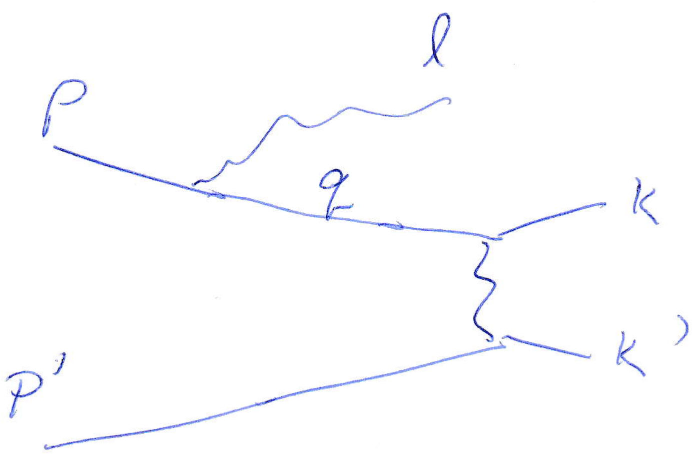
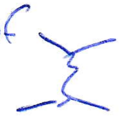
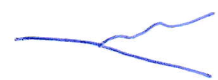
Finish Splitting

Consider



Break q propagator into $\frac{u\bar{u}(q)}{q^2-m^2} = u \frac{1}{q^2-m^2} \bar{u}$

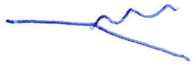
part of  part of 

Last time: $\int \frac{d^4 l}{(2\pi)^4} 2\pi\delta(l^2) = \int \frac{dl^0}{\cancel{2\pi}} \frac{dl_z}{2\pi} \int \frac{d^2 l_\perp}{(2\pi)^2} \cancel{2\pi}\, \delta(l_0^2 - l_z^2 - l_\perp^2)$

$1/2l_0 = 1/2zP$

$\frac{dl_z}{P} = \int dz$ momentum fraction

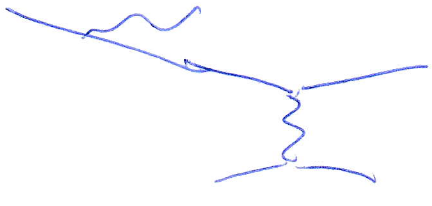
so $\to \int_0^1 \frac{dz}{z} \frac{1}{4\pi} \frac{1}{4\pi} \int d(l_\perp^2)$

Last time:  produced $|M|^2 = \frac{2e^2 l_\perp^2 (1+(1-z)^2)}{(1-z)z^2}$

$\left(\frac{1}{q^2-m^2}\right)^2 = \frac{z^2}{(l_\perp^2)^2}$ ~~Ah~~

Finally, $\frac{1}{2p\,2p'}|M|^2_{pp'\to kk'} \to \frac{1}{2p\,2p'}|M|^2_{qp'\to kk'} = \frac{(1-z)}{2q^0\,2p'^0}|M|^2_{qp'\to kk'}$

~~Ah~~

Combining 

emitting phase space

$$\int \frac{d\ell^0 \, d\ell_z \, d^2\ell_\perp}{(2\pi)^4} \, 2\pi\delta(\ell^2)$$

Matrix element propagator matrix element

$$\downarrow$$

$$\int \frac{dz}{z} \frac{1}{16\pi^2} \int d(\ell_\perp^2) \frac{2e^2(1+(1-z)^2)\ell_\perp^2 \cancel{z^2}}{\cancel{z^2}(1-z)\;(\ell_\perp^2)^2} (1-z) \times (\text{usual } 2\to 2 \text{ calc.})$$

$$= \int \frac{dz}{z} \underbrace{\int \frac{d(\ell_\perp^2)\,\ell_\perp^2}{(\ell_\perp^2)^2}} \left(\frac{e^2}{8\pi^2}\right)(1+(1-z)^2) \times \sigma_{(1-z)p,p'\to kk'}$$

Do first. Max value of $\ell_\perp^2$ is zs

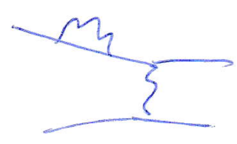
min value is $z m_e^2$ – the $\frac{1}{\ell_\perp^2}$ ignored mass effects which emerge here.

