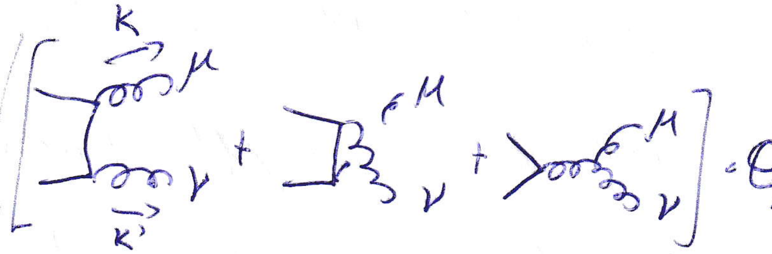
Q: What is cross-section for $\bar{q}q \to gg$?

A1: Calculate $\sum_{\lambda_1,\lambda_2}$



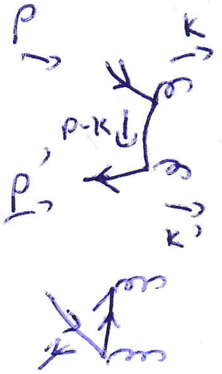
$\cdot\, \epsilon^*_{\lambda_1\mu}\epsilon^*_{\lambda_2\nu} \Big|^2$

With ϵ_{λ_1} the two polarizations ⊥ to k

ϵ_{λ_2} " " " " " k'

That is, call $\mathcal{M} = \mathcal{M}^{\mu\nu}\epsilon^*_{1\mu}\epsilon^*_{2\nu}$

↳ matrix element w. μ, ν external 4-vectors



$$\left.\begin{aligned}&(ig)^2\,\bar{v}(p')\Big[\gamma^\nu T^B \frac{i}{\not p - \not k - m}\gamma^\mu T^A \\ &\qquad + \gamma^\mu T^A \frac{i}{\not p - \not k' - m}\gamma^\nu T^B\Big]u(p) \\ &+ ig f_{ABC} T^C\, \bar{v}(p')\gamma_\rho v(p) \frac{-i}{(p+p')^2 + i\epsilon} \\ &\quad \Big[g^{\mu\nu}(k'-k)^\rho + g^{\nu\rho}(-k-2k')^\mu + g^{\mu\rho}(2k+k')^\nu\Big]\end{aligned}\right\} \mathcal{M}^{\mu\nu}$$

Summing over 2 transverse polarizations →

$$\overline{\mathcal{M}}^2 = 2\frac{C_2^2[R]}{d_R}\left(\frac{u}{t}+\frac{t}{u}\right) - 2\frac{C_2[R]C[A]}{d_R}\left(\frac{t^2+u^2}{s^2}\right)$$

A2: Same but $\sum_\lambda \epsilon^*_\mu \epsilon_{\mu'} = g_{\mu\mu'}$ to simplify polariz.

QED: this works. QCD: $\overline{\mathcal{M}}^2 = 2\frac{C_2^2[R]}{d_R}\left(\frac{u}{t}+\frac{t}{u}\right) - \frac{2C_2[R]C[A]}{d_R}\left(\frac{t^2+u^2}{s^2} - \frac{tu}{2s^2}\right)$

Wait, what?

Calculation with $\sum_\lambda \varepsilon_\lambda^{\mu'} \varepsilon_\lambda^{\mu *} \neq$ calc with $g^{\mu\nu}$

Why not? Isn't $M^{\mu\nu}$ transverse?

~~A~~ Answer: define $\varepsilon^+_\mu \propto k_\mu$ $\varepsilon^-_\mu \propto \bar{k}_\mu$

if $k^\mu = (1\,0\,0\,1)$, $\varepsilon_1^\mu = (0\,1\,0\,0)$] spacelike ⊥
$\varepsilon_2^\mu = (0\,0\,1\,0)$] polariz.

