Theoretische Physik I: Klassische Mechanik - Übungsblatt

Prof. Dr. Guy Moore

TECHNISCHE UNIVERSITÄT DARMSTADT

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Übungsblatt 4

Deadline: 13.05 23 Uhr online

Aufgabe 4.1: Coriolis on rockets. 3p

Consider a rocket, fired into space, to achieve orbit. It will orbit 400km above the surface of the Earth, around the Equator.

4.1a) 1p

How fast must the rocket move, to maintain a circular orbit? The Earth's radius is 6370km, gravity at the surface is 9.8 $\mathrm{m/s}^2$, and the strength of gravitational acceleration falls as $1/r^2$.

4.1b) 1p

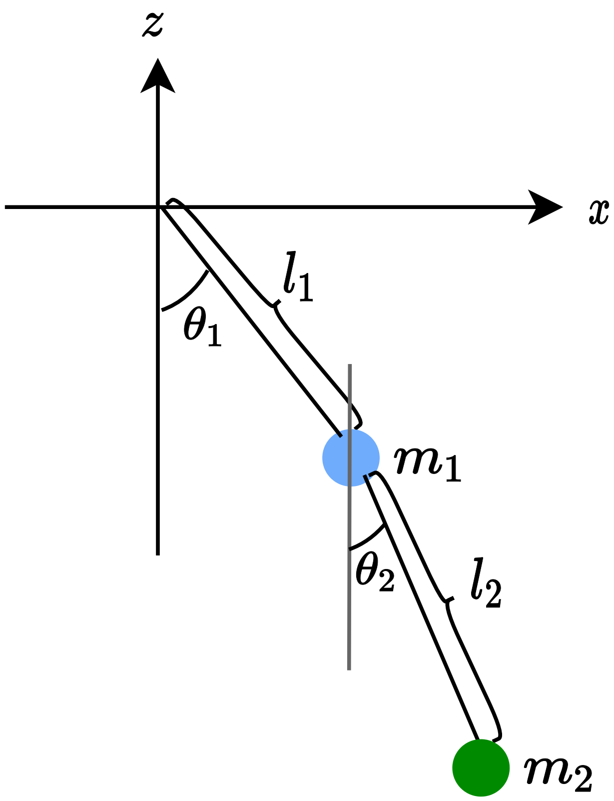
How fast is the rocket moving over the surface of the Earth, viewed in the Earth's frame, if the rocket is flying east?

4.1c) 1p

What if the rocket is flying west? In which direction is it cheaper to put a rocket in orbit?

Aufgabe 4.2: Double pendulum. 8p

A pendulum swings in the (x,z) plane. The string has a length l_1 and the bob has a mass m_1; the string has negligible mass and does not stretch, and the attachment at the top does not move. A second pendulum is attached onto the end of the first. It also moves in the (x,z) plane, and has length l_2 and mass m_2.



4.2a) 1p

Using the two angles θ_1 and θ_2 as generalized coordinates, calculate the speeds of each mass as functions of $(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2)$.

4.2b) 2p

Use your results from the previous part to write the kinetic energy T as a function of $(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2)$.

4.2c) 1p

Write an expression for the potential energy due to the gravitational force.

4.2d) 4p

Write down the Lagrange function and derive the equations of motion for each angle.

Hint: the speed of the second mass depends on *both* $\dot{\theta}_2$ and $\dot{\theta}_1$. You might find it easier to compute its velocity by using Cartesian coordinates, although you may be able to do so without reference to Cartesian coordinates.

Aufgabe 4.3: Satellites. 9p

Consider a satellite of mass m in orbit around the Earth. The gravitational potential energy is $V = -G_N Mm/r$, where M is the mass of the Earth, G_N is Newton's constant, and r is the distance from the satellite to the center of the Earth.

4.3a) 2p

Write the Lagrangian of the system in spherical coordinates, $L(r, \theta, \varphi, \dot{r}, \dot{\theta}, \dot{\varphi})$, and derive the Euler-Lagrange equations for each coordinate.

4.3b) 3p

Rewrite the Lagrangian in polar coordinates, $L(\rho, z, \varphi, \dot{\rho}, \dot{z}, \dot{\varphi})$. Derive the equations of motion for these coordinates.

4.3c) 4p

Write the relation between the coordinate pairs (r, θ) and (ρ, z), and between $(\dot{r}, \dot{\theta})$ and $(\dot{\rho}, \dot{z})$. Substitute these relations into your polar-coordinate equations of motion to re-express these equations in terms of spherical coordinates. Show that they reduce to the same equations of motion which you found in spherical coordinates.