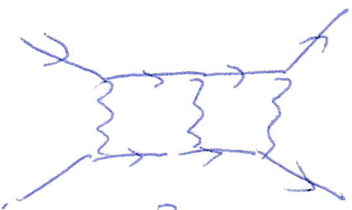
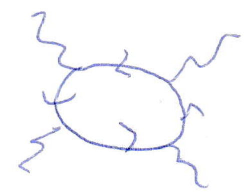
Power Counting

Consider a diagram  or  or...

How do I know if it is divergent?

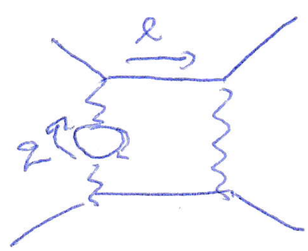
Possibility 1: when you $\int d^D q_1 \, d^D q_2 \ldots$ over loop momenta, making them <u>all</u> bigger & comparable increases/decreases contrib. (including phase space)

"Superficial Degree of Divergence"

Poss. 2: when a subset of loop $\int$'s get large, eg, $\int d^D q_1 \, d^D q_2$ with q_1 huge but q_2 modest, contribution gets bigger/smaller

"Divergent/convergent subdiagram"

eg.

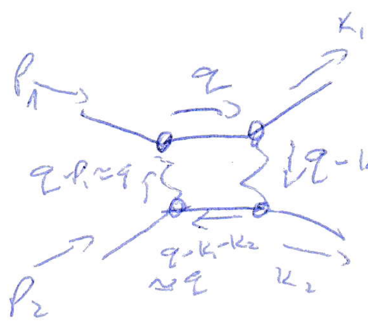


l, q both large: OK! (as we will see)

q large at fixed l: problem! (as we will see)

If I can figure out Superficial DOD, I can apply methodology to subdiagram (say, lines q flows through)

Consider box in Scalar QED



$$\underbrace{\int d^D q}_{4=D} \ \underbrace{\frac{g^{\mu\nu} \, g^{\alpha\beta}}{(q^2) \, (q^2)} \, \frac{1}{q^2} \, \frac{1}{q^2}}_{-8=(-2)\times 4} \ \underbrace{(2q)_\mu (2q)_\nu (2q)_\alpha (2q)_\beta}_{4} = q^0$$

Sum of $4 = D$ from each loop

L1P2

-2 from boson, -1 fermion propagators (internal only)

$+1$ vertex with ∂_μ, 0 vertex without

~~Positive~~: Negative $\int d^4q \frac{1}{q^6} \sim \int \frac{dq}{q^3}$ UV convergent - safe

(IR: worry about m^2 and P factors)

Zero: $\int dq/q$ log divergent

~~negative~~: Positive $\int dq\, q^{\#-1}$ power divergent

This quantity, $\overset{4}{D} \cdot (\#\overset{L}{\text{loops}}) - 2 \cdot (\#\text{bos-propagators})^{P_b}$

$- 1 \cdot (\#\text{fermion propagators})^{P_f}$

$+ 1 \cdot (\#\, \partial\text{-vertices})^{V_\partial}$

Called "Superficial degree of divergence" D_{sup}

Note: we previously saw $L = 1 + P_f + P_b - V$ $\quad V =$ all vertices

$$D_{sup} = D + (D-2)P_b + (D-1)P_b \underbrace{- DV + V_\partial}_{-DV_{no\partial} - (D-1)V_\partial}$$

Aside: think of $\frac{1}{\not p}$ as $\frac{\not p}{p^2} = \not p^{1/2} \frac{1}{p^2} \not p^{1/2}$ half a $\not p$ goes on each vertex, and $1/p^2$ for propagator.

3-pt vertex: either has a ∂ or has two fermions.

