Teilchenphysik:

Lecture 17: Baryons

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Three quarks fit together into a baryon.
Why is this harder to figure out than mesons?

- Quarks are fermions: the total wave function must be **antisymmetric** on exchange of any two particle labels.
- Mesons: q and $\bar{q}$, different. Antisymmetry played no role
- Baryons: qqq and 3 independent exchanges must be considered
- Three kinds of labels:
  - Spin (up, down)
  - Color (red, green, blue)
  - Flavor (u, d, s, c, b)

Correctly handling all labels for 3 particles is tricky!

2: Our goal: Baryon wave-function

Each quark has 4 labels! $\vec{x}$, $s \in \{\uparrow, \downarrow\}$, $c \in \{rgb\}$, $f \in udscb$
Total wave function: $\Psi(x_1, x_2, x_3)$ is linear combination of each spin, color, flavor combination:

$$\Psi(x_1, x_2, x_3) = \sum_{s_1, s_2, s_3; c_1, c_2, c_3; f_1, f_2, f_3} \psi_{s_1, s_2, \ldots}(x_1, x_2, x_3)$$

$\psi_{s_1, s_2, \ldots}$ is wave function, in (x_1, x_2, x_3) product-space, of the spin-s_1, s_2, s_3, color-c_1, c_2, c_3, and flavor f_1, f_2, f_3 component of the total wave function.

Antisymmetry: trade $x_1 \leftrightarrow x_2$, $s_1 \leftrightarrow s_2$, $c_1 \leftrightarrow c_2$, $f_1 \leftrightarrow f_2$
and Ψ must pick up overall $-$ sign.

3: Simplifications!

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Thankfully!

- Colorless: total color must be antisymmetric! $\psi_{\ldots,c_1,c_2,c_3,\ldots} \propto \epsilon_{c_1 c_2 c_3}$ when all other labels are held fixed.
  Already found our wave-function antisymmetry!
  All other labels must combine to be exchange-symmetric.
- **If** we just examine $\ell = 0$ (S-wave) baryons, then x_1, x_2, x_3 dependence is symmetric. Don't know what it is, but decouples from the rest of the problem.
- Considering only $\ell = 0$ also tells us our states will be parity-even, $P = +$ (C and G are no longer useful; baryons cannot be their own antiparticle)

We cannot solve QCD, and just don't know what space-dependence of wave function is (if it even makes sense). But doesn't stop us from trying to understand how the other labels can behave.

Now we just need totally-symmetric combinations of spin and flavor.

4: Warm-up problem

What if there were just two colors? Then I need to combine *two* quarks into totally-antisymmetric wave function

- Combine colors antisymmetrically
- Combine space indices symmetrically, *S*-wave
- Combine flavors and spins somehow!

Let's also just do up, down.

- Spin: spin-1 $|\uparrow\uparrow\rangle$ etc. symmetric, or spin-0 $\frac{|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle}{\sqrt{2}}$ antisymmetric
- Flavor: **EITHER** $|uu\rangle$, $\frac{|ud\rangle+|du\rangle}{\sqrt{2}}$, $|dd\rangle$ symmetric (isospin triplet)
  OR $\frac{|ud\rangle-|du\rangle}{\sqrt{2}}$ (isospin singlet) symmetric

To be totally symmetric, I can choose symmetric-symmetric (spin-1 isotriplet) or anti-anti (spin-0 isosinglet)

5: Warm-up Problem: not purely hypothetical

What if I have one heavy quark?

- If it's one color, other two quarks must be other two colors
- Heavy quark's spin almost irrelevant: combine light spins first.
- There will be spin-spin interaction but suppressed by $1/M$

The other two quarks act like 2-quark problem.

