Quantum Field Theory II
Homework 4

Do Three out of Four Problems! Due 9 February 2024

1 Strong Interactions with Symmetric Tensors

Your crazy friend Jim (based on a real person) has a particle physics model with new quark species which are in the 6 (symmetric tensor or $\square\square$) representation. In his model, these have a mass of a few GeV. You are trying to figure out how such particles would change QCD, in order to persuade Jim that they are unlikely to be experimentally viable.

1. What effect would N_6 such quark species have on the beta function of QCD (at one loop or lowest order)? How many species can there be before asymptotic freedom is lost?

2. What would be the simplest and lightest new hadronic bound states containing one of these new quarks Q? What would the expected spin and isospin be?

3. Suppose these new hadrons are to carry an integer electric charge. What are the allowed electric charges of the new Q particles? What is the smallest possible contribution to R the ratio of hadronic to muonic states in e^+e^- annihilation, above the Q mass threshold?

For extra points, see if you can find any literature or other source which presents experimental limits on such particles.

2 Beta function to the UV

The strong coupling in the 5-quark MSbar scheme at the scale $\mu = 91$ GeV is $\alpha_s = 0.118$. Use the 1-loop beta function to determine the value of α_s at $\mu = 173$ GeV, the top quark mass.

At this scale, switch to the 6-quark scheme to include top quark effects. Evolve the coupling to the scale $\mu = 10^{16}$ GeV, where the strong coupling may take the same value as the weak coupling and the properly rescaled hypercharge coupling.

3 quark-ghost scattering

Calculate the initial-state spin averaged, final-state spin summed squared matrix element for a quark scattering against a ghost. Look up the Feynman rules and remember that the ghost is a spin-scalar and in the adjoint representation. Express your answer in terms of Mandelstamm variables. Remember that there is a minus sign due to the ghost loop which means that your result is actually negative.

4 Momentum fraction in gluons

This problem takes some work, but maybe it's worth it.

Consider the Altarelli-Parisi equations of QCD, Peskin Eq.(17.128) to (17.130). Assume that four flavors of quark are light and are relevant in our discussion (*udsc*).

It has been observed that about half of the energy/momentum of a proton is carried by the gluons and about half by the quarks and antiquarks. Is this reasonable? Should we expect gluon radiation to turn the proton into purely glue at high resolution scale Q, or do the evolution equations favor a balance between quarks and gluons?

To investigate this, define the total fraction of the proton's energy which is carried by gluons to be

$$X_g \equiv \int_0^1 x\, f_g(x, Q)\; dx = \int_0^1 x\, g(x, \mu)\; dx \tag{1}$$

following either Peskin's notation or the more common one where the PDF is written as $g(x, \mu)$. Similarly, the energy fraction in quarks and antiquarks is

$$X_q \equiv \int_0^1 x \left(\sum_f f_{q_f}(x, Q) + f_{\overline{q}_f}(x, Q) \right) dx = \int_0^1 x \left(\sum_f q_f(x, \mu) + \overline{q}_f(x, \mu) \right) dx\,. \tag{2}$$

Here the sum is over quark flavors. Note that X_g, X_q are functions of Q (or μ depending on notation). We want to explore this scale dependence.

Use the Altarelli-Parisi equations to determine $dX_g/d(\ln(Q))$ and $dX_q/d(\ln(Q))$. Show that $dX_g/d(\ln(Q)) + dX_q/d(\ln(Q)) = 0$ which is conservation of energy. Which one grows and which one shrinks, or does the answer depend on the values of X_g and X_q? Do you find that there is a value of X_g for which the momentum fraction is stable with scale? Is it a UV attractor?

(These quantities X_g and X_q are called the *first Mellin moments* of the PDFs. One can define Mellin moments with any nonnegative integer power of x and analyze their evolution, and for some problems they are simpler and more directly relevant than the PDFs themselves. The moment with x^0 counts the total number of partons; it grows without limit as Q is increased, indeed I think its Q-derivative contains log divergences – total parton number is not well defined. The x^3 moment for quarks is relevant for neutrino scattering on nucleons; these moments are finite and decrease with increasing Q....)