Theoretische Physik I: Klassische Mechanik - Übungsblatt

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Übungsblatt 5

Deadline: 20.05. 23 Uhr online

Aufgabe 5.1: Spherical pendulum

Consider the spherical pendulum. A mass m hangs from an inextensible string which is threaded through a narrow hole at the coordinate origin. That allows a monkey/robot/perverse scientist to pull the string longer or shorter as they see fit; so the string's length $L(t)$ is a known but nontrivial function of time. Gravity acts on the pendulum so the potential energy is $V = gmz$. (z will be negative.)

5.1a) 1p

Write the Lagrangian in spherical coordinates. Derive the equations of motion.

5.1b) 2p

Is the Hamiltonian conserved? Is there a conserved quantity other than the Hamiltonian, and if so, what is it and what symmetry does it correspond to?

5.1c) 3p

Rotational symmetry could have provided 3 conserved quantities; the angular momentum about each axis. Why do we not find 3 conserved quantities?

Aufgabe 5.2: The Earth and Moon system

Consider the Earth and the Moon, moving through space. Their masses are M_e and M_m, and the potential energy is $V = -GM_eM_m/|\vec{r}_e - \vec{r}_m|$.

5.2a) 1p

Write the Lagrangian for this system.

5.2b) 7p

There are 6 symmetries for this Lagrangian: three translational symmetries and three rotational symmetries. Write down the transformation rules on the coordinates $\vec{r}_e$, $\vec{r}_m$ (in the original Cartesian coordinates for this system) associated with each symmetry. Verify that the Lagrangian is actually unchanged under each transformation. Find the conserved current for each using Noether's Theorem.

5.2c) 3p

Suppose we used the center of mass coordinate $\vec{r}_s$ and the relative coordinate $\vec{r}_r$ instead. Now what are the transformation rules for each symmetry, and the associated conserved current? Careful when you consider rotations.