- Anti-Anti: $I = 0, S = 0 \quad \Rightarrow \quad I(J^P) = 0(\frac{1}{2}^+)$ state. Lighter.
- Symm-Symm: $I = 1, S = 1 \quad \Rightarrow \quad I(J^P) = 1(\frac{1}{2}^+)$ and $1(\frac{3}{2}^+)$

Ask the PDG:

- Λ_c^+ : $0(\frac{1}{2}^+)$ with $m = 2286$MeV (udc)
  Σ_c : $1(\frac{1}{2}^+)$ with $m = 2453$MeV, and $1(\frac{3}{2}^+)$ with $m = 2518$MeV
- Λ_b^0 : $0(\frac{1}{2}^+)$ with $m = 5619$MeV (udb)
  Σ_b : $1(\frac{1}{2}^+)$ with $m = 5810$MeV, and $1(\frac{3}{2}^+)$ with $m = 5830$MeV

6: Back to baryons: combine 3 spins

How do I combine 3 spins? Combine 2, then add the third

$$\left(\frac{1}{2} \otimes \frac{1}{2}\right) \otimes \frac{1}{2} \quad = \quad (1 \oplus 0) \otimes \frac{1}{2} \quad = \quad \frac{3}{2} \oplus \frac{1}{2} \oplus \frac{1}{2}$$

There is *one* spin-$\frac{3}{2}$ and *two different* spin-$\frac{1}{2}$ combinations!

$$\frac{3}{2} = \left\{ \uparrow\uparrow\uparrow,\ \frac{\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow}{\sqrt{3}},\ ... \right\} = \psi_s \qquad \text{Symmetric!}$$

$$\frac{1}{2} = \left\{ \frac{(\uparrow\downarrow - \downarrow\uparrow)\uparrow}{\sqrt{2}},\ \frac{(\uparrow\downarrow - \downarrow\uparrow)\downarrow}{\sqrt{2}} \right\} = \psi_{12} \qquad \text{Mixed!}$$

$$\frac{1}{2} = \left\{ \frac{\uparrow(\uparrow\downarrow - \downarrow\uparrow)}{\sqrt{2}},\ \frac{\downarrow(\uparrow\downarrow - \downarrow\uparrow)}{\sqrt{2}} \right\} = \psi_{23} \qquad \text{Mixed!}$$

You can build ψ_{13} but it's a linear comb. of ψ_{12}, ψ_{23}.

7: Combine 3 flavors???

First, review combining 3 spins. Start with state with maximum S_z: $\uparrow\uparrow\uparrow = \left(\frac{3}{2}\ \frac{3}{2}\right)$

$$\text{Lowering-op:} \quad J_- \equiv \frac{J_x - iJ_y}{2} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \qquad J_- \uparrow = \downarrow, \quad J_- \downarrow = 0$$

Remember that J_- acts on *each* arrow in *turn*, eg, 3 terms.

$$J_- \uparrow\uparrow\uparrow = \downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow = \sqrt{3}\left(\frac{3}{2}, \frac{1}{2}\right)$$

$$J_- (\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow) = 2\downarrow\downarrow\uparrow + 2\downarrow\uparrow\downarrow + 2\uparrow\downarrow\downarrow = \sqrt{12}\left(\frac{3}{2}, \frac{-1}{2}\right)$$

$$J_- (\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow) = 3\downarrow\downarrow\downarrow = 3\left(\frac{3}{2}, \frac{-3}{2}\right)$$

$$J_- \downarrow\downarrow\downarrow = 0$$

Gives (un-normalized) entries in $3/2$ rep.

8: The other reps

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Start with $\left(\frac{3}{2}, \frac{1}{2}\right)$. Look for *orthogonal mixtures* which will be spin-$\frac{1}{2}$ reps:

$$\left(\frac{3}{2}, \frac{1}{2}\right) = \downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow$$

orthogonal to

$$\left(\frac{1}{2}, \frac{1}{2}\right) = \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow$$

and also

$$\left(\frac{1}{2}, \frac{1}{2}\right) = \downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow - 2\uparrow\uparrow\downarrow$$

All need to be properly normalized, sorry :-)
Use J_- to build each into a multiplet.
Last slide wrote a different (non-orthogonal) linear-combination.

9: Start with 2 flavors

We can work out the *isospin multiplets* with no *s*-quarks.
The flavor doublets combine just like spin:

$$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{3}{2} \oplus \frac{1}{2} \oplus \frac{1}{2}$$

The $3/2$ is totally symmetric. Aha:
Totally symmetric in spin and in flavor is totally symmetric!
Spin-$3/2$ Isospin $3/2$, eg, $(uuu), (uud + udu + duu), (udd + dud + ddu), (ddd)$
These are $\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-}$

Partly antisymmetric in spin and in flavor is ... what exactly?

