R_ξ Gauge

The Problem: ~~Fixing~~

Consider scalar field φ, Vac. value φ_0 (eg $\begin{bmatrix}\varphi_1\\ \varphi_2\end{bmatrix}$, $\begin{bmatrix}0\\ v/\sqrt{2}\end{bmatrix}$)

Consider group generators T^A for which $T^A\varphi_0 \neq 0$
Associated A^A_μ will become massive.

Unitary Gauge strategy: choose gauge-fix function
to force all $T^A\varphi_0 \to 0$, eg. $G^A = \cancel{\varphi^\dagger T^A \varphi_0}\ \varphi_0^\dagger T^A \varphi$

$\int \mathcal{D}A_\mu \mathcal{D}\varphi\, e^{i\int d^4x \mathcal{L}}\ \delta(\varphi_0^\dagger T^A \varphi(x))$ one δ-func. per gauge freedom ✓

Problem – UV fluctuations – for large momenta $p \gg |v|$,
fluct. bigger than VEV $|\varphi_0|$...

Alternative – go back to $G^A = \partial^\mu A^A_\mu$?

Or some mix $G^A = \partial^\mu A^A_\mu + C_* \varphi_0^\dagger T^A \varphi$?

Hint: think again about $\mathcal{L}$ and problem of inverting kin. terms
to find propagators

$$\mathcal{L} = A_\mu \left[g^{\mu\nu}\partial_\alpha\partial^\alpha - \partial^\mu\partial^\nu \right] A_\nu \quad + \text{cubic} + \text{in fields}$$
$$\underbrace{-\varphi^*\left[D^\mu D_\mu \right]\varphi}_{\text{look at this more carefully}} - V(\varphi^*\varphi)$$

$-\varphi^* D^\mu D_\mu \varphi \qquad D_\mu = \partial_\mu - igT^A A^A_\mu$

Vev-squared: $-\underbrace{\varphi_0^* (-ig)^2 T^A T^B \varphi_0}_{\text{mass term for some A-fields}} A^A_\mu A^{\mu B} \equiv \frac{1}{2} M^2_{AB} A^A_\mu A^{B\mu}$

Linear in Vev: $-\varphi_0^*(-igT^A \varphi\, \partial_\mu A^{\mu A})$

$-\varphi(-igT^A \varphi_0\, \partial_\mu A^{\mu A}$

$-2ig\,\varphi_0^* T^A A^\mu_A \partial_\mu \varphi$ → can ∫ by parts onto A^μ as well

These terms <u>mix</u> A^A_μ with φ – propagator not diagonal.

Nondiagonal propagator – pain in neck.

When we <u>add</u> $\frac{1}{\xi} G^A G^A$ – choose c such that these terms are eliminated!

$G^A = \partial_\mu A^\mu - \xi ig\, \varphi_0^* T^A \varphi$

$\frac{1}{\xi} G^A G_A = \underbrace{\frac{1}{\xi} \partial_\mu A^\mu \partial^\nu A_\nu}_{\text{what we needed before}} - \underbrace{2\partial_\mu A^\mu ig\, \varphi_0^* T^A \varphi}_{\text{cancels terms from } -\varphi^* D_\mu D^\mu \varphi} - \xi g^2 |\varphi_0^* T^A \varphi|^2$

↓ masses for φ-fluct.

Gauge fields:

$$\frac{1}{2} A^A_\mu \underbrace{\left[\delta^{AB} g^{\mu\nu}\partial_\alpha\partial^\alpha - \partial^\mu\partial^\nu + g^{\mu\nu} M^2_{AB} + \frac{\delta^{AB}}{\xi}\partial^\mu\partial^\nu \right]} A^B_\nu$$

Inverse is $G_{\mu\nu}(p) = \dfrac{-i\left(g_{\mu\nu} + (\xi - 1)\dfrac{p_\mu p_\nu}{p^2 - \xi M_A^2}\right)}{p^2 - m_A^2 + i\epsilon}$ (in basis which diagonalizes M^{AB} matrix)

Check: <u>nontrivial</u>

Each broken T^A has goldstone boson — now massive due to $\xi g^2 |\varphi_0^* T^A \varphi|^2$. $\text{Mass}^2 = M_A^2$ same as above

note

Propagator: "Would-Be Goldstone boson" $\dfrac{i}{p^2 - \xi M_A^2 + i\epsilon}$

Ghosts? $\bar{c}_A [\,\cdots\,] c_B$

$\hookrightarrow \dfrac{\delta G^A}{\delta \Theta_B}$ gauge dependence of G^A function, which was chosen to <u>be</u> gauge-dep.

As usual $\dfrac{\delta A^\mu_A}{\delta \Theta_B} = D^\mu_{AB}$ Adj. covariant deriv.

But now also $\dfrac{\delta \varphi}{\delta \Theta_B} = i g T^B \varphi \rightarrow \xi g^2 \varphi_0^* T^A T^B \varphi_0$

$= \xi M_A^2$ same mass as gauge bosons

Ghost-propagator $\dfrac{i}{p^2 - \xi M_A^2 + i\epsilon}$

Crazy: propagators have ξ-dependent masses!

$$G_{\mu\nu} = -i \, \frac{g_{\mu\nu} + (\xi - 1)\frac{P_\mu P_\nu}{p^2 - \xi M_A^2}}{p^2 - M_A^2}$$

Longitudinal mode
Ghost
WB Goldstone
all not valid external states.
Don't sum - just like BRST story.

WB Goldstone $\rightarrow \dfrac{i}{p^2 - \xi M_A^2}$

Ghosts "

R_ξ gauge

Limits: $\xi \to 0$: $G_{\mu\nu}$ purely transverse, Ghost & WBG massless

$\xi \to 1$: $G_{\mu\nu}$ simple, all masses $= M_A^2$. R_ξ - Feynman.

$\xi \to$ large : WB Gold, Ghost decouple - except in loops

$$G_{\mu\nu} \to \frac{g_{\mu\nu} + \xi \frac{P_\mu P_\nu}{-\xi M_A^2}}{p^2 - M_A^2} = \frac{g_{\mu\nu} - \frac{P_\mu P_\nu}{M_A^2}}{p^2 - M_A^2}$$

$\xi \to$ large approaches unitary gauge.
Must keep ξ finite inside loops

Note: Ghosts have $\bar{c}_A (\partial_\mu D^\mu_{AB} + i g^2 \varphi_0 T^A T^B \varphi) c$
incl. fluct incl. fluct.

 and  couple to Gauge & Would-be Goldstone and to Higgs

Fermions? Sure. Yukawas induce masses, Higgs-couplings, and WB Goldstone couplings!