Is QCD a theory of Quarks & Gluons?

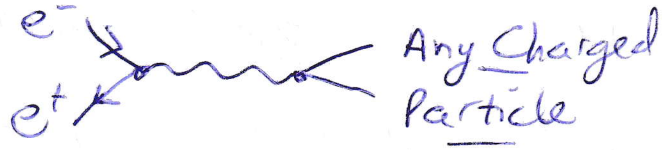
Yes in the sense that A_μ, ψ are in $\mathcal{L}$

But are the particles q, g ??

Answer - No, but it's complicated.

q, g have something to do with QCD states.

Consider $e^+e^- \to \gamma^* \to$ stuff



Process automatically generates all chg'd part's
w. mass $m^2 < s/4$ so 2 part's can be made.

$$\text{Rate} = \frac{4\pi\alpha^2}{3s} Q_e^2 Q_f^2 \times \begin{cases} 1 \text{ per Dirac fermion species} \\ 1/4 \text{ per } \mathbb{C} \text{ scalar species} \end{cases}$$

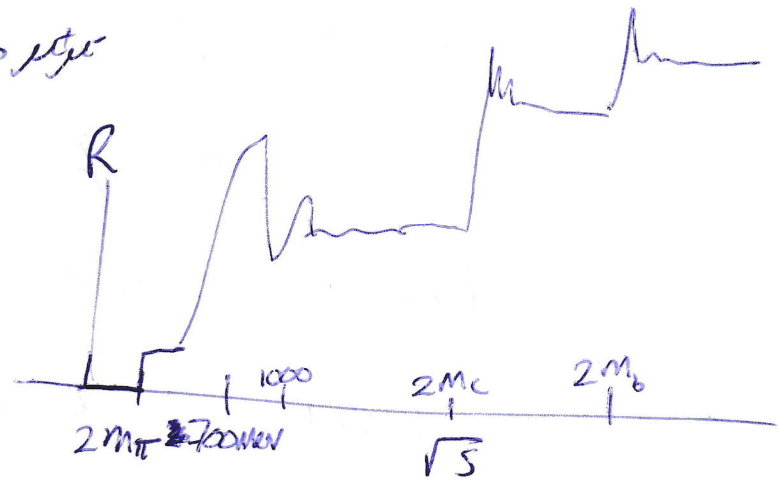
Q_e^2 = 1 for e^-; Q_f^2 = squared electric charge

all in $s \gg 4m_f^2$ approximation. "Threshold" region $s \simeq 4m_f^2$ is complicated (bound states...)

- Lets us find all charged states
- Lets us "count" Number $\times$ charge^2 of states.
  What happens above QCD scale?

Define ratio $R = \dfrac{\sigma_{e^+e^- \to \text{strong-int states}}}{\sigma_{e^+e^- \to \mu^+\mu^-}}$

$= \sum_{\substack{\text{all} \\ \text{strong-charged} \\ \text{states}}} Q_f^2$



Plateaus! Height of first: uds $\frac{4}{9} + \frac{1}{9} + \frac{1}{9} = \sum Q^2 = 2/3$

Plateau ht? $\simeq 2 = 3\sum Q^2$. 3?? $\sum$ includes rgb

Next plateau: udsc $\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} = \frac{10}{9}$ $(\times 3)$ =

Plateau $\simeq 10/3$ ✓

Next: add $Q_b^2 = 1/9 \to 11/3$ ✓

actually $\simeq 10\%$ higher.
We'll see why.

Looks like production of quark-antiquark pairs.

QCD cannot be thy of q, g "states"

They aren't gauge invariant!

Rather - consider correl. func. to learn about ψ-prop, to "determine mass of a quark" from pole in prop. or δ-func. in spectral function:

$$S(x) = \langle \psi(0) \bar{\psi}(x) \rangle$$

T-ordered for Feynman
Unordered - Wightman
Commutator - spectral func.

Either 1) Fix gauge to ξ-... gauge
Perturb $\to$ $S(x)$ is ξ-dependent
Nonperturb $\to$ issues defining gauge-fix procedure

2) Don't fix gauge - integrate all gauge fields

$$S(x) = \int \mathcal{D}(A_\mu \bar{\psi} \psi) e^{i\int \mathcal{L}(A \bar{\psi} \psi) d^4 y} \psi(0) \bar{\psi}(x)$$

Just for fun, write $\int$ over all gauge choices, divided by total measure of that int.

$$S(x) = \frac{1}{N} \int \mathcal{D}\theta \underbrace{\int \mathcal{D}(A \bar{\psi} \psi) e^{i\int \mathcal{L}(\cdot)}}_{} \psi(0) \bar{\psi}(x)$$

Change variables: $A \to A_\theta$, $\psi \to \psi_\theta$ gauge transform of old var.

