Teilchenphysik:

Lecture 11: Real QED Calculations

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Last time we defined QED and wrote its Feynman rules

- How fermions enter, exit, propagate
- How photons enter, exit, propagate
- Their interaction: $ig_e\gamma^\mu$

We started to build tools for spin-summed/averaged $|\mathcal{M}|^2$.
Here we will push those tools through to get real differential cross-sections and we will see what happens beyond leading order!

2: But first – evaluation

You can now evaluate this course:
`http://evaluation.tu-darmstadt.de/evasys/online.php?pswd=6L2UV`

Also the homeworks (for Jillur):
`http://evaluation.tu-darmstadt.de/evasys/online.php?pswd=WVDG5`

3: Electron-Muon Scattering

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Leading order:
One diagram
Two incoming spins:
$\frac{1}{4}\sum_{s_1,s_2}$
Two outgoing spins:
$\sum_{s_3,s_4}$

4: Continuing

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$$\frac{1}{4}\sum_{s_1 s_2 s_3 s_4} \mathcal{M}^* \mathcal{M} = \frac{g_e^4}{4} \sum_{s_1 s_2 s_3 s_4} \frac{g_{\mu\nu} g_{\alpha\beta}}{t^2} \operatorname{Tr} u\bar{u}(p_3, \sigma_3)\gamma^\mu u\bar{u}(p_1, s_1)\gamma^\alpha$$
$$\times \operatorname{Tr} u\bar{u}(p_4, \sigma_4)\gamma^\nu u\bar{u}(p_2, s_2)\gamma^\beta$$

$$= \frac{8g_e^4}{t^2}\left[p_1 \cdot p_2 p_3 \cdot p_4 + p_1 \cdot p_4 p_2 \cdot p_3 - M^2 c^2 p_1 \cdot p_3 - m^2 c^2 p_2 \cdot p_4 + 2m^2 M^2 c^4\right] \to 2g_e^4 \frac{s^2 + u^2}{t^2}$$

5: Total cross-section?

In the relativistic limit,

$$t \to \frac{1-\cos\theta}{2}s\,, \qquad u \to \frac{1+\cos\theta}{2}s\,, \qquad |\bar{\mathcal{M}}|^2 \to 2g_e^4\frac{5+2\cos(\theta)+\cos^2(\theta)}{(1-\cos(\theta))^2}$$

Angular integral is

$$\int d\Omega = \int_0^{2\pi} d\phi \int_0^{\pi} \sin(\theta)d\theta = \int_0^{2\pi} d\phi \int_{-1}^{1} d[\cos(\theta)]$$

But the integral $\int^1 d[\cos(\theta)]/(1-\cos(\theta))^2$ diverges *as a power*.
Is this wrong? Or is it right?
It is right!

- Very small angle scattering without “identity change” hard to constrain
- Arises from large impact parameter $\to$ sensitive to environment
- Changes to environment (in an atom? In a beam pipe?) become relevant

6: Møller Scattering: Interference

When $e^- e^- \to e^- e^-$, two diagrams contribute:

Final result: $|\overline{\mathcal{M}}|^2 \to 2g_e^4 \left(\frac{s^2 + u^2}{t^2} + \frac{s^2 + t^2}{u^2} + 2\frac{s^2}{tu} \right)$

7: Annihilation to Muons: Crossing

Consider $e^- e^+ \to \mu^- \mu^+$:

Just take $e^- \mu^- \to e^- \mu^-$ answer and replace
$(p_1, p_2, -p_3, -p_4) \to (p_1, -p_4, p_2, -p_3)$. $|\overline{\mathcal{M}}|^2 = 2g_e^4(t^2 + u^2)/s^2$.

8: Compton: photon final states

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Consider $e^-\gamma \to e^-\gamma$, two diagrams:

We need to compute $\sum_\lambda \epsilon_\lambda^{\mu *} \epsilon_\lambda^\nu(q) = -g^{\mu\nu} + q^\mu \bar{q}^\nu + \bar{q}^\mu q^\nu$ ...

