Teilchenphysik:

Lecture 15: Mesons

TECHNISCHE
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A **Hadron** is the term for a bound state of strongly interacting particles. Last time we saw that quarks and gluons should *always* bind together in this way.

Mesons are $q\overline{q}$ combinations.

Barons are qqq combinations.

We will talk about

- Quarkonium: quark-antiquark pairs where both are heavy
- Heavy-light mesons
- Light mesons
- Baryons (next lecture!)

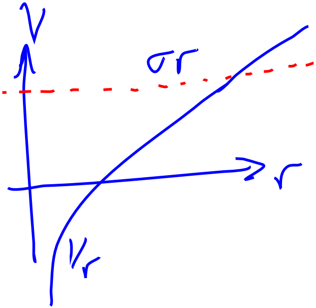
2: Are there others?

Are there other possibilities? Yes but we won't discuss them

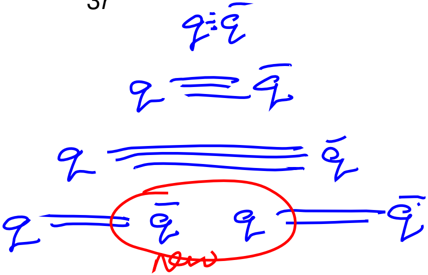
- **Glueballs**: 2+ gluons. Heavy and very unstable
- **Exotics**: quark+antiquark+gluon or 3 quarks + gluon.
  Also heavy and very unstable.
- **Tetraquarks**: $qq\overline{q}\overline{q}$. Exist, but probably more like meson molecules
  That is, loosely bound pairs of mesons or resonances between mesons
- **Pentaquarks**: $qqqq\overline{q}$. Probably meson-baryon molecules.
- **Multiquarks**: 6, 9, or more quarks. Only long-lived ones are atomic nuclei
  which can be understood as loosely bound associations of baryons

3: Reminder: strings and string breaking

If you couldn't create new $q\overline{q}$ pairs, $q\overline{q}$ potential would be approximately



$$V(r) \sim -\frac{4\alpha_s}{3r} + \sigma r$$



As r increases, energy always rises enough to make $q\overline{q}$ pair.
You always end up with bound states!

4: How can we figure out the bound states?

There are several options:

- Experiments. Very precise. Struggles with some of the states.
  Right answers, but might not tell you *why*.
- Rigorous QCD methods. There really only is one:
  Lattice QCD. Shows that QCD theory agrees with experiment
  But also doesn't give much direct intuition about "why" And struggles with resonances....
- Models, such as the Quark Model.
  Gives a clear physical picture.
  Fairly accurate but by no means exact, difficult to improve systematically

It's nice to know that experiment and lattice QCD are there, but
Let's look at the quark model

5: Quarkonium: heavy-heavy bound states

Think of Hydrogen or Positronium. System size

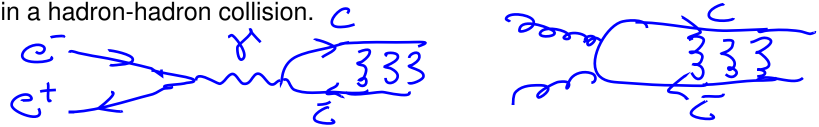
$$r \sim a_{\text{Bohr}} = \frac{1}{\alpha m} \quad \text{or} \ \frac{\hbar}{\alpha m c}$$

If m is big enough (compared to 300MeV scale where potential goes from $1/r$ to linear) then $1/\alpha_s m$ is small.
The bound state occurs where $V \simeq -4\alpha_s/3r$.
Momentum big enough – α_s not small, but not yet huge.
Velocity $v \sim \alpha c$ still smaller than speed of light.

This could be (barely) *nonrelativistic* system with
(approximately) $1/r$ type potential. Hydrogen- or positronium-like.

6: Charmonium

The charm quark is pretty heavy, $m_c \sim 1.3\,\mathrm{GeV}$.
A c and $\overline{c}$ are produced in pairs either from γ (as in $e^+e^- \to \gamma \to c\overline{c}$) or[1] from gg in a hadron-hadron collision.



