QCD21P1

What is QCD?

1) $SU(3) = G$. So $\square$, $\begin{smallmatrix}\square\\\square\end{smallmatrix}$, $\square\square$, etc allowed...

2) Vectorlike/Dirac fermions "quarks" all (convention) in $\square$

3) No scalars or Yukawas

6 quarks $\{u\,d\,c\,s\,t\,b\}$: $(u\,c\,t)$ charge $+2/3$
"flavor" $(d\,s\,b)$ charge $-1/3$

gen1 gen2 gen3

flavor : Vanilla, Chocolate, ~~Ra~~ strawberry

$$\mathcal{L} = \frac{1}{4\underbrace{g^2}_{g \text{ or } g_s (\text{or } g_3)}} G^A_{\mu\nu} G^{\mu\nu}_A + \frac{1}{4 e^2} F_{\mu\nu} F^{\mu\nu} + \sum_q \bar{q}(i\not{D} - m_q) q$$

(EM)

$$\not{D} = \gamma^\mu (\partial_\mu - i g T^A B^A_\mu + i Q_q e A_\mu)$$

if absorb...

$Q_{u,c,t} = 2/3$

$Q_{d,s,b} = -1/3$

m_q : Parameter of $\mathcal{L}$: <u>not</u> "number in GeV"

eg $m_q(\bar{\mu}_{\overline{MS}})$ depends on $\bar{\mu}_{\overline{MS}}$ Ren. pt.

Parameters
g_s or Λ_{QCD} we'll see
m_u
m_d at some $\bar{\mu}$. Often defined as
$\vdots$ $m(\mu = m)$ for heavy t, b, c
m_t $m(\mu = 2\,\mathrm{GeV})$ for light s, d, u

7 parameters

Best values: $\alpha_s(\bar{\mu} = M_Z) = 0.1182(12)$

$m_u(2\,\mathrm{GeV}) = 2.14\,\mathrm{MeV}$ $\quad m_u/m_d = .465$ $\quad m_c(\bar{\mu} = m_c) = 1.27\,\mathrm{GeV}$

Is the theory asymptotically free?

$$\mu^2 \frac{\partial}{\partial \mu^2} \underbrace{\frac{Ng^2}{16\pi^2}}_{\equiv a} = -a\left[b_0 a + b_1 a^2 + b_2 a^3 + b_3 a^4 + b_4 a^5 + \ldots\right)$$

$3b_0 = 11 - \frac{2N_f}{N_c}$ positive for $N_f < \frac{11 N_c}{2} = \frac{33}{2}$ eg, 0..16 but not 17

$9b_1 = 102 - \left(\cancel{40}\,30 - \cancel{8}\,18 \frac{N^2-1}{2N^2}\right)\frac{N_f}{N_c}$

$27b_2 = \frac{2857}{2} + \frac{N_f}{N_c}\left(27\left(\frac{N^2-1}{2N^2}\right)^2 - \frac{615}{2}\frac{N^2-1}{2N^2} - \frac{1415}{2}\right)$

$81b_3 = \ldots$ has ζ_3 in it

$243b_4 = \ldots$ has $\zeta_3, \zeta_4, \zeta_5$ in it $\quad \zeta_n = \sum_{m=1}^{\infty} m^{-n}$

$\overline{MS}$ specific

$N_c < 16$: Asympt. Free

N_c large but < 16: Banks-Zaks fixed pt: at finite a, b_0 & b_1 can cancel

N_c close to 16: know this reliably (b_0 accidentally small $b_0 \approx b_1$ but $b_2 \ldots \ll$ possible)

N_c of 6 ... definitely Asympt. Free.

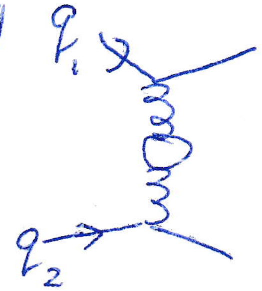
Coupling grows in IR. Scale where 1 (2)-loop β-func. predicts $\alpha \to \infty$ is named Λ_{QCD}. Depends on q-mass handling

$N_c = 3$

What is N_f ??

Sounds dumb but it's not.

Ans. 1: $N_f = 6$ dummy!