$\varepsilon^{\mu +} = (1\,0\,0\,1)$

$\varepsilon^{\mu -} = (-1/2\,0\,0\,+1/2)$

$$\sum_{\lambda=1,2} \varepsilon_\lambda^\mu \varepsilon_\lambda^\nu + \varepsilon^{\mu +} \varepsilon^{\nu -} + \varepsilon^{\mu -} \varepsilon^{\nu +} = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} = -g^{\mu\nu}.$$

QED: $M_{\mu\nu} \varepsilon^{\mu +}_{(k)} = 0$. ~~k~~

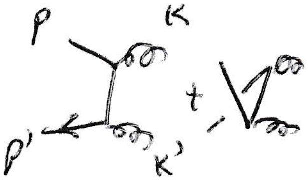
QCD $M_{\mu\nu} \varepsilon^{\mu +}_{(k)} \varepsilon^{\nu +}_{(k')} = 0$ but not just

So you cannot replace $\sum_\lambda \varepsilon_\lambda^\mu \varepsilon_\lambda^\nu \to -g^{\mu\nu}$.

See it explicitly

QED: Replace $\varepsilon^\mu \to k'^\mu$

add $\not p + m$ - gives 0 on $u(p)$



$$(ig)^2 \bar{v}(p')\left[\gamma^\mu T^A \frac{i}{\not p - \not k' - m} \not k' T^B + \not k' T^B \frac{i}{\not k' - \not p' - m} T^A \gamma^\mu\right] u(p)$$

add $-\not p' - m$ - gives 0 on $\bar v(p')$

$$= i(ig)^2 \bar{v}(p') \left[\underbrace{\gamma^\mu T^A \cdot 1 \cdot T^B - T^B \cdot 1 \cdot T^A \gamma^\mu}_{\text{cancel except } T^A T^B \text{ vs } T^B T^A \,!!} \right] u(p)$$

$[T^A, T^B] = i f_{ABC} T^C$

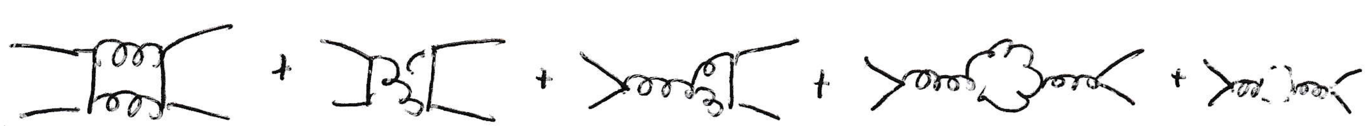
Purely nonabelian leftover piece.

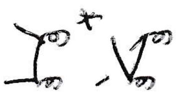
Two answers differ. Which is right?

Total cross-sec: Im part of $\begin{smallmatrix} p \to \\ p' \to \end{smallmatrix} \left(\quad \right) \begin{smallmatrix} \to p \\ \to p' \end{smallmatrix}$

Forward scatt (Optical Thm)

Include <u>all</u> diagrams at 1-loop



 squared    squared

Ghost.

All diagrams, Feynman gauge, ~~...~~ using $-g_{\mu\nu}$ on external propagators

Plus square of process 

which contributes negative probability.

Specifically, $$|\bar{\mathcal{M}}|^2_{\text{ghosts}} = -\frac{C_2[R]C[A]}{d_R}\frac{tu}{s^2}$$

Cancels extra term in A2, giving A1

Ghosts give negative prob's, which cancel positive prob. from unphysical longitudinal polarizations.

Can we make this systematic?

Yes: Becchi Rouet Stora Tyutin BRST

Remnant of gauge symm. in gauge-fixed thy ensures unphys. $\left\{\begin{matrix}\text{gluons}\\ \text{ghosts}\end{matrix}\right\}$ exactly cancel at all orders

BRST Symmetry

Rewrite (sorry) $e^{-\frac{i}{2\xi}(\partial_\mu A^\mu)(\partial_\nu A^\nu)}$

$$= \int \mathcal{D}B^A \, e^{+i\left(\xi \frac{B^2}{2} + iB\, \partial^\mu A_\mu\right)}$$

(shift to complete square, Gauß int, ...)