Particle physicists consider each possible $c\overline{c}$ bound state to be a new type of particle. They are all unstable, but some have $M \gg \Gamma$.

Quark model – expect energies, J^{CP} like positronium but with α much bigger.

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[1] You can make $c\overline{c}$ from one gluon, but then it's colored and can't make a colorless $c\overline{c}$ meson.

7: Charmonium states

Naming convention: $n^{(2s+1)}\ell_j$ with $(2s+1) = 1$ for spin-singlet or 3 for spin-triplet, $\ell = (SPDF)$ for $\ell = (0123)$:

- Ground state: $n = 1$, $\ell = 0$, spin-0 so $j = 0$: 1^1S_0
- Spin-1 hyperfine-split version: $n = 1$, $\ell = 0$, spin-1 so $j = 1$: 1^3S_1
- Radial excited states: 2^1S_0 and 2^3S_1
- Angular excited states ($\ell = 1$): 2^1P_1, 2^3P_0, 2^3P_1, 2^3P_2

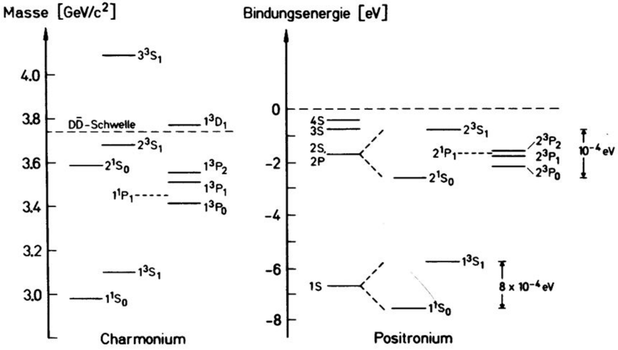
Stupid historical reasons: 1^1S_0, 2^1S_0 called η_c, η_c' and 1^3S_1 called J/ψ

For $n = 3$ or higher, a decay $c\overline{c} \to (c\overline{q})(q\overline{c})$ is allowed, and the states are short-lived “resonances”

8: Charmonium vs Positronium

Spectra of positronium and charmonium

(Povh et al.. Particles & nuclei)



Much larger fine splittings ($\alpha_s^2 \gg \alpha^2$) and $2S$ higher than $2P$ ($V \neq 1/r$)

9: Bottomonium

The bottom quark is even heavier: $m_b \sim 4.3\,\mathrm{GeV}$.
Quark model works even better, spectrum more like positronium, and $n = 3$ states also below $(b\overline{q})(q\overline{b})$ threshold.
States called Υ for historical reasons.

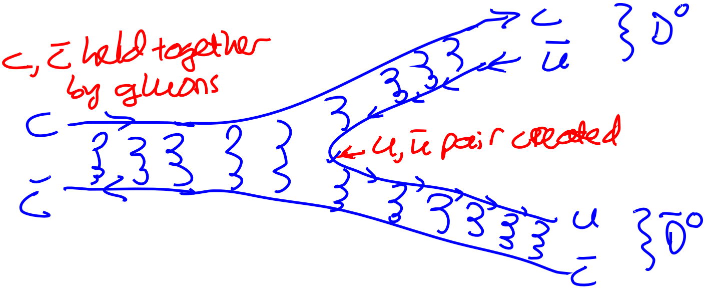
The 4*s* states are extra interesting, because

- they almost always decay to $B^0, \overline{B}^0$ $(d\overline{b}$ and $b\overline{d})$ or B^+, B^- $(u\overline{b}, b\overline{u})$
- large enhancement in $e^+e^- \to BB$ because resonance is there

Used in “B factories” to study large number of $B^0, \overline{B}^0$ pairs.

10: How do quarkonium decay?