Measure, action same, so

$$\frac{1}{N} \int \mathcal{D}\theta \int \mathcal{D}(A_\theta \bar{\psi}_\theta \psi_\theta) e^{i\int \mathcal{L}(A_\theta \bar{\psi}_\theta \psi_\theta)} \psi_\theta(0) \bar{\psi}_\theta(0)$$

"$\mathcal{D}A\bar{\psi}\psi$" $e^{i\int \mathcal{L}(A\bar{\psi}\psi)}$

$$S(x) = \langle \psi(0)\, \bar{\psi}(x) \rangle$$

$$= \int \mathcal{D}(A_\mu \bar{\psi}\psi)\, e^{i\int \mathcal{L}(A\bar{\psi}\psi)} \times \left(\frac{1}{N} \int \mathcal{D}\theta \;\; \psi_\theta(0)\, \bar{\psi}_\theta(x) \right)$$

$$\psi_\theta(0) = \exp[i\theta^A_{(0)} T^A]\, \psi(0) = U(0)\, \psi(0)$$

$$\bar{\psi}_\theta(x) = \bar{\psi}(x) \exp\{-i\theta^A_{(x)} T^A\} = \bar{\psi}(x)\, U^\dagger(x)$$

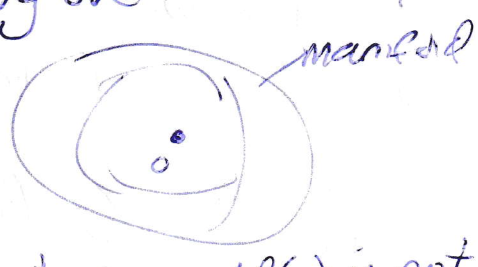
$$= \int \mathcal{D}(A_\mu \bar{\psi}\psi)\, e^{i\int \mathcal{L}\ldots} \times \frac{1}{(\text{group volume})^2} \int dU(x)\, dU(0)$$

$$\bar{\psi}(x)\, U^\dagger(x)\, U(0)\, \psi(0)$$

separate indep. $\int$ over gauge parameter at 0 and at x.

$U(1)$: $\int dU(0)\, U(0) = \int_0^{2\pi} d\theta\; e^{i\theta}$. This $\int$ is 0

$SU(N)$: $\underset{\text{group manifold}}{\int dU(x)}\;\; U(x) = 0$ avg over manifold $= 0$



$S(x) = 0$. Any correl. func. where a $U(x)$ is not compensated by a $U^\dagger(x)$ integrates to 0.

W/O gauge fixing, all gauge-dependent correl. func's $= 0$

Only gauge-inv. corr. func. are nonzero.

So back to gauge-fixed. Maybe Landau-gauge correl. are "right" ones?

No. A brief calculation in ξ-gauge finds the anomalous dimensions at 1-loop to be

$$\gamma_q = +\xi C_2[R]\frac{\alpha}{4\pi} \qquad \left(\alpha = \frac{g^2}{4\pi}\right)$$

$$\gamma_g = -\left(\frac{13}{6}N_c - \frac{\xi}{2}N_c - \frac{2}{3}N_f\right)\frac{\alpha}{4\pi} \underline{\underline{<}} 0$$

ξ-dependence. maybe $\xi = 0$ Landau gauge is "Right Gauge"??

But $\gamma_g < 0$. Recall

$$\langle A_\mu(0) A_\nu(x)\rangle \propto x^{-2-2\gamma_g}$$

or in p-space $G_{\mu\nu}(p) \propto p^{-2+2\gamma_g}$

But $G_{\mu\nu}(p) = \int_0^\infty ds\ \rho(s) \times \frac{1}{p^2+s}$ / falls slower than $1/p^2$

spectral func.

$\gamma_g < 0$: $G_{\mu\nu}(p)$ falls faster than $1/p^2$.

Only possible if $\rho(s)$ not uniformly nonnegative.

That is, $\rho(s) < 0$ in some s-range.

If $A_\mu|0\rangle = |\text{gluon}\rangle$ physical states,

that gives phys. states w. negative norm — not unitary.

Remark: QED also has gauge-fixing issues.

$S(x) = \langle 0 | \psi(0) \bar{\psi}(x) | 0 \rangle = 0$ without gauge fixing.

Less severe - Landau gauge basically works, as $\gamma_\gamma > 0$

Alternately, use Wilson lines to make $S(x)$ sensible, "does little damage" to physical interpretation.

But ultimately QED also "not really" thy of $e^+ e^- \gamma$.

You can never have pure e^- without some very low-E γ's.

We will revisit this issue soon.

If
- take QCD
- keep M_q fixed at MeV scale
- make $\alpha_s = 1/50$ at 100 MeV scale

then it's "practically" thy of quarks, coupling to gluons like e^+ couple to γ's, except gluons re-interact.
well defined hydrogen-like bound states, "quasi" free quarks

At large scales, g interact in complex way (unlike γ) but...

Reason QCD "confines" - M_q not well separated from Λ_{QCD} scale where α_s becomes large.