9: Current conservation

What's the difference between

$$\sum_\lambda \epsilon_\lambda^{\mu*}\epsilon_\lambda^\nu(q) = -g^{\mu\nu} + q^\mu \bar{q}^\nu + \bar{q}^\mu q^\nu \quad \text{and} \quad -g^{\mu\nu} \text{ ?}$$

Functionally, nothing! q^μ gives zero in rest of diagram...

$q_\mu \ \ldots (\not{p}_1 + m)\gamma^\mu(\not{p}_3 + m)$ with $q^\mu = p_1^\mu - p_3^\mu$

$= (\not{p}_1 + m)(\not{p}_1 - m - \not{p}_3 + m)(\not{p}_3 + m)$ but $(\not{p} + m)(\not{p} - m) = \not{p}\not{p} - m^2 = p^2 - m^2 = 0$

$= 0$

This always works, essentially because the photon couples to J_μ and $q^\mu J_\mu = 0$ by current conservation.

10: Higher order effects: box diagram

Finite. Contribution is of order $\frac{g_e^2}{4\pi} = \alpha = \frac{1}{137}$ **times** c/v.
For small velocities, becomes more important.
For $v \sim \alpha c$, profoundly changes physics, introduces bound states.

11: Higher order effects: vertex at small q

Leading-order: $\epsilon_\mu \bar{u}\gamma^\mu u$ couples with gyromagnetic factor $g = 2$.
NLO correction changes form of coupling: small (finite) effective $\bar{u}F_{\mu\nu}\sigma^{\mu\nu}u$. Acts like magnetic dipole, changes gyromagnetic factor:

$$\frac{g}{2} = 1 + \frac{1}{2}\frac{\alpha}{\pi} - .328479\left(\frac{\alpha}{\pi}\right)^2 + 1.18124\left(\frac{\alpha}{\pi}\right)^3 - 1.913\left(\frac{\alpha}{\pi}\right)^4 + 7.8\left(\frac{\alpha}{\pi}\right)^5$$

Precision test of QED...

12: High-order effects: deep cancelations

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13: The one which doesn't cancel

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$$\text{Integral is divergent: } \simeq (...)e^2 \int \frac{d^4 r}{(2\pi)^4} \frac{1}{(r^2 - m^2)((r-q)^2 - m^2)}$$
$$\simeq \frac{e^2}{12\pi^2} \times \begin{cases} \ln \frac{\Lambda^2}{m^2} & q^2 \ll m^2 \\ \ln \frac{\Lambda^2}{|q^2|} - \frac{5}{3} & |q^2| \gg m^2 \end{cases}$$

Requires **Regulator** Λ.
Occurs **everywhere** a propagator propagates.
The e^2 we measure is the e^2 including this effect $\rightarrow$ **Renormalization**

14: Renormalization

- Divergences only occur in very simple sub-diagrams
- These correspond 1-to-1 with terms in Lagrangian
- What I *measure* is *combination* of Lagrangian and these effects
- Reorganize (add-and-subtract) to improve convergence of e^2 series
- Leaves nontrivial *momentum-dependence* in results
- $\ln \frac{\Lambda^2}{m_e^2} \to \ln \frac{\Lambda^2}{q^2}$ means coupling *effectively gets larger* logarithmically with scale above the scale m_e.
- Coupling “constant” really (weakly) scale-dependent coupling.

15: Summary

- Feynman rules tell how to draw diagrams, write $\mathcal{M}$ expressions
- Trace tricks help us evaluate them efficiently
- Subtleties: small-angle scattering divergences
- Subtleties: interference
- Subtleties: crossing
- Subtleties: external photons, current conservation
- Higher order: bound states from boxes and ladders
- Higher order: spin and gyroscopic ratio, testing QED
- Higher order: renormalization