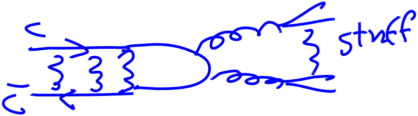
Above meson-pair threshold (n = 3 for charm, n = 4 for bottom):



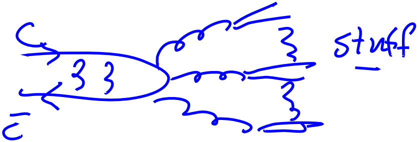
create $q\overline{q}$ pair, turn into two heavy-light mesons (they come next)

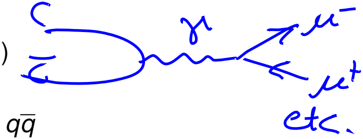
11: Quarkonium decay 2

Below meson-pair threshold: cannot form two heavy-light mesons.
Instead, $c\bar{c}$ want to annihilate!



or





- Works best for $\ell = 0$ (s) states ($2P \to 1S + \pi$)
- 1^3S_1 can go to $3g$ or 1γ.
  The γ is off-shell, becomes e^+e^- or $\mu^+\mu^-$ or $q\bar{q}$
  Going to $3g$ is suppressed – lots of α_s, small phase-space
- 1^1S_0 can go to $2g$ or 2γ.
  Decay to $2g$ (to several light mesons) is much much faster

12: Heavy-light mesons

What about $c\bar{u} = D^0$, $c\bar{d} = D^+$, $u\bar{b} = B^+$, $d\bar{b} = B^0$ etc?
Heavy-light is Hydrogen-like system.
Momentum scale of Hydrogen is $1/\alpha m_e$ set by light particle.
Bound state size is where $V \neq -4\alpha_s/3r$, light quark is relativistic.
Neither weak-coupled nor nonrelativistic!

$n = 2$ states $\sim$ 400MeV heavier than $n = 1$, short-lived ($D_{2p} \to D_{1s}\pi$)

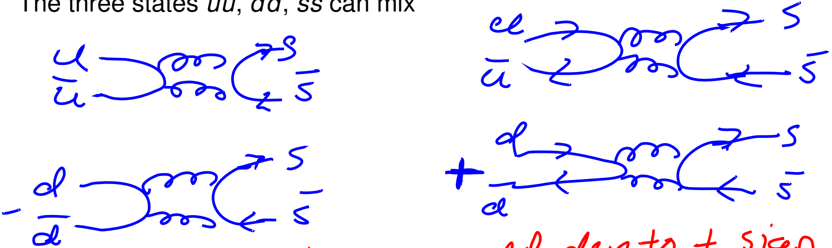
Lightest state, like $1S$ Hydrogen state, has spin-spin (hyperfine) splitting.
Though α_s^2 not small, $1/M$ is still a suppression!
$J = 1$ state (D^*, B^*) heavier than $J = 0$ state (D, B) but splitting fairly small

- m_{D^0} = 1864MeV, but $m_{D^{0*}}$ = 2006MeV, 142MeV heavier $\propto 1/m_c$
  $D^{0*} \to D^0\pi$ (barely energetically allowed) or $D^0\gamma$
- m_{B^0} = 5279MeV, but $m_{B^{0*}}$ = 5324MeV, 45MeV heavier $\propto 1/m_b$
  Only decay $B^{0*} \to B^0\gamma$ is allowed.

$$\frac{142}{45} \simeq \frac{m_b}{m_c} \checkmark$$

13: Light-light mesons

Combine a $q = (u, d, s)$ with $\overline{q} = (\overline{u}, \overline{d}, \overline{s})$.
Let's just try to understand $1s$ states.
Some combinations are clear: $u\overline{d}$, $u\overline{s}$, etc. BUT
The three states $u\overline{u}$, $d\overline{d}$, $s\overline{s}$ can mix



cancel due to − sign

add due to + sign

Can we predict what mixtures will be energy eigenstates? Yes we can.

