Standard Model P1

Gauge Group

	(G_μ) $SU_C(3)$	$\times$ (W_μ) $SU_W(2)$	$\times$ (B_μ) $U_Y(1)$	
Scalar	$\mathbb{1}$	2	$+1/2 = Q_h$	doublet, $Q=1/2$
$P_L Q_f$	3	2	$+1/6$	chiral, eg, Majorana fermionic fields.
$P_R U_f$	3	1	$2/3$	
$P_R D_f$	3	1	$-1/3$	
$P_L L_f$	1	2	$-1/2$	Each in 3 copies "generations" $f=1\,2\,3$
$P_R E_f$	1	1	-1	

Just look at leptons first.

$$\mathcal{L} = \mathcal{L}_{gauge} + \bar{L}\gamma^\mu\left[\partial_\mu - igW_\mu^a\frac{\tau^a}{2} + i\frac{g'}{2}B_\mu\right]P_L L$$

$$\cancel{+ h.c.\ eg,\ + i g W \ldots + i\frac{g'}{2}}$$

$$+ \bar{E}\gamma^\mu\left[\partial_\mu + ig'B_\mu\right]P_R E$$

$$+ y_{ff'}\, h\, \bar{L}_f P_R E + h.c.$$

h is doublet

✓ $\bar{L}$ is $*$ of doublet contract into singlet

$Q=1/2 \quad Q=1/2 \quad Q=-1$ ✓

$Y_{ff'}$. $\quad h = \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix}$ spont. breaks. $\quad \bar{L} = [\bar{\nu}\ \ \bar{e}_L]$

neutrino electron$_L$

Higgs couples e_L to E_R → electron

Problem: Anomalies

$$\partial_\mu J^\mu_L = \frac{g^2}{32\pi^2} W^A_{\mu\nu} \tilde{W}^{A\,\mu\nu} \neq 0 \quad \text{net } L\text{-\# generated by W-boson } E\cdot B$$

Anomaly - creation of an odd # of L's - violates Q

violates SU(2) singlet.

Need more fields - an odd # of L-handed fields

with total E-charge = +1/2

That's what $P_L Q$: 3 colors, chg 1/6 - does !!

New Yukawas: $Y_{D ff'} \, h \, \bar{Q}_f P_R D_{f'}$

$$Y_{uff'} \, h^+_\alpha \epsilon^{\alpha\beta} \bar{Q}_{\beta f} P_R U_{f'}$$

h : chg +1/2

h^+ : chg −1/2

charges OK!

$$Q = \begin{bmatrix} u_L \\ d_L \end{bmatrix}$$

$$\bar{Q}_f h = \frac{v}{\sqrt{2}} [0 \;\; d_L]$$

$$\bar{Q}_{f\beta} h^+_\alpha \epsilon_{\alpha\beta} = \frac{v}{\sqrt{2}} [u_L \;\; 0]$$

upper/lower Q entry "higgs" couple to U, D.

→ up, down quarks.

$Y_D \frac{v}{\sqrt{2}}$ = down-quark mass matrix

$Y_u \frac{v}{\sqrt{2}}$ = up " " "

not simultaneously diagonal - CKM matrix....

Gauge Bosons, Masses

SM3

$$h = \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} + \text{fluctuations}$$

$$D_\mu h = \left(\partial_\mu - ig W_\mu^a \frac{\tau^a}{2} - i\frac{g'}{2} B_\mu\right) \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} \partial_\mu - i\left(\frac{gW^3 + g'B}{2}\right) & -\frac{ig}{2}(W_\mu^1 - iW_\mu^2) \\ -\frac{ig}{2}(W_\mu^1 - iW_\mu^2) & \partial_\mu - i\left(\frac{g'B_\mu}{2} - \frac{gW_\mu^3}{2}\right) \end{bmatrix} \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix}$$

Mass matrix $\frac{v}{\sqrt{2}} \left| \begin{matrix} -\frac{igW^1}{2} - \frac{gW^2}{2} \\ -\frac{i}{2}(g'B - gW^3) \end{matrix} \right|^2$

W^1: mass^2 $\frac{g^2v^2}{4}$. Same for W^2.