$\longrightarrow \ln\left(\frac{s}{m_e^2}\right) + \mathcal{O}(1)$. We have to think about meaning

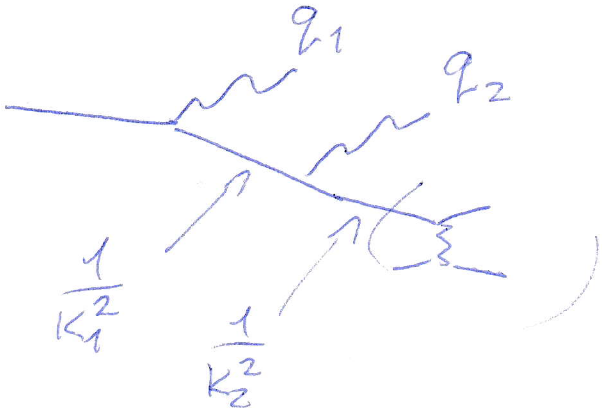
$$\underset{1-\gamma\text{ emit}}{\sigma_{scatt}} = \left(\frac{\alpha_{EM}}{2\pi}\right)\left(\ln\frac{s}{m_e^2}\right)\int_0^1 dz \left(\frac{1+(1-z)^2}{z}\right) \underset{\text{Energy }(1-z)p}{\sigma_{scatt,\,no\,\gamma}}$$

Meanwhile

$$\underset{\text{no }\gamma\text{ – 1 loop correction}}{\sigma_{scatt}} = \quad '' \quad '' \quad '' \quad \underset{\text{Energy } p}{\sigma_{scatt,\,no\,\gamma}}$$



What about 2 γ emit?



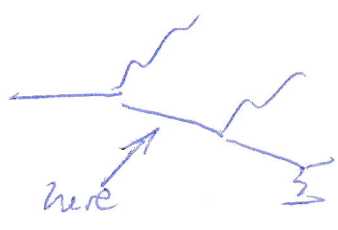
eg if $q_{1\perp}^2 \ll q_{2\perp}^2$

Claim: $\underline{\underline{\text{If}}}$ $\frac{1}{k_1^2} \ll \frac{1}{k_2^2}$ I get iteration of last calculation

$$\int_{m^2}^{s} \frac{d q_{1\perp}^2}{(2\pi)^2} \frac{1}{q_{1\perp}^2} \int_{q_{1\perp}^2}^{s} \frac{d q_{2\perp}^2}{(2\pi)^2} \frac{1}{q_{2\perp}^2}$$

But ~~if~~ the $q_{2\perp}^2 \ll q_{1\perp}^2$ part of $\int$ does not contribute much

why? if $q_{1\perp}^2 \ll q_{2\perp}^2$, e^-



may as well be on-shell
Same calc applies.

opposite case - it's $\underline{\text{not}}$ almost on-shell - say, $k_1^0 = \sqrt{\vec{k}_1^{\,2}} + \delta k_1^0$

then $\int d^4k_2 \int dq^0\, dq_z\, d^2q_\perp\, \delta^4(k_1 - q_2 - k_2)\, \delta(q_2^2)$

has $\int dq^0\, \delta(q_0^2 - q_z^2 - q_\perp^2)$ set $q^0 = q_z + \frac{q_\perp^2}{2q_z}$ as before ← small

then $k_2^0 = k_1^0 - q^0 = \delta k_1^0 + k_z - q_z + (\text{small})$

So enhancement from propagator doesn't make up for $q_\perp^2$ phasespace

$$\int \frac{d^2 q_{\perp 2}^2 \, q_{\perp 2}^2}{(q_{\perp 1}^2)^2} \quad \text{for } q_{\perp 1}^2 \gg q_{\perp 2}^2 \quad \text{no large contrib.}$$

With this in mind we can solve probs to have energies from "outside - in"

$$\sigma_{scatt} = \int_0^1 dz \left(\frac{d\text{Prob to have } e^- \text{ or } \gamma \text{ w. fraction } z}{dz} \right) \cancel{d}\sigma_{zp\ldots}$$

for 1 emission we found this was $\frac{\alpha}{2\pi}\left(\frac{1+(1-z)^2}{z}\right)\ln s/m_e^2$

for photon, $z \leftrightarrow (1-z)$ for electron ...