14: u, d only: $SU(2)$ isospin

To get the idea, we start with mesons containing only (u, d).
These have an approximate symmetry: Isospin

$$\begin{bmatrix} u \\ d \end{bmatrix} \to \begin{bmatrix} V_{uu} & V_{ud} \\ V_{du} & V_{dd} \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix} \quad \text{or} \quad q \to Vq$$

Here $V \in SU(2)$. The antiquarks transform under V^*:

$$\begin{bmatrix} \overline{u} \\ \overline{d} \end{bmatrix} \to \begin{bmatrix} V^*_{uu} & V^*_{ud} \\ V^*_{du} & V^*_{dd} \end{bmatrix} \begin{bmatrix} \overline{u} \\ \overline{d} \end{bmatrix} \quad \text{or} \quad \overline{q} \to V^* \overline{q}$$

Trick: $V^* = V^\top = (-i\sigma_2) V (i\sigma_2)$. Therefore

$$\begin{bmatrix} -\overline{d} \\ \overline{u} \end{bmatrix} \to \begin{bmatrix} V_{uu} & V_{ud} \\ V_{du} & V_{dd} \end{bmatrix} \begin{bmatrix} -\overline{d} \\ \overline{u} \end{bmatrix} \quad \text{or} \quad \epsilon \overline{q} \to V \epsilon \overline{q} \,, \quad \epsilon \equiv \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

15: Isospin and mesons

How can (u, d) combine with $(\overline{u}, \overline{d})$?
Combine (u, d) with $(-\overline{d}, \overline{u})$ just like $(\uparrow, \downarrow)$ and ordinary spin.

$$\frac{1}{2} \otimes \frac{1}{2} = \mathbf{1} \oplus \mathbf{0} \qquad \mathbf{1} = \left(|\uparrow\uparrow\rangle,\ \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}},\ |\downarrow\downarrow\rangle \right) \qquad \mathbf{0} = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$$

$$\frac{1}{2} \otimes \frac{1}{2} = \mathbf{1} \oplus \mathbf{0} \qquad \mathbf{1} = \left(-|u\overline{d}\rangle,\ \frac{|u\overline{u}\rangle - |d\overline{d}\rangle}{\sqrt{2}},\ |d\overline{u}\rangle \right) \qquad \mathbf{0} = \frac{|u\overline{u}\rangle + |d\overline{d}\rangle}{\sqrt{2}}$$

A triplet of states π^+, π^0, π^- and a singlet η_3.
The pions should have almost the same mass: $m_{\pi^0} = 134.98\text{MeV}$, $m_{\pi^\pm} = 139.57\text{MeV}$. Because they are so light, they have no strong decays.

The $(u\overline{u} + d\overline{d})$ mixes with $s\overline{s}$, the π^0 does not.

16: Isospin and spin

Each q or $\overline{q}$ also has spin $\frac{1}{2}$.
They can combine into spin-0 or spin-1.
Spin-spin interactions lower energy for spin-0 and raise for spin-1 (like positronium)

- Spin-0 combinations: (π^+, π^0, π^-) and η
  Masses $\simeq$ 138MeV and (heavier)
- Spin-1 combinations: (ρ^+, ρ^0, ρ^-) and ω
  Masses $\simeq$ 770MeV and 782MeV.

If m_u, m_d were zero, pions would become massless, but ρ, ω would not.

Because $m_\rho > 2m_\pi$, the ρ decay rapidly, $\rho \to \pi\pi$, $\Gamma \sim$ 150MeV
The ω is G-parity-odd[2] and decays to $\omega \to \pi\pi\pi$. less phase space, $\Gamma \sim$ 8.5MeV

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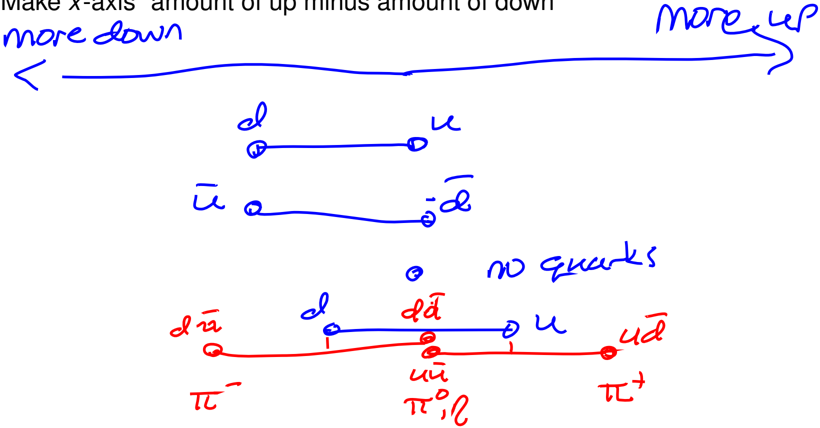
[2] G-parity is $Ce^{i\pi I_2}$. Pion is G-odd, ρ is G-even, ω is G-odd...

