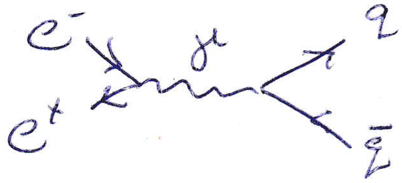
Consider again $e^+e^- \to$ (hadrons)

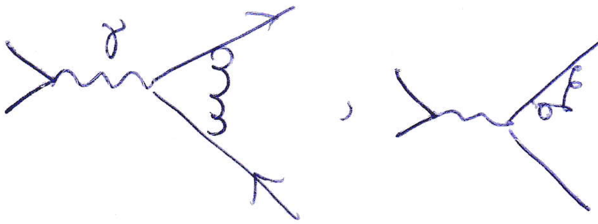
Leading:



$$\sigma = \frac{4\pi\alpha^2}{3s}\cdot 3Q_q^2 \qquad (3 = N_c,\ \tfrac{4\pi\alpha^2}{3s} = \sigma_0)$$

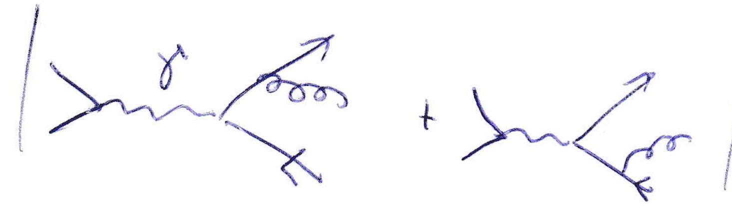
summed on q species.

NLO:



interfere with ↑ reducing σ.

And new process



to compute.

Phase space of 3 particles:
5 int's, 3 angular & 2 energy

$$\int \frac{d^3q_1\, d^3q_2\, d^3k}{(2\pi)^9\, 2q_1^0\, 2q_2^0\, 2k^0}(2\pi)^4\delta^4(P_{in}-q_1-q_2-k)$$

(somework)

$$= \frac{P_{in}^2}{128\pi^3}\int_0^1 dx_1\int_{\cancel{1/2 - x_1}\ 1-x_1}^1 dx_2$$

$x_1 = \frac{q_1^0}{(P_{in}/2)}$, $x_2 = \frac{q_2^0}{(P_{in}/2)}$ energies as frac. of beam energy ($E_{cm} = 2E_{beam}$)

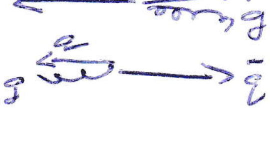
$(P_{in}/2) = E_{beam}$

More work:

$$\sigma_{q\bar{q}g} = \sigma_{q\bar{q}} \times \frac{2\alpha_s}{3\pi}\int_{x_1+x_2>1}^{1} \frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}\, dx_1\, dx_2$$

→ approx form for ~~large~~ small $1-x$

oops - diverges as $x_1 \to 1$
or $x_2 \to 1$



(and $\log^2$ as both $\to 1$)

Very large rate for process w. gluon.

Goodnews: regulate (eg, cut off g energy $> \varepsilon$)

Div cancel

QCD4 P2

Message:

1) Inclusive rates are finite, IR-safe

2) Exclusive have IR & collinear divergences

~~2 gluons~~ 1 gluon: $\alpha_s \log^2(q^2/\text{IR scale})$

similarly, large gluon energy region of calc. is safe.

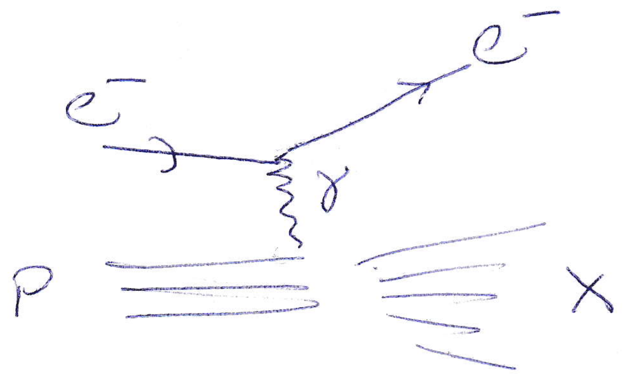
oops - $\log(q^2/\text{IR scale}) \sim 1/\alpha_s$ if IR scale $= \Lambda_{QCD}$

More than compensates α_s.

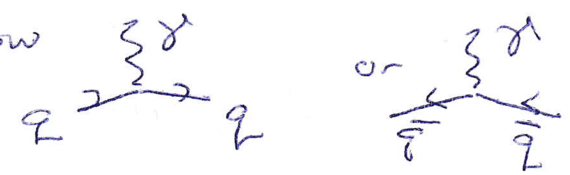
2 gluons: $\alpha_s^2 \log^4$ .... clearly no convergence

Message quite general. Try not to ask exclusive questions.

Example 2: $\sigma_{\text{differential}}$ for $e^- p \to e^-(\text{anything})$



I know



as those are only vertex which exists for γ to attach

For this process $\frac{d\sigma}{d\hat{t}}(e^- q \to e^- q) = \frac{2\pi\alpha^2 Q_f^2}{\hat{s}^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$ $\hat{s}, \hat{t}, \hat{u}$ Mandel. for q, e system.

But is there a q inside a proton?

$g_{\mu\nu}/q^2$: q set by $k'-k$ diff.