$2(P_f + P_b) + (N_b + N_f) = 3V_3 + 4V_4$

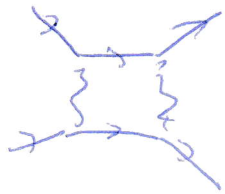
in 4-dim

~~Ba~~ $D_{sup} = \left(\overset{4}{\cancel{D}} + \overset{2}{\cancel{\tfrac{D-2}{2}}}\right)(P_b+P_f) \underbrace{- 4V_4 - 3V_3}_{-2(P_b+P_f)-(N_b+N_f)} \underbrace{- \tfrac{1}{2}N_f}_{}$

$p^{1/2}$ which are external momenta

$$D_{super} = 4 - N_b - \frac{3}{2}N_f$$

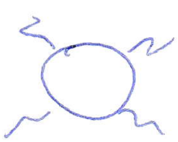
Let's try it:



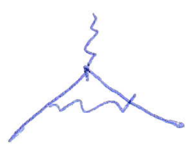
$= 4 - \frac{3}{2}\cdot 4$

QED $D = -2$ UV finite ✓

$D = 4-2 = 2$ Power divergent

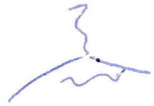


$4-4$ log divergent



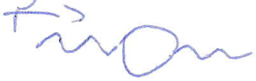
$4 - \frac{3}{2}\cdot 2 - 1 = 0$ log divergent

Also ~~If I want the mth power of~~ I may want to expand result with $q \gg P_{ext}$ in powers of P_{ext}'s, eg.

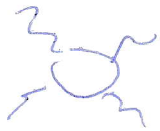
$\frac{\partial}{\partial P_{ext}}\Big($  $\Big)$ or  $= \text{const} + P_\mu (\;)^\mu + P_\mu P_\nu (\;)^{\mu\nu} + \cdots$

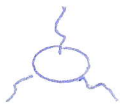
Each power of ext. momentum is one which could have been a q in a loop. So $D_{super} = 4 - N_b - \frac{3}{2}N_f - m$ powers of external mom. makes it more convergent.

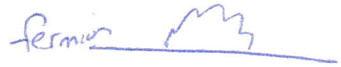
In some cases we know diagram <u>must</u> be prop. to external momentum. Gauge invariance (Ward identities) etc


$\propto p^2$ Log div.

$\propto p$ Log-div.

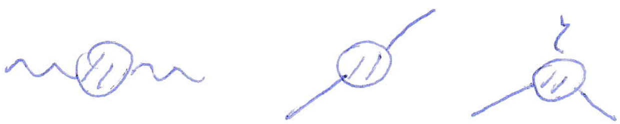

$\propto p^4$ finite


actually 0



$\propto p$ or m (chiral symm.)

So in QED, <u>only</u> div. diagrams are


log divergent

<u>Scalar</u> QED


quad. divergent*

log divergent

In dim $\neq 4$, story can be very different.

Divergent diagrams in 1-1 correspondence w. Lagrangian terms.

Always true in Renormalizable theories.

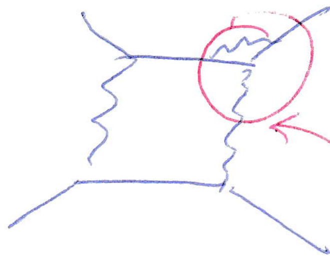
(why? Another way to count:


fields & deriv's and a $\delta^4(\Sigma p)$

call $\phi \sim q^1$ $\psi \sim q^{3/2}$. IF # fields & deriv's add to ≤ 4, brings in no positive powers.

$\langle\phi\phi\rangle \sim q^2$ because $\int d^4q \frac{1}{q^2} \sim q^2$ One $\delta^4(\Sigma p)$ lost to <u>overall</u> energy-mom. conservation

$\langle\psi\psi\rangle \sim q^3$ because $\int d^4q \frac{1}{q} \sim q^3$ Each external line is ϕ or ψ, Subdivergences

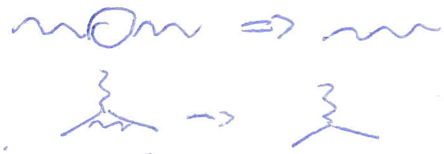


superficially finite.
But subdiagram
is <u>not</u>

<u>Divergent</u>, but only due to this sub-diagram.

But for this to happen, sub-diagram must have ~~same conditions as~~ superficial degree of divergence ≥ 0.

If this does not happen (or if we replace loops w. $D_{div} \geq 0$ with points,



)

diagram called "skeleton diagram" & integration finite.