Consider massless thy, eg,

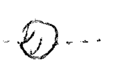
no i $\quad +\delta m$

$$\mathcal{L}(\psi,\phi) = Z_1 \bar{\psi}(\cancel{\partial} + \cancel{m+\delta m})\psi + \frac{Z_2}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{(\lambda+\delta\lambda)}{4!}Z_2^2\phi^4$$
$$- (y+\delta y)Z_1 Z_2^{1/2}\,\phi\bar{\psi}\psi \qquad - \delta M^2 \phi^2/2$$

with ψ, λ, y, ϕ renormalized quantities, $Z_1, Z_2, \delta\lambda, \delta y, \delta M^2, \delta m$ renorm. consts

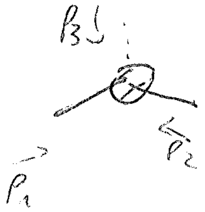
Choose $Z, \delta\lambda$, etc based on renorm. prescription

$$\Sigma = \frac{d}{d\cancel{p}}\Sigma = 0 \quad \text{at } p^2 = -M^2$$



$$\Pi = \frac{d}{dp^2}\Pi = 0 \quad \text{at } p^2 = -M^2$$

note, all at spacelike external momenta, described by introduced scale M.



$$\langle \phi\psi\psi \rangle_{amp} = -iy \quad \text{at } (P_1+P_2)^2 = (P_1+P_3)^2 = (P_2+P_3)^2 = -M^2$$

$$\langle \phi^4 \rangle_{amp} = -i\lambda \quad \text{at} \quad ''$$

Unique mapping between bare $\phi_0\, \psi_0\, \lambda_0\, y_0 \rightarrow$ ren $Z_1\, Z_2\, \lambda\, y$

But mapping is M dependent!

How must I change $Z\, \lambda\, y$ if I change M?

Holding what fixed? Physics $\rightarrow$ bare quantities & their correl. func's.

what happens when I change scale M? Renorm. $\lambda\, y\, Z_1\, Z_2$ change.

L8P2

Specifically consider bare correl. func.

$$G_0^{(n)} \equiv \langle 0|T[\phi_0(x_1)\ldots\phi_0(x_n)]|0\rangle_{conn}$$ or some ψs in there, eg, $G^{\psi\psi\phi}$ or $G^{\bar\psi\psi\bar\psi\psi}$ ...

$$G^{(n)} = \langle 0|T(\phi(x_1)\ldots\phi(x_n))|0\rangle_{conn} = Z^{-n/2} G_0^{(n)}$$

\# ψs, \# ϕs

or $$G^{(n_1)(n_2)} = Z_1^{-\frac{n_1}{2}} Z_2^{-\frac{n_2}{2}} G_0^{(n)}$$

$M\frac{\partial}{\partial M}$ Now $G_0^{(n)}$ is M-independent but it looks M-dependent from logs and chosen values of counterterms, eg,

$$X + \times = \lambda - \frac{\lambda^2}{32\pi^2}\left(\frac{3}{\varepsilon} + \ln\frac{\mu^6}{stu} + c\right) + c.t. = \lambda \text{ for } s=t=u=M^2$$

so $$c.t. = \frac{\lambda^2}{32\pi^2}\left(\frac{3}{\varepsilon} + \ln\frac{\mu^6}{M^6} + c\right)$$

$$X + \times = \lambda - \frac{\lambda^2}{32\pi^2}\left(\ln M^6/stu\right)$$ renormalized expression with our renorm. prescription.