0 emission : $\delta(z-1)\,[1 - \text{chance to have 1 emission}]$

Name these prob's $e_e(z)$, $\gamma_e(z)$. Also depend on s.

chance to have e / γ

if you start with e

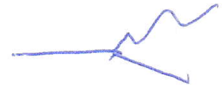
and look at mom frac z

Call intermediate quantity : $e_e(z)$ counting emissions with $q_\perp^2$ up to some value μ^2,

$e_e(z,\mu^2)$ book uses f_e

Then way $\int d^2 q_\perp^2$ int. worked, and nested nature of ints,

tells us that

As you increase μ^2,  process should turn e's into $e\gamma$ pairs.

Evolving w μ^2 (like RG) captures iterated emissions?

Pick μ_2^2 slightly larger than μ_1^2

$$e_e(z, \mu_2^2) = e_e(z, \mu_1^2) + \ln\frac{\mu_2^2}{\mu_1^2} * \text{coeff. of } \int \frac{dq_\perp^2}{q_\perp^2} \text{ of all ways } z'\text{-}e^- \text{ could turn into } z\text{-}e^-$$

$$- \ln\left(\frac{\mu_2^2}{\mu_1^2}\right) * \text{coeff of } \int \frac{dq_\perp}{q_\perp^2} \text{ of all ways } e^-(z) \text{ could turn into } \gamma, e \text{ of smaller mom frac.}$$

$$\mu^2 \frac{d}{d\mu^2} e_e(\cancel{x}, \mu^2) =$$

think about it

$$= \int_{\cancel{x}}^{1} \frac{dz}{z} \left(\frac{1+z^2}{(1-z)}\right) \frac{\alpha}{2\pi} e_e\left(\frac{x}{z}, \mu^2\right)$$ ways someone else makes me

$$- \underbrace{\int_0^1 dz \left(\frac{1+(1-z)^2}{z}\right) \frac{\alpha}{2\pi}}_{\text{ways of } \to e\gamma} e_e(x, \mu^2)$$

$$\mu^2 \frac{d}{d\mu^2} \gamma_e(x, \mu) = \int_x^1 \frac{dz}{z} \left(\frac{1+(1-z)^2}{z}\right) \frac{\alpha}{2\pi} e_e\left(\frac{x}{z}, \mu^2\right)$$

QCD6 P6

Subtlety 1: canceling divergences

$$\int_x^1 \frac{dz}{z}\left(\frac{1+z^2}{1-z}\right)\frac{\alpha}{2\pi}\, e_e\!\left(\frac{x}{z},\mu^2\right)$$

diverges as $z \to 1$. where $e_e(\frac{x}{z} \to x)$ same as in other term!

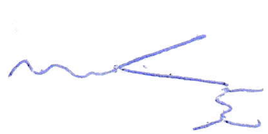
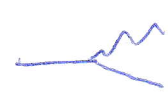
Define $\frac{1}{(1-x)_+}$ as " $\frac{1}{1-x}$ but with negative δ-func. at $x=1$ "

such that $\int_0^1 dx \frac{f(x)}{(1-x)_+} = \int_0^1 dx \frac{f(x)-f(1)}{1-x}$ is well defined.

(or $\lim_{\varepsilon\to 0} \frac{1}{1-x}\Theta(1-x-\varepsilon) - \delta(1-x)\int_0^{1-\varepsilon} dx'/(1-x')$)

note, $\int_0^1 dx \frac{1+x^2}{(1-x)_+} = -3/2$ (show!)

New $$\mu^2\frac{d}{d\mu^2} e_e(x,\mu^2) = \int_x^1 \frac{dz}{z}\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]\frac{\alpha}{2\pi}\, e_e\!\left(\frac{x}{z},\mu^2\right)$$

Next: realize that  also possible.  $|M|^2$ by crossing from 

$$\mu^2\frac{d}{d\mu^2} e_e(x,\mu^2) = \frac{\alpha}{2\pi}\int_x^1 \frac{dz}{z}\Big[P_{e\leftarrow e}(z)\, e_e(x/z,\mu^2) + P_{e\leftarrow\gamma}(z)\, \gamma_e(x/z,\mu^2) \Big)$$

similar for γ

$$P_{e\leftarrow e} = \frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \qquad P_{e\leftarrow\gamma} = z^2 + (1-z)^2$$