Consider scattering 

s, t, u a few GeV, say

$q_1 \neq q_2$: only this diagram at LO

$$\cancel{\frac{d\sigma}{dt} = \frac{4\pi\alpha_s^2}{9s^4}\Big(\frac{1}{4}\sum_{\sigma\sigma'\sigma''\sigma'''} \mathcal{M}^*\mathcal{M}} = \frac{\pi\alpha_s^2}{s^2}\cdot\frac{4}{9}\,\frac{s^2+u^2}{t^2} = \frac{d\sigma}{dt}$$

at leading order. At NLO,

$$\alpha_s \to \alpha_s - \frac{\alpha_s^2}{4\pi}\left(\frac{11}{9}N_c\left(\ln\frac{q^2}{\bar{\mu}^2} - \overset{5/6\,??}{5/3}\right) - \sum_q \frac{2}{3}\begin{cases}\ln\frac{q^2}{\bar{\mu}^2} - 5/3 & \text{if } q^2 \gg m^2 \\ \ln m^2/\bar{\mu}^2 & \text{if } q^2 \ll m^2\end{cases}\right)$$

Pick $\bar{\mu}^2 \simeq q^2$ to cancel log up to small correction

Unless $m^2 \gg q^2$. Then $\ln\left(\frac{m_q^2}{\bar{\mu}^2}\right) \gg 1$ still in there.

But if $m_q^2 \gg q^2$ why is this quark (eg. top q) in calc. at all?

Below scale $\mu = m_q$ work in simpler thy with 1 fewer quark

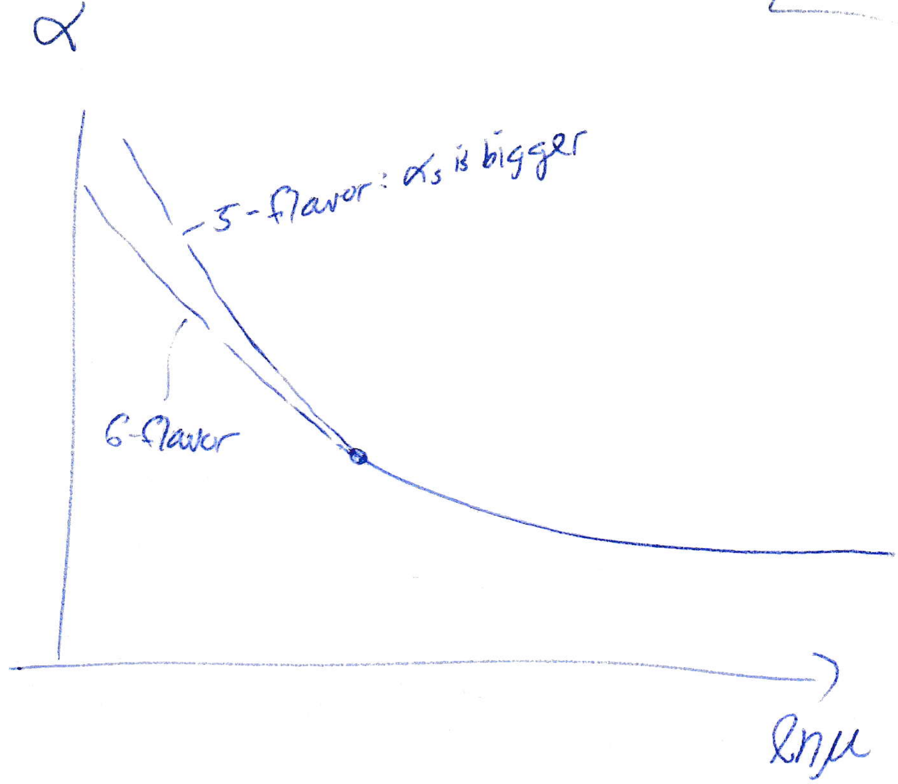
(simplest example of EFT)

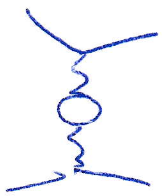
$\alpha_s(\bar{\mu}, 5\text{-}q) \neq \alpha_s(\bar{\mu}, 6\text{-}q)$ after all, β func's differ!

Choose α_s so answer to some phys. question same in both thys

Then difference in running compensates large logs

That is,



 $= \alpha - \frac{\alpha^2}{4\pi}\left(\text{identical logs} - \frac{2}{3}\times\begin{cases}\log m^2/\mu^2 & \text{6-fl.}\\ 0 & \text{5-fl.}\end{cases}\right)$

α: bigger in 5-flavor

6 fl: loop w. top q exists
5 fl: no such loop.
Bigger in 6-flavor.

Theories agree at each loop level. But 5-q thy missing large

$\alpha^{n+1} \ln^n \frac{m^2}{\mu^2}$ corrections — better convergence if you use 5-q thy.

Match:

1) write down full & reduced theories
2) Pick $\bar{\mu}$ around "cutoff" scale m_q (m_t say)
3) Calculate some deeply IR phenomenon(a), eg, $d\sigma/dt$ for $q^2 \ll M_t^2$
4) Choose parameter(s) so answer(s) same in