Theoretische Physik I: Klassische Mechanik - Übungsblatt

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Übungsblatt 6

Deadline: 27.05 23 Uhr online

Aufgabe 6.1: A new planet. 2p

A small new planet is found, orbiting between Jupiter and Saturn. Its semi-major axis is the average (arithmetic mean) of the values for Jupiter and Saturn.

6.1a) 2p

What is its orbital period in years? (Jupiter orbits the Sun every 11.9 years and Saturn every 29.5 years. That's the only data you should need.)

Aufgabe 6.2: Two quarks. 5p.

Two quarks interact as a two-body problem with central potential. The strong interaction between them is well approximated by $V(r) = ar$ with a a constant and r the separation between the quarks. The masses are m and M with $M \gg m$.

6.2a) 2p

For the case of a circular orbit, compute the orbital period T as a function of m, r, a. What is the angular momentum P_φ as a function of m, r, a?

6.2b) 3p

Consider a nearly circular orbit – circular plus small oscillations. What is the oscillation frequency of the radial separation ω_{rad}? Is $\omega_{rad} \cdot T = 2\pi$ (as it is for a $1/r$ potential)?

Aufgabe 6.3: Central force. 12p

Consider the central force problem with potential $V = kr^2/2$, so the Lagrange function is

$$L(r, \dot{r}, \varphi, \dot{\varphi}) = \frac{m}{2}(\dot{r}^2 + r^2\dot{\varphi}^2) - \frac{1}{2}kr^2 \tag{6.3.1}$$

6.3a) 2p

Calculate the period T for a circular orbit as a function of m, k, r. Does the result depend on r?

6.3b) 2p

Calculate the frequency of radial oscillations ω_{rad} for nearly circular orbits, as a function of m, r, k.

6.3c) 2p

How do we know that the angular momentum p_φ and energy $E = T + V$ are conserved?

6.3d) 4p

Show that

$$E_x = \frac{m}{2}(\dot{r}\cos\varphi - r\sin(\varphi)\,\dot{\varphi})^2 + \frac{k}{2}r^2\cos^2\varphi \tag{6.3.2}$$

is conserved. (There are several ways to do this. One is to find the equations of motion, to explicitly compute dE_x/dt, and to use the equations of motion to show that it is zero.)

6.3e) 2p

Rewrite the problem in Cartesian coordinates. Show that the Lagrange function can be written as

$$L = L_x(x, \dot{x}) + L_y(y, \dot{y}) \tag{6.3.3}$$

Show that the Euler-Lagrange functions for x and for y are independent of each other, so the x-motion and y-motion are completely independent. Does that help explain your previous results?

6.3f) Bonus! 5p

Find the most general possible solution. Show that the orbit follows an ellipse, but with the origin at the midpoint of the ellipse, rather than a focus.

Aufgabe 6.4: Earth's orbit. 3p.

The Earth's orbit is in fact elliptical, with eccentricity $e = 0.0167$.

6.4a)

What is the ratio between the distances at aphelion and perihelion?

6.4b)

What is the ratio of the Earth's fastest velocity to its slowest velocity? Hint: angular momentum is conserved.

6.4c)

What is the ratio b/a of semi-minor to semi-major axis?