$\frac{W^1 - iW^2}{\sqrt{2}}$: W^+ "raising"

$\frac{W^1 + iW^2}{\sqrt{2}}$: W^- "lowering"

$$Z_\mu \equiv \frac{g'B_\mu - gW_\mu}{\sqrt{g^2+g'^2}} \qquad M_Z^2 = \frac{(g^2+g'^2)v^2}{4} > M_W^2$$

We define $\frac{g'}{\sqrt{g^2+g'^2}} = \sin\theta_W$ $\quad \frac{g}{\sqrt{g^2+g'^2}} = \cos\theta_W$ mixing angle showing how B, W^3 mix into Z

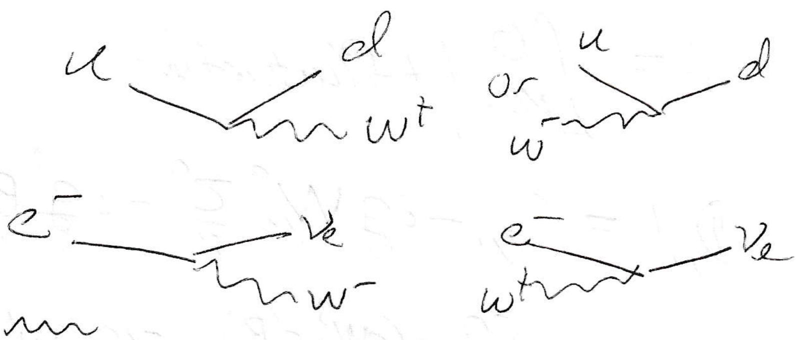
orthogonal comb. $A_\mu = \frac{gB_\mu + g'W_\mu^3}{\sqrt{g^2+g'^2}}$ is massless (count DOF!) Photon

$e \equiv \frac{gg'}{\sqrt{g^2+g'^2}}$ its coupling strength.

$$B_\mu = \frac{gA_\mu + g'Z_\mu}{\sqrt{g^2+g'^2}} \qquad W_\mu^3 = \frac{g'A_\mu - gZ_\mu}{\sqrt{g^2+g'^2}}$$

$W^\pm$ couple upper–lower components SM4

up-to-down
e^--to-ν_e



W^-, W^+ are just same ~
viewed in opposite ⇄ sense.

Allow violation of u, d, s, c, b, t – numbers

E couples to $B_\mu = \dfrac{gA_\mu - g' Z_\mu}{\sqrt{g^2+g'^2}}$

$$D_\mu E = (\partial_\mu + i g' B_\mu) E = \partial_\mu - i\left(\underbrace{\frac{gg'}{\sqrt{g^2+g'^2}} A_\mu}_{-eA_\mu} + \frac{g'^2}{\sqrt{g^2+g'^2}} Z_\mu\right) E$$

$-eA_\mu$: $e = \dfrac{gg'}{\sqrt{g^2+g'^2}}$ and $Q = 1$

Similarly $\underset{\text{up}}{U}$, $\underset{\text{down}}{D}$ have $Q = +2/3$ and $-1/3$ ✓

And L ?
$$\begin{bmatrix} \frac{gW^3 + g'B}{2} & \\ & \frac{-gW^3 - g'B}{2} \end{bmatrix} \begin{bmatrix} \nu \\ e_L \end{bmatrix}$$

in terms of A, Z :
$$\underbrace{\frac{gg'}{2}A}_{\text{from } W_3} - \underbrace{\frac{gg'}{2}A}_{\text{from } B} \to 0$$

$$\underbrace{-\frac{gg'}{2}A}_{W_3} - \underbrace{\frac{gg'}{2}A}_{B} \to -1$$

ν has $Q = 0$

e_L has $Q = -1$

Z-couplings are... computable.

SM5

Z-boson - flavor diagonal, L-R asymmetric, species-dependent coupling

A-boson : $Q_e = -1$ $\quad Q_u = 2/3$ $\quad Q_d = -1/3$ $\quad Q_\nu = 0$

W-boson : universal-strength u-d, ν-e couplings. ← not flavor diagonal.

$$e = \frac{g g'}{\sqrt{g^2+g'^2}}$$

g controls W-coupling

$$M_W^2 = \frac{g^2 v^2}{4} \qquad M_Z^2 = \frac{(g^2+g'^2)v^2}{4}$$

Masses, charges & couplings inter-related - testable ✓✓

Higgs couples ∝ mass, fermions & bosons. ✓

Higgs mass from $V(h^* h)$ - not determined from g, g'

Now well measured $M_H = 125$ GeV

Full Feynman rules : Burgess & Moore inside cover

R_ξ gauge - Appendix D