Theoretische Physik I: Klassische Mechanik - Übungsblatt

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Übungsblatt 7

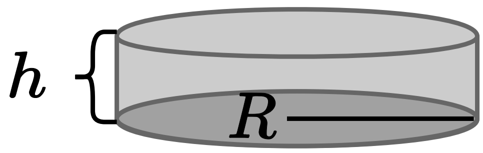
Deadline: 03.06. 23 Uhr online

Aufgabe 7.1: A serving bowl. 6p.

Consider a serving bowl, consisting of a thin flat circular plate of uniform mass per area and radius R, plus a cylindrical shell with the same radius and mass-per-area, rising up from the edge of the plate to a height $h = R$. The total mass is M.

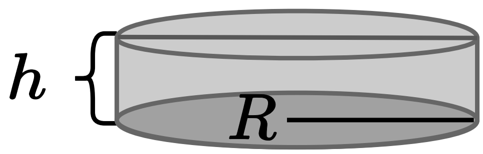
7.1a) 3p

Find the center of mass. Pick a set of coordinates and compute the angular momentum tensor. (Hint: pick simple set of coordinates.) Is the result triaxial, symmetric, or spherical?



7.1b) 3p

Add a thin bar onto the top of the bowl (maybe to hold it by?), running along a diameter of the circle, with a mass equal to $M/4$. With the addition of this bar, what is the moment of intertia tensor? Is it triaxial, symmetric, or spherical? What are the principal axes?



Aufgabe 7.2: Instable rotations. 4p

In class we saw that a book with moment of inertia components $I_1 < I_2 < I_3$ obeys the evolution equations

$$\begin{cases} I_1\dot{\omega}_1 & = (I_2 - I_3)\omega_2\omega_3 \\ I_2\dot{\omega}_2 & = (I_3 - I_1)\omega_3\omega_1 \\ I_3\dot{\omega}_3 & = (I_1 - I_2)\omega_1\omega_2 \end{cases} \tag{7.2.1}$$

and we showed that, if $|\omega_1| \gg |\omega_2|, |\omega_3|$ or $|\omega_3| \gg |\omega_1|, |\omega_2|$, that the evolution is stable – the dominant ω stays dominant. But if ω_2 dominates, there is an instability and the smaller components grow.

But what happens if we reduce I_2 and increase I_1? How does the stability change?

To investigate this, we will consider the case where $I_1 = I_2$. (This would happen if the book has a square cover, that is, the book is as wide as it is tall.) The same evolution equations still apply, but they imply something different now.

Consider $|\omega_3|$ to be small but not zero, and *initially* ω_1 is large and $\omega_2 = 0$.

7.2a)

Show that ω_3 does not change. Show that this will remain true even if ω_1, ω_2 are both nonzero.

7.2b)

Show that, initially (for short times, so long as $|\omega_2| \ll \omega_1$), ω_2 grows *linearly* with time. This behavior is *intermediate* between oscillatory behavior and exponentially growing behavior (the behaviors we would find if $I_2 > I_1$ or if $I_2 < I_1$).

7.2c)

Show that the equations for ω_1 and ω_2 are solved, for all times, by

$$\begin{cases} \omega_1 & = \omega_0 \cos(\tilde{\omega}t) \\ \omega_2 & = \omega_0 \sin(\tilde{\omega}t) \end{cases} \tag{7.2.2}$$

where $\omega_0 = \omega_1(t = 0)$ the initial value of ω_1, and where $\tilde{\omega}$ is some constant (time-independent) frequency which you will need to solve for.

Aufgabe 7.3: The top. 10p.

Consider the heavy top, with principal angular momentum components $I_1 = I_2 = I$ and $I_3 \neq I$. It sits in the gravitational field of the Earth. The origin of coordinates in both the inertial and the body-centered coordinate systems is at the tip of the top.

7.3a) 2p

Explain why the Lagrange function of the problem is

$$L = \frac{I}{2}(\dot{\varphi}^2 \sin^2(\theta) + \dot{\theta}^2) + \frac{I_3}{2}(\dot{\varphi}\cos\theta + \dot{\psi})^2 - mgl\cos\theta. \tag{7.3.1}$$

Here m is the mass, l is the distance from the center of mass to the tip, g is the strength of gravity, and φ, θ, ψ are the Euler angles.

7.3b) 2p

Which variables are cyclic? What are the associated canonical momenta?

7.3c) 2p

Write the energy in terms of the cyclic canonical momentum/momenta and the noncyclic variables and their time derivatives.

7.3d) 2p

Compute the Euler-Lagrange equation for the angle θ.

7.3e) 2p

Show from the previous part that nutation-free precession is possible, that is, $\dot{\theta} = 0$ but $\theta \neq 0$. What occurs for the cyclic variable? Discuss carefully the possible solutions for the precession frequency $\dot{\varphi}$.