Explicitly M-dep. from counterterm choice
Implicitly M-dep. from $\lambda = \lambda(M)$ depends on renorm. scale
But $G_0^{(n)}$ has no actual M-dependence.

$$M\frac{d}{dM} G_0^{(n)} = 0 = \left(M\frac{\partial}{\partial M} + \frac{M\partial\lambda}{\partial M}\frac{\partial}{\partial\lambda} + \frac{M\partial y}{\partial M}\frac{\partial}{\partial y} + \frac{M\partial Z_1}{Z_1\partial M}\frac{Z_1\partial}{\partial Z_1} + \frac{M\partial Z_2}{Z_2\partial M}\frac{Z_2\partial}{\partial Z_2}\right) Z_1^{\frac{n_1}{2}} Z_2^{\frac{n_2}{2}} G^{(n_1,n_2)}$$

Define $\beta_\lambda = \frac{M\partial\lambda}{\partial M}$

$\gamma_\phi = \frac{1}{2}\frac{M\partial Z_2}{Z_2\partial M}$

$$0 = \left(M\frac{\partial}{\partial M} + \beta_\lambda\frac{\partial}{\partial\lambda} + \beta_y\frac{\partial}{\partial y} + n_1\gamma_\psi + n_2\gamma_\phi\right) G^{(n_1,n_2)}.$$

Callan-Symanzik Eq

How to compute? For me, Wick rotate first (Euclidean methods)

$$\langle \phi(x)\phi(0)\rangle = \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot x} \frac{1}{p^2}$$ no i's !! $p^2 = \vec{p}^2 + p_0^2$ positive metric $g_{\mu\nu} = \delta_{\mu\nu}$

Our renorm. convention defined here anyways.

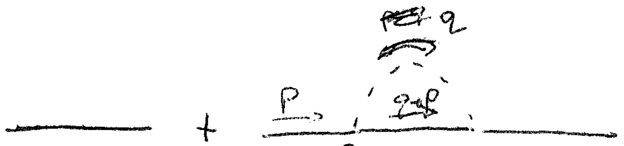
$$\int \mathcal{D}\phi\bar{\Psi}\Psi \exp - \int d^4x \left[\bar{\Psi}(\not{\partial})\Psi + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{4!}\phi^4 + y\phi\bar{\Psi}\Psi \right]$$

every term positive - positive action for convergence

no i if $\{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu} = \delta^{\mu\nu}$

$-\lambda$ $\quad -y$ due to this - sign.

Consider first Z_1:



$$\frac{1}{i\not{p}} + \frac{1}{i\not{p}}\underbrace{\left[(-y)^2\int\frac{d^Dq}{(2\pi)^D}\frac{\not{q}+\not{p}}{i(q+p)^2 q^2}\right]}_{\Sigma}\frac{1}{i\not{p}} = \frac{1}{i(\not{p}-\Sigma(p))}$$

from i^2 sorry.

$$\Sigma = \frac{-y^2}{16\pi^2}\int_0^1 dx \left(\frac{\mu^2}{4\pi}\right)^{4-D/2}\int (q^2)^{\frac{D}{2}-1} dq^2 \frac{(x\not{p}+\not{q})}{(q^2+x(1-x)p^2)^2}$$

$$= \not{p}\left(\frac{-y^2}{32\pi^2}\right)\left(\frac{1}{\varepsilon} + 2\int_0^1 dx\, x \ln\left(\frac{\mu^2}{x(1-x)p^2}\right) = 2 + \ln\frac{\mu^2}{p^2}\right)$$

Prescription: $p^2 = M^2$ above $= 0$.

Same as $\mu^2 = M^2 e^{-2}$

$$\boxed{\Sigma = \frac{-y^3}{32\pi^2}\ln\frac{M^2}{p^2}}$$

$$\boxed{\langle \Psi_r \bar{\Psi}_r\rangle = \frac{1}{\not{p} + \frac{y^2}{32\pi^2}\ln\frac{M^2}{p^2}\not{p}}} = Z_1^{-1}(M)\langle \Psi_0\bar{\Psi}_0\rangle$$

(since $Z_1\bar{\Psi}\Psi = \bar{\Psi}_0\Psi_0$)

$$\langle \Phi_0 \Phi_0 \rangle = \frac{Z(M)}{\cancel{p^2}\left(1+\frac{y^2}{32\pi^2}\ln\frac{M^2}{p^2}\right)}$$

$$\frac{M\partial}{\partial M}\ '' = 0 = \frac{\cancel{Z}}{\cancel{p^2}}\Big(\underbrace{\frac{M\partial Z}{Z\partial M}}_{2\gamma_\psi} + \frac{y^2}{16\pi^2}\Big)$$

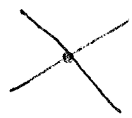
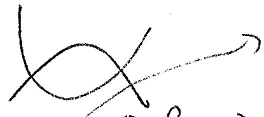
$$\boxed{\gamma_\psi = +\frac{y^2}{32\pi^2}}$$

opposite sign on ϕ in Σ

But wait: there's also $M\frac{\partial}{\partial M}y$ ... but that's higher order.

