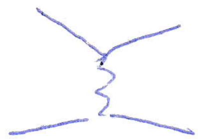
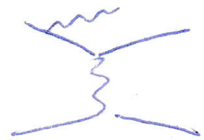
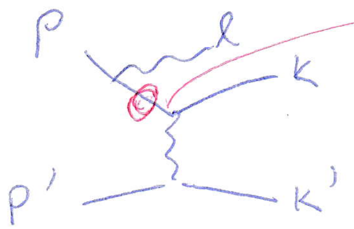
Consider (QED to make life easier)



LO,



NLO



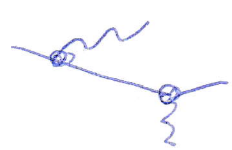
propagator $\frac{i}{\not p + \not l - m} = i \frac{\not p + \not l + m}{(p+l)^2 - m^2}$

But $p^2 = +m^2$, $l^2 = 0$, so $(p+l)^2 - m^2 = 2p \cdot l =$

Denominator gets small → $\mathcal{M}$ large, if $l \to 0$ and/or p, l become parallel

effects: 1) Amplitude, prob. of emission large,

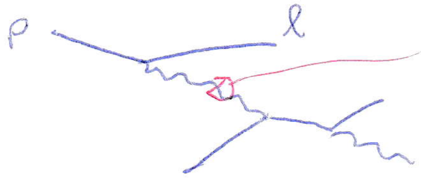
2) physical separation of



vertices large

Emission, then scattering.

Also process,

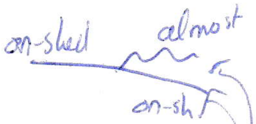


$q = p - l$

$$\frac{1}{q^2} = \frac{1}{(p-l)^2} = \frac{1}{p^2 + l^2 - 2p \cdot l} = \frac{1}{2m^2 - 2p \cdot l}$$

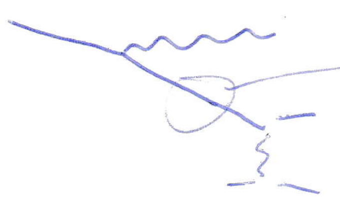
At high energy m^2 small, $p \cdot l \to 0$ if collinear process, again large amplitude.

Idea: think of as 2-step process

1)  (or )

2) one of these undergoes scatt.

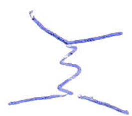
Key:



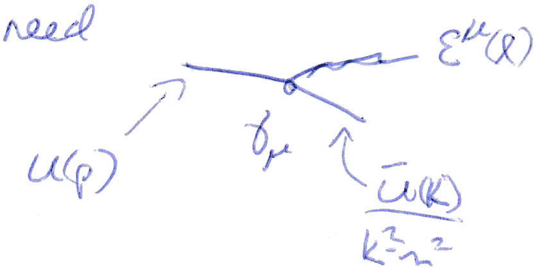
call mom. k.

$$\frac{i\not{k}+m}{k^2-m^2}$$

small m, almost on-shell k: $\not{k} \simeq \sum_\sigma u(k)\,\bar{u}(k)$

Put the $\bar{u}(k)$ into  – like normal external scatt.

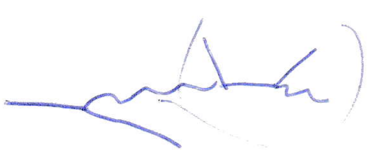
I need



$\bar{u}(k)\,\not{\varepsilon}\,u(p)$ like $\mathcal{M}$ for 1–2 process

$\frac{1}{k^2-m^2}$ part of weight for what k may occur – will $\int d^4k$ the square of this.

