z is the vertical direction starting at ground level, x is the forward direction (as seen by the goalie), and the origin (Ursprung) is at the goal line at the center of the goal.

1.2b)

In the coordinate system where the axes point in the same directions, but the origin is at the field's corner. (The field is 70 meters wide.)

1.2c)

In the coordinate system where x is the distance east of the field's corner and y is the distance north of the field's corner. The long edge of the field extends southeast from the corner, and the short side (with the goal on it) extends northeast from the corner.

1.2d)

In the frame of reference (Bezugssystem) where, *initially*, the origin is at the goal line at the center of the goal, **but** the ball is initially at rest and the *goalie* is moving with velocity components $-v_z$ and $-v_h$. In this frame, describe *both* the trajectory of the ball, *and* the trajectory of the goalie. Show that the x and z separation of the ball from the goalie evolves just like in a), but the interpretation is different.

Aufgabe 1.3: Rethinking the starting conditions 5p

In this problem we see that we can solve a problem based on the initial and final position, rather than the initial position and velocity. You *may* find it simple, or you *may* find it tricky – I am not sure.

Consider problem 2, part 1. But instead of knowing the initial velocity of the ball, suppose you instead know that the ball lands 4 seconds after being kicked, a distance 64 meters in front of the point where it was kicked.

Leave v_z and v_h as unknowns. Solve for the ball's trajectory, to write $\vec{r}(t)$. Then use the final position data to determine the initial velocities, and therefore the full trajectory of the ball.

This problem is an example that, when solving a 2'nd order differential equation, one can either use $\vec{r}(t_0)$ and $\dot{\vec{r}}(t_0)$ (initial position and velocity) as data, or $\vec{r}(t_0)$ and $\vec{r}(t_1)$ (initial and final position data).

Aufgabe 1.4: Our first Zwangsbedingung

1.4a)

In this problem we deal with a constraint (Zwangsbedingung). Consider a pendulum, suspended from the coordinate origin (Ursprung), which can swing in the (x, z) plane under the influence of gravity with strength g. That is, the potential is $V(\vec{r}) = gz$ (note that z will generally be negative).

The pendulum consists of a small ball of mass M, on a light, stiff string of length L. Therefore, the tension in the string will take whatever value is needed to keep $\sqrt{x^2 + z^2} = L$. Neglect everything annoying (the physical size and moment of inertia of the ball, the mass and stretchability of the string, air resistance, friction, any "give" in the pendulum mount).

Write down all forces acting on the pendulum (gravity and tension), and find a differential equation for how (x, z) evolve with time.

1.4b)

Suppose we hang a second pendulum from the bottom of the first pendulum. The second pendulum has a ball of mass M_2 and a string of length L_2. This string is also stiff, so it always stays the same length.

Try to write down a complete set of equations describing how the coordinates of the first ball, (x, z), and of the second ball, (x_2, z_2), change with time. Get as far as you can. If you feel ambitious and you get differential equations (2 extra points if you do!), try to solve them. What makes this part of the problem difficult?