Write $\mathcal{L} = \underbrace{-\frac{1}{4} F^A_{\mu\nu} F^{\mu\nu A}}_{1} + \underbrace{\bar\psi (i\not{D} - m)\psi}_{2} + \underbrace{\frac{\xi}{2} B^2}_{3} + \underbrace{B^A \partial^\mu A^A_\mu}_{4} - \underbrace{\bar c\, \partial^\mu D_\mu c}_{5}$

Consider Fermionic symm transform, Grassmann param ε

$\delta A^A_\mu = \varepsilon D^{AB}_\mu c^B$ — like gauge trans $\delta A = D_\mu \theta$ but $\theta = c$ ghost.

$\delta \psi = ig\, \varepsilon c^A T^A \psi$ — like " " $\delta\psi = igT^A\theta^A$ but $\theta = c$ ghost

$\delta \bar\psi$ similar

$\delta c^A = -\frac{1}{2} g \varepsilon f^{ABC} c^B c^C$

$\delta \bar c^A = \varepsilon B^A$

$\delta B^A = 0$

Claim: this is a symmetry!

Terms 1, 2: gauge transform on A, ψ, and these are gauge inv. ✓

Term 3: $\delta B = 0$ so OK.

Term 4 → $\varepsilon B^A \partial^\mu D_\mu c$ cancels $\delta \bar c$ from term 5

Term 5: δA and δc cancel. ✓ Namely, $\delta(D_\mu c) = 0$.

Claim: Apply transform twice — get $\delta(\delta ..) = 0$. "Nilpotent"

What BRST symm does

Call BRST-operator (transformer on fields) Q

eg $QA_\mu = D_\mu c$. $Q^2=0$ as operator - nilpotent.

$[Q,H]=0$ as transform on $\mathcal{L}$ gave 0.

Eigenstates of H are E-states of Q.

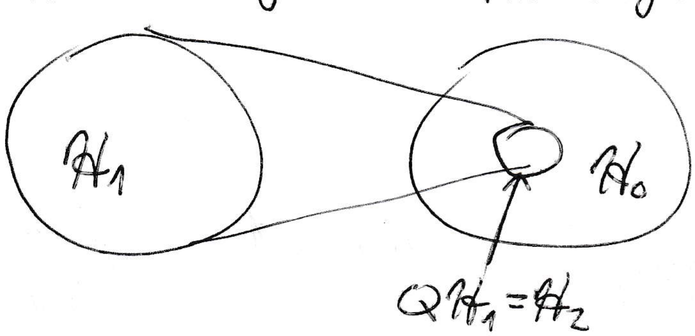
3 Types of States:

1) $|\psi\rangle$ such that $Q|\psi\rangle \neq 0$. "$\mathcal{H}_1$"

2) Images of $\mathcal{H}_1$: $|\chi\rangle$ such that $|\chi\rangle = Q|\psi\rangle$ "$\mathcal{H}_2$" $\rightarrow$ $Q|\chi\rangle=0$ as $QQ=0$...

3) Annihilated by Q but not in $\mathcal{H}_2$: "$\mathcal{H}_0$"

Not Annih. by Q Annih. by Q



Time evolution: $\mathcal{H}_0 \to \mathcal{H}_0$, $\mathcal{H}_1 \to \mathcal{H}_1$, $\mathcal{H}_2 \to \mathcal{H}_2$

Free thy: $\bar{c}$, A^+ lie in $\mathcal{H}_1$. Q on these $\neq 0$

c, A^- lie in $\mathcal{H}_2$; c from QA^+

~~A from~~

Transverse A in $\mathcal{H}_0$ -

Physical states in $\mathcal{H}_0$, and time evol. keeps them separate, also at inter. level.