Theoretische Physik I: Klassische Mechanik - Übungsblatt

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Übungsblatt 8

Deadline: 10.06. 23 Uhr online

Aufgabe 8.1: A well known pair. 4p.

To get used to differential equations numerically, consider the following pair of coupled ODEs: the functions s and c depend on time t through

$$\begin{cases} \dfrac{\mathrm{d}c(t)}{\mathrm{d}t} = -s(t) \\ \\ \dfrac{\mathrm{d}s(t)}{\mathrm{d}t} = +c(t) \end{cases} \tag{8.1.1}$$

with initial conditions $c(0) = 1$ and $s(0) = 0$.

8.1a)

Solve these equations numerically. What are the correct analytical answers? Compare your numerical results at $t = 2$ to the known analytical result.

Aufgabe 8.2: Chaos. 10p.

Consider a massive particle in the (x, y) plane, with Lagrangian

$$L(x, y, \dot{x}, \dot{y}) = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2\right) - a(x^4 + y^4) - bx^2y^2 \tag{8.2.1}$$

Here a, b are real parameters.

8.2a)

Write down the Euler-Lagrange equations for x and for y. Re-express these as a system of four 1'st order ODEs for the four variables $x, y, \dot{x}, \dot{y}$.

8.2b)

Solve these equations numerically for the initial conditions: $x(0) = 1.0, y(0) = 0.0, \dot{x}(0) = 0.0, \dot{y}(0) = 0.5$ as well as for the same but $y(0) = 0.0001$, for each of the following parameter choices:

- $m = 1, \quad a = 1, \quad b = -1$
- $m = 1, \quad a = 1, \quad b = 0$
- $m = 1, \quad a = 1, \quad b = 2$
- $m = 1, \quad a = 1, \quad b = 8$

For which choices is the evolution chaotic? For which does it appear NOT to be chaotic?

8.2c)

For the cases which are NOT chaotic, each case has an additional conserved quantity besides the energy. For each case, what is the additional conserved quantity? (The presence of 2 conserved quantities with 2 coordinates is sufficient to ensure that the evolution is not chaotic.)

8.2d)

Predict *another* b value for which the evolution is *not* chaotic. What is the new conserved quantity? (Note: this may be hard, don't worry if you can't do it.)

Aufgabe 8.3: A marble. 10p

A marble rolls without slipping in a parabolic dish under the influence of gravity, $V(z) = gmz$. The fact that it rolls just means $T = \frac{1}{2}mv^2$ is replaced with $T = \frac{7}{10}mv^2$. That it is a parabolic dish means that $z = (x^2 + y^2)/(2R)$ with R the focal length of the parabola.

8.3a)

Write the Lagrangian for this system in radial coordinates.

8.3b)

Find the canonical momenta. Is there a cyclic coordinate?

8.3c)

Write the equations of motion for r and for θ.

8.3d)

Find the Hamiltonian $H(r, \theta, p_r, p_\theta)$. Careful.

8.3e)

Find the Hamilton's equations. Show that they are equivalent to the Euler-Lagrange equations.