17: Isospin in pictures

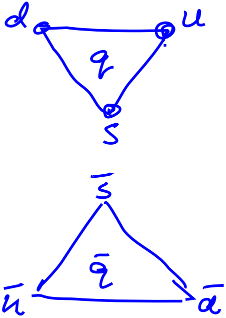
Make *x*-axis “amount of up minus amount of down”

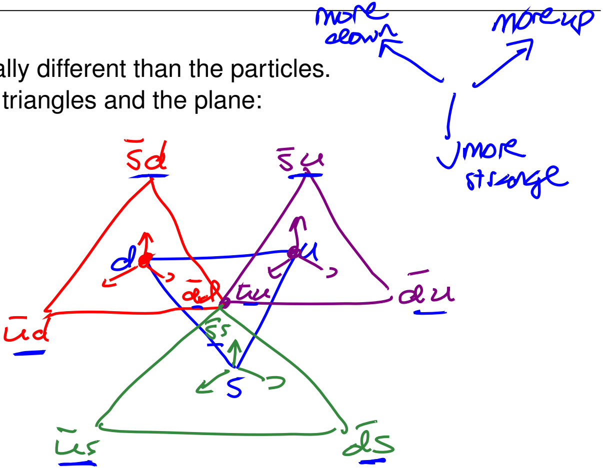


18: Add in the strange quark

The antiparticles are now really different than the particles.
Same “picture” now involves triangles and the plane:

Ingredients





9 total states

19: Octet and Singlet

One finds 8 “octet” combinations:

$$K^0 = d\bar{s} \qquad K^+ = u\bar{s}$$

$$\pi^- = d\bar{u} \qquad \pi^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}} \qquad \pi^+ = u\bar{d}$$

$$\eta = \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}}$$

$$K^- = s\bar{u} \qquad \overline{K}^0 = s\bar{d}$$

and one singlet:

$$\eta' = \frac{u\bar{u} + d\bar{d} + s\bar{s}}{\sqrt{3}}$$

These are “just” like the λ^α and λ^9 if we rearrange (λ^1, λ^2), (λ^4, λ^5), (λ^6, λ^7) in “raising/lowering” combinations

20: Masses, spin-1 versions

The spin-1 versions are named:

$$\pi \to \rho, \qquad K \to K^*, \qquad \eta \to \omega \qquad \eta' \to \phi$$

If $m_u = m_d = m_s$ then the octet members would be degenerate and the singlet would be different. $m_\pi = m_K = m_\eta$ but $m_{\eta'}$ heavier

Since m_s is rather different, the states with more s are heavier, and the exact mixing of (η, η') and ω, ϕ are different.

$$\eta \simeq \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}} \qquad \underline{\text{but}} \qquad \omega \simeq \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$

$$\eta' \simeq \frac{u\bar{u} + d\bar{d} + s\bar{s}}{\sqrt{3}} \qquad \phi \simeq s\bar{s}$$

why? long story, not today.

21: Summary

- q and $\overline{q}$ stick together into mesons.
- The resulting states are vaguely like hydrogen or positronium.
- For heavy-heavy, the analogy to positronium is close, but hyperfine effects are big
- For heavy-light, the spin-spin effect is suppressed by $1/M$
- For light-light, isospin and $SU(3)$ flavor help organize the states
- Isospin multiplets have the same mass up to a few MeV.
- Spin-0 is always lighter than spin-1. For light-light, difference is large.

On to baryons!