Amplitude for scatt:

$$\langle X e^-_{k'} | (\bar{e}\gamma^\mu A_\mu e)(\bar{q}\gamma^\nu A_\nu q) | p\, e^-_k \rangle$$

A bit more explicit: for <u>given</u> k' of e^-_{final},

$$\int d^4x\, d^4y \; \langle X e^-_{k'} | (\bar{e}\gamma^\mu A_\mu e)^{(x)} (\bar{q}\gamma^\nu A_\nu q)^{(y)} | p\, e^-_k \rangle$$

need $\int d^4x \langle e^-_{k'} | \bar{e}\, e(x) | e_k \rangle$ nonvanishing →

write $\int d^4x\; \bar{e}\gamma^\mu e(x) A_\mu(x) = \int dx \underbrace{\int d^4q\; A_\mu(q) e^{-iq\cdot x}}_{A_\mu(x)\text{ is}\dots} \bar{e}\gamma^\mu e(x)$

$= \int d^4q\; A_\mu(q) (\bar{e}\gamma^\mu e)(-q)$ as $\int d^4x\, e^{-iq\cdot x} f(x) = f(-q)$

I <u>need</u> $\langle e_{k'} | \bar{e}\gamma^\mu e(-q) | e_k \rangle$ to have $q = k - k'$

So I really want $\int d^4y\; e^{iq\cdot y} \langle X | \bar{q}\gamma^\nu q(y) | p \rangle$

$\bar{q}\gamma^\nu q$ operator at <u>mom</u> q.

Since q-op looks for q content, I can ~~write~~ insert complete set of states containing a quark:

$$|p\rangle = \sum_{X'} \int d\bar{p}\; |q_{\bar{p}} X'\rangle \langle q_{\bar{p}} X' | p\rangle$$

X': any other stuff which might be in a proton. $\bar{p}$: momentum of q in proton.

$\bar{p} = \xi\, p_{proton}$ + (small transverse part we ignore)

Now I can compute $\displaystyle\sum_X \left| \langle X | \bar{q}\gamma^\nu q(q) | q_{\xi p} X' \rangle \right|^2$

total cross-sec. for q, mom. ξp, to scatter, summed over final states

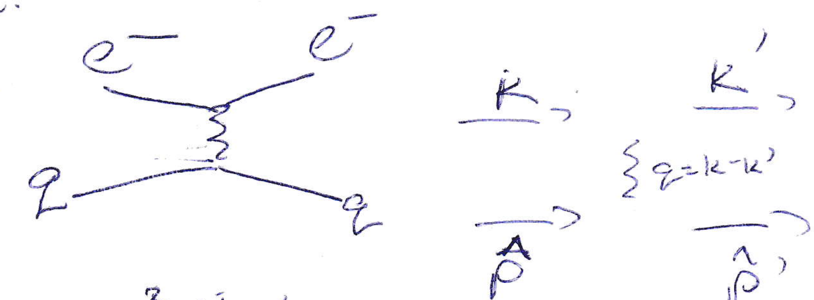
$$\int d\bar p\; \delta(\bar p^0 - \xi p^0)\, |\langle q_{\xi p} X' | p \rangle|^2$$

$=$ prob. per unit ξ, that proton contains q of momentum frac. ξ.

Call this $q(\xi)$ (usually $q(x)$) "Parton Dist Func."

x directly measurable!

Consider elastic scatt



$$-q^2 = -(k-k')^2 \text{ like } t$$

$$2\hat p\cdot q = 2\hat p\cdot(k-k') = 2\hat p\cdot(\hat p' - \hat p) = 2\hat p\cdot\hat p' = -q^2$$

$$\text{so } \frac{-q^2}{2\hat p\cdot q} = 1. \quad \text{so } \frac{\hat p}{p} = \frac{\hat p\cdot q}{p\cdot q} \quad \text{so } \frac{-q^2}{2p\cdot q} = \overbrace{\frac{-q^2}{2\hat p\cdot q}}^{1}\left(\frac{\hat p\cdot q}{p\cdot q}\right)$$

$$\frac{-q^2}{2p\cdot q} \equiv x$$ is same as ξ en. frac. in quark.

$\vec q$ measured from $\vec k, \vec k'$ so I get this from beam energy & e^- kinematics.

At lowest order, I can say

Scattering rate $=$ (prob. p contains q of frac x) $*$ (scatt. of e^- w. q of mom. xp)

Also I expect (find) $\int_0^1 dx\,\big(u(x)+d(x)-\bar u(x)-\bar d(x)\big) = 3$ total q content

$$\int_0^1 dx\; x\cdot\big(u(x)+d(x)+s(x)+\dots+g(x) + \bar u(x)+\bar d(x)+\bar s(x)\big) = 1$$ total energy = total energy.

$pp \to e^+e^- + \text{stuff}$

$$\langle e^- e^+ X | \underbrace{\bar{e}\gamma^\mu A_\mu e}_{\text{to get } e^+e^-} \; \underbrace{\bar{q}\gamma^\nu A_\nu q}_{\text{to get } A_\mu} | P_1 P_2 \rangle$$

Again, $A_\mu(q)$ now timelike 4-mom of "dilepton"

$\bar{q}q$ must add up to 4mom q

Kinematics: must be $\bar{q}$ from one p, q from other.

Again, $|q\bar{q}X'X''\rangle\langle q_{x_1 p}\, X' | P_1 \rangle \langle \bar{q}_{x_2 p'}\, X'' | P_2 \rangle$

and

$$\langle X | \bar{q}\gamma^\mu A_\mu q | q_{xP_1} \bar{q}_{x_2 P_2} X' X'' \rangle$$

same as total ways $q\bar{q}$ can get absorbed onto γ-line.

$$\int dx_1 dx_2 \; \sigma(q(P_1 x_1)\bar{q}(P_2 x_2) \to e^+e^-) \; \underbrace{q(x_1)\bar{q}(x_2)}_{\text{PDFs.}}$$

At leading-order.

Similarly for jet prod...