Similarly

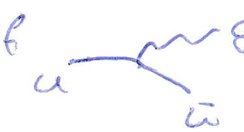


$$\frac{-ig^{\mu\nu}}{k^2}$$

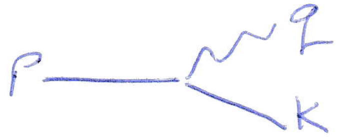
$$g^{\mu\nu} = -\sum_\lambda \epsilon^\mu_\lambda \epsilon^{\nu *}_\lambda + \varepsilon^\mu \bar{\varepsilon}^\nu + \bar{\varepsilon}^\mu \varepsilon^\nu$$

$\propto k^\mu$. Vanishes by on-shell...

$$-ig^{\mu\nu} \simeq i\sum_\lambda \varepsilon^\mu \varepsilon^{\nu *}$$

Use ε^μ as part of , $\varepsilon^{\nu *}$ part of  $\mathcal{M}$ for 1–2 ...

So let's compute. Choose



$$p = (p, 0, 0, p) \quad (p0 0p)$$

$$q = (zp, q_\perp, zp) + (\delta q^0, 0, 0, 0)$$

$$k = ((1-z)p, -q_\perp, (1-z)p) + (\delta k^0, 0, 0, 0)$$

for q on-shell, $(zp + \delta q^0)^2 = q_\perp^2 + z^2 p^2$

$$2zp\,\delta q^0 \simeq q_\perp^2 \qquad \delta q^0 \simeq q_\perp^2 / 2zp$$

Cons. of momentum: $\delta k^0 = -\delta q^0$. But to be onshell we would need

$$\delta k^0 = q_\perp^2 / 2(1-z)p$$

so $$\delta k^0 - \delta k^0_{onshell} = \frac{q_\perp^2}{2p}\left(\frac{1}{1-z} + \frac{1}{z} = \frac{1}{z(1-z)}\right)$$

Similarly if k on-shell, $\delta q^0 - \delta q^0_{on\text{-}sh} = \frac{q_\perp^2}{2p}\frac{1}{z(1-z)}$

former: $k^2 = 2k^0 \delta k^0$, $\frac{1}{k^2} = \frac{1}{q_\perp^2}\frac{z(1-z)\cdot 2p}{(1-z)2p}(= 2k^0) = \frac{z}{q_\perp^2}$

latter: $\frac{1}{q^2} = \frac{(1-z)}{q_\perp^2}$

Neglect $\delta q^0, \delta k^0$ in computing matrix element

$$i\mathcal{M} = \bar{u}_\sigma(k)\, \not{\varepsilon}^*_q\, u(p)$$

Consider incoming left-handed - $\bar{u}_\sigma$ must also be left, $u = \sqrt{2p^0}\begin{pmatrix}\xi(p)\\0\end{pmatrix}$ $\xi(p) = \begin{pmatrix}0\\1\end{pmatrix}$

$$\varepsilon_L^{*i} = \frac{1}{\sqrt{2}}\left(1, i, -\frac{p_\perp}{zp}\right) \quad \text{mixing of } p_x \ldots$$

$u(k) = \sqrt{2k^0}\begin{pmatrix}\xi(k)\\0\end{pmatrix}$ $\xi(k) = \begin{pmatrix}\frac{p_\perp}{2(1-z)p}\\1\end{pmatrix}$

$$\gamma_\mu = \begin{pmatrix}0 & \sigma_\mu\\ \bar{\sigma}_\mu & 0\end{pmatrix}$$

$\varepsilon^{*i} = \frac{1}{\ldots}(1, \ldots, \ldots)$

Note $\frac{1}{k^2} \sim \frac{1}{k_\perp^2}$ happens in $\mathcal{M}$ and $\mathcal{M}^*$

leading to $\int dk^0 dk_z d^2k_\perp \, f(k^2) \frac{1}{(k_\perp^2)^2}$

$\frac{d^2 k_\perp}{k_\perp^4}$ diverges power!

But at $k_\perp = 0$ level,

$(0\ 1)(\sigma_x, \sigma_y)\begin{pmatrix}0\\1\end{pmatrix}$ give 0.

only nonzero due to $\sigma_z \propto -\frac{q_\perp}{zp}$ and $q_\perp / 2(1-z)p$ term in $\xi(k)$

...

$$\frac{2e^2 k_\perp^2}{z(1-z)} \Bigg(\underbrace{1}_{\text{one } \gamma \text{ pol}} + \underbrace{(1-z)^2}_{\text{other } \gamma\text{-pol}} \Bigg/ z \Bigg)$$