Theoretische Physik I: Klassische Mechanik - Übungsblatt

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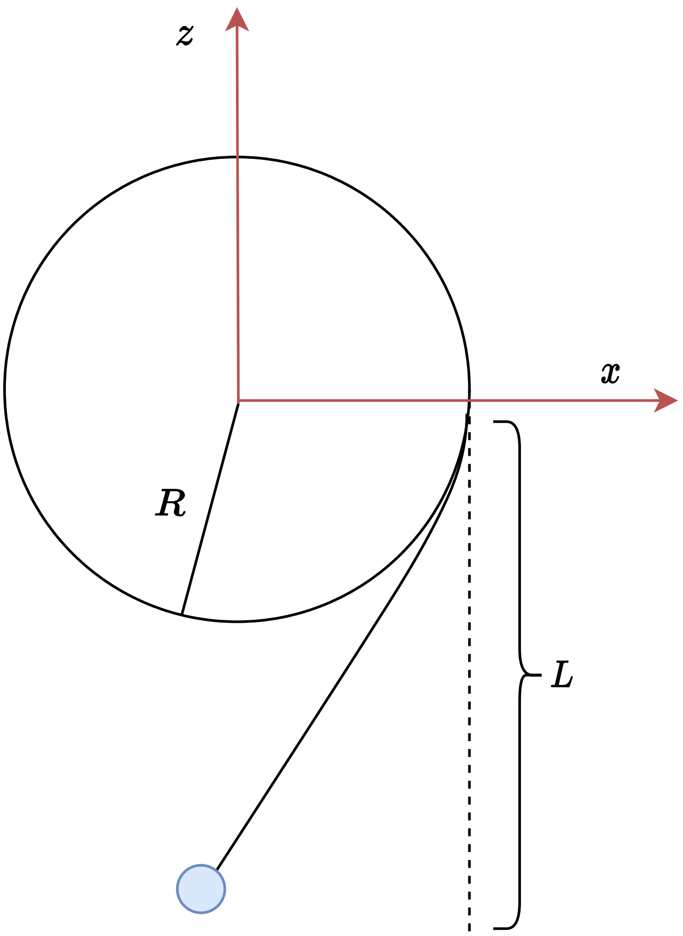
Übungsblatt 2

Deadline: 28.04. in Lecture or 29.04. 23 Uhr online

Aufgabe 2.1: A complicated pendulum

Consider a pendulum. The pendulum mass M moves in the (x, z) plane, but the string, rather than hanging from a point, is wrapped around a dowel (a cylinder) of radius R. When the pendulum hangs straight down, the string has length L (so the mass is at $(x, z) = (R, -L)$). As the pendulum swings back and forth, the string winds or unwinds around the dowel, making it shorter or longer.



2.1a) 5p

Write the constraint in Cartesian coordinates. Then, try to find a set of generalized coordinates where the constraint takes a simpler form.

2.1b) 2p

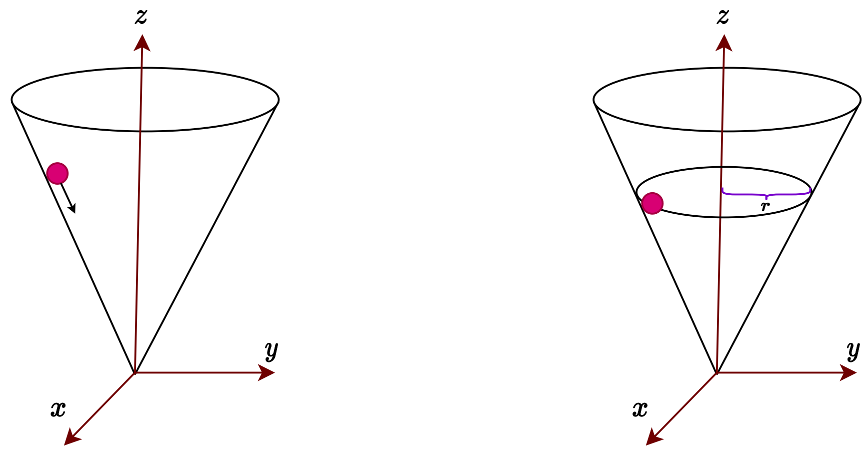
Is the constraint holonomic or nonholonomic? If it is holonomic, is it scleronomic or rheonomic?

Aufgabe 2.2: A particle in a cone

Consider a small mass m which slides without friction on the inside surface of a cone. The point of the cone is at the origin, and the height of the cone is given by $z = \sqrt{x^2 + y^2}$. The mass feels the force of gravity with $V = mgz$ and a normal force due to the cone.

2.2a) 5p

Consider first a trajectory where the mass slides straight down the side of the cone towards the tip (see figure 1a). What is the trajectory? How strong is the normal force?



(a) Mass sliding straight down. (b) Mass following a circle.

2.2b) 5p

Next consider a trajectory where the mass follows a circle of radius r (see figure 1b). The object's speed is just enough to keep it going around in this circle. What is the velocity, and how strong is the normal force? Is the normal force weaker than, equal to, or stronger than the force of gravity? How does it compare to the strength in part A?

Aufgabe 2.3: Like a rolling sphere. 6p

Consider a sphere with mass M and radius R. Its surface is painted in a complicated way, so that we can tell one point on the surface from another.

2.3a) 1p

If the sphere is flying through space, how many generalized coordinates does it take to describe its motion?

2.3b) 2p

Suppose the sphere rests on a flat surface, but the surface is perfectly slippery. The sphere can slide in any direction and can spin in any way without any friction force acting on it. How many constraints act on the sphere, and are they holonomic or nonholonomic?

2.3c) 3p

Now suppose that the sphere is moving on the inside of the cone from the previous problem. The inside is made of rubber, so that the sphere rolls without sliding. How many constraints are there now?

If the sphere rolls around a circle of radius $r \neq R$ (like the mass in the previous problem), when it comes back to the point where it started, is the same or a different point on the sphere touching the cone? Based on your answer, are some of the constraints non-holonomic? These are simple yes/no answers. Do not try to figure out the constraint in full detail, we have not talked about the coordinates needed to describe the sphere's orientation.