Similarly  gives $\gamma_\phi = y^2(\#)$ Find it! Return to...

If I take $Z(\Lambda_{cutoff}) = 1$, $\frac{1}{2}\frac{M}{Z}\frac{\partial Z}{\partial M} = \gamma_\psi < 0$ then $Z(\text{lowscale}) > 1$

Finally $\langle \phi^4_{ren} \rangle_{conn} =$  $+$ 

$-\lambda$ $\qquad + \lambda^2\int(\dots)$

$$-G^{(4)} = \underbrace{\left(\frac{1}{p^2\left(1+\# y^2\ln\frac{M^2}{p^2}\right)}\right)^{\text{for each leg}} \times \left(\lambda - \frac{3\lambda^2}{32\pi^2}\left(\frac{1}{\varepsilon}+\ln\frac{M^2}{(stu)^{1/3}}\right)\right)}$$

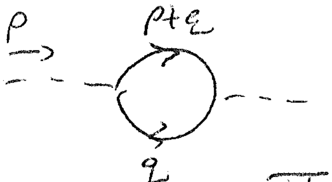
$$\left[M\frac{\partial}{\partial M} + \beta_\lambda\frac{\partial}{\partial\lambda} + 4\gamma_\phi\right] = 0$$

on M^2 in λ^2 term $\qquad$ on M^2 in propagators

$$\left(\frac{1}{\pi p^2}\right)\left(-\frac{3\lambda^2}{16\pi^2}\right) + \frac{1}{\pi p^2}\beta_\lambda + 4\gamma_\phi\frac{\lambda}{\pi p^2} - 8\gamma_\phi\frac{\lambda}{\pi p^2} = 0$$

$$\boxed{\beta_\lambda = +\frac{3\lambda^2}{16\pi^2} + 4\lambda\gamma_\phi}$$

$\rightarrow$ gives rise to λy^2 term



has lessons

why?

$$\Pi(p^2) = -y^2 \int \frac{d^Dq}{(2\pi)^D} \frac{\mathrm{Tr}\, \not{q}(\not{p}+\not{q})}{i q^2\, i (p+q)^2}$$

fermionloop

another − sign. But $\not{q}\not{q} = +q^2$.
OR no i's but $\not{q}\not{q} = -q^2$.

$\mathrm{Tr} \to 4(q^2 + p\cdot q)$

shift: $\displaystyle\int dx \frac{q^2+p\cdot q}{(xq^2+(1-x)(q^2+2p\cdot q+p^2))^2} = \frac{}{\left[(q+(1-x)p)^2 + x(1-x)p^2\right]^2}$

shift $q \to q-(1-x)p$

$-x(1-x)p^2 \;''\; -(1-x)p^2$

$q^2+p\cdot q \to q^2 + (2x-1)\,p\cdot q + (1-x)^2 p^2$

$$-\frac{4y^2\,\Gamma(D/2)}{(4\pi)^{D/2}} \int dx \int (q^2)^{\frac{D-2}{2}} dq^2 \, \frac{q^2 + \overbrace{(2x-1)}^{\to 0 \text{ on x-int}}\, p\cdot q + (1-x)^2 p^2}{(q^2+x(1-x)p^2)^2}$$

$$\Gamma(\varepsilon-1) = \frac{1}{(\varepsilon-1)}\Gamma(\varepsilon) = (-1-\varepsilon)\Gamma(\varepsilon) = \left(\frac{1}{\varepsilon} - \gamma_E\right)$$

Careful: $\displaystyle\int \frac{q^2}{(q^2+A)^2}(q^2)^{\frac{D}{2}-1} dq^2 = (A)^{1-\varepsilon}\, \frac{\Gamma(\varepsilon-1)\,\underbrace{\Gamma(3-\varepsilon)}_{\simeq 2}}{\Gamma(2)}$

q^2 term $\to (x(1-x)p^2)^{1-\varepsilon}\left(-\frac{1}{\varepsilon} + \gamma_E - 1\right)(2+\mathcal{O}(\varepsilon))$

q-integral $\to \left(\frac{1}{\varepsilon} + \text{const.}\right)\left(-x(1-x)^2 p^2 - 2x(1-x)p^2\right)\left(1+\varepsilon \ln\frac{\mu^2}{p^2}\right)$

$\displaystyle\int_0^1 dx\; (1-x)^2 = 1/3 \qquad x(1-x) = 1/6 \qquad\qquad \downarrow\; -\frac{1}{2}p^2$

$$= -+\frac{2\, y^2 p^2}{16\pi^2}\left(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{p^2} + \text{const.}\right)$$

Note: q^2 term also turned into p^2 effect. No m^2 generated.

~~opposite~~ same sign as Σ. $\quad \gamma_\phi = \frac{2\, y^2}{16\pi^2}$

$$\beta_\lambda = \frac{3\lambda^2}{16\pi^2} + \frac{8y^2\lambda}{16\pi^2}$$

Also: bad way

$$q\cdot(p+q) = \left(\tfrac{1}{2}q^2 + p\cdot q + \tfrac{p^2}{2}\right) + \tfrac{1}{2}q^2 - \tfrac{1}{2}p^2$$

$$= \frac{(q+p)^2 + q^2 - p^2}{2}$$

$$\frac{q\cdot(p+q)}{q^2(p+q)^2} = \frac{\cancel{(p+q)^2}}{2q^2\cancel{(p+q)^2}} + \frac{\cancel{q^2}}{2\cancel{q^2}(p+q)^2} - \frac{p^2}{2q^2(p+q)^2}$$

Now $\int \frac{d^Dq}{(2\pi)^D} \frac{1}{(p+q)^2} = \int \frac{d^Dq}{(2\pi)^D}\frac{1}{q^2}$ shift. IN $\overline{MS}$, this is zero!

Because, dimensions but no dimensionful quantity!

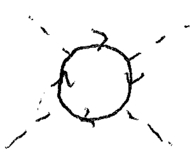
So, left with $-\frac{p^2}{2}\int \frac{d^Dq}{(2\pi)^D} \frac{1}{q^2(p+q)^2}$

$$= -\frac{p^2}{2}\frac{1}{(16\pi^2)}\left(\frac{1}{\varepsilon} + \int_0^1 \ln\frac{\mu^2}{x(1-x)p^2}\,dx = 2 + \ln\frac{\mu^2}{p^2}\right)$$

$$= -\frac{p^2}{2}\left(\frac{1}{16\pi^2}\right)\left(\ln\frac{M^2}{p^2}\right) \text{ after counterterm removal.}$$

$$\Pi = -\frac{2y^2}{16\pi^2}p^2\ln\frac{M^2}{p^2}$$

Also I forgot!



let's just find $1/\varepsilon$ - enough to get anom. dim etc.

$$-\lambda + (-)(y)^4\int\frac{d^Dq}{(2\pi)^D}\,\mathrm{Tr}\left(\left(\frac{1}{i\not q}\right)^4\right) \simeq -\lambda - 4y^4\int\frac{d^Dq}{(2\pi)^D(q^2\ldots)^2} \to \frac{1}{16\pi^2}\left(\frac{1}{\varepsilon} + \ln\frac{M^2}{p^2}\right)$$

opposite sign as λ^2 term we found!!

$$\beta_\lambda = +\frac{3\lambda^2}{16\pi^2} - \frac{4y^4}{16\pi^2} + \frac{8\lambda y^2}{16\pi^2}$$ hmmm.