10: Partly antisymmetric combination

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Totally antisymmetric in spin and in flavor is totally symmetric.
But spin-$\frac{1}{2}$ isn't totally symmetric. However:

- ψ_{12} is antisymmetric in the first two entries. Then

$$(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)\,(udu - duu)$$

  Symmetric on exchanging the first two entries, at least
- Same is true if I use ψ_{13} in spin and in flavor, or ψ_{23}.
- Combine them all!

$$(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)(udu - duu) + (\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)(uud - duu) + (\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)(uud - udu)$$

is symmetric on exchange of $(1, 2)$ (first term directly, second and thrid turn into each other), on $(1, 3)$ (middle term directly...) and $(2, 3)$.

Spin-$\frac{1}{2}$, isospin-$\frac{1}{2}$ is total 2 spin-$\frac{1}{2}$ particles: (N, P) Neutron and Proton

11: Three flavors?

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Spin-$\frac{1}{2}$ had $J_{-} = (\lambda_1 - i\lambda_2)/2$ replaces $\uparrow\rightarrow\downarrow$ or $u \rightarrow d$ and $J_{+} = (\lambda_1 + i\lambda_2)/2$ replaces $\downarrow\rightarrow\uparrow$ or $d \rightarrow u$.

- To do 3-flavors, name those J_{ud} and J_{du}.
- I also need $J_{us} = (\lambda_4 - i\lambda_5)/2$ and $J_{su} = (\lambda_4 + i\lambda_5)/2$ turns $u \rightarrow s$ and $s \rightarrow u$ respectively
- And $J_{ds} = (\lambda_6 - i\lambda_7)/2$ and $J_{sd} = (\lambda_6 + i\lambda_7)/2$ turn $d \rightarrow s$ and $s \rightarrow d$ respectively

12: Combine two triplets!

$$3 \otimes 3 = 6 \oplus \overline{3}$$

13: Three triplets

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$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

14: What are these reps?

The 10 is the totally symmetric combination. s-free part is the 4 Δ states we already saw.
Combine 10 with spin-$\frac{3}{2}$ – everyone symmetric.
Spin-$\frac{3}{2}$ decuplet!

The two 8's have s-free parts corresponding to the two independent isospin-$\frac{1}{2}$ combinations we needed to build the single isodoublet (N, P).
Use J_{ds}, J_{us} to finish out the *spin-*$\frac{1}{2}$ *Octet*

That's all the light $\ell = 0$ baryon multiplets!

15: Decuplet and Octet Labeled

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16: Masses? Magnetic moments?

Quark model includes a crude fit (not really calculation) of the masses of all these states. Idea is as follows:

- Mass of light quarks is small: $m_u \sim$ 3MeV, $m_d \sim$ 6MeV, $m_s \sim$ 100MeV
- But quark always relativistic, held in small volume by strong interaction, generating large environmental gluonic field.
- In practice, presence of quark $\rightarrow$ few-hundred MeV energy

I will therefore define:

- **Current** quark mass: small Lagrangian mass
- **Constituent** quark mass: includes energy of inevitable glue, kinetic energy, "stuff" accompanying quark. $\sim$ 300MeV

The constituent quark masses are the ones to use in figuring out meson and baryon masses....

17: Masses in constituent quark model

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Constituent masses m_u, m_d, m_s (so far unknown)
Spin-spin couplings with some effective coefficient:

$$M_{\text{baryon}} = m_1 + m_2 + m_3 + A' \left[\frac{S_1 \cdot S_2}{m_1 m_2} + \frac{S_1 \cdot S_3}{m_1 m_3} + \frac{S_2 \cdot S_3}{m_2 m_3} \right]$$

Spin dot-products must be evaluated explicitly from wave-function.
Larger for spin-$3/2$ than for spin-$1/2$ states.
They differ between Σ^0 and Λ, which makes Λ lighter.

4-parameter fit: $(m_u, m_d, m_s, A') = (363, 363, 538, (2m_u/\hbar)^2 50) MeV$,
Fits all masses in 10 and 8-plet to within around 10 MeV.

Can also make pretty good magnetic-moment predictions!

18: Summary

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- Listing possible baryon states more complicated than with mesons!
- The main challenge: wave function in space, color, spin, flavor
- Total wave-function antisymmetric. color-anti, $(x + s + f)$ symmetric
- Spin combines into symmetric $3/2$ and two mixed-symmetric $\frac{1}{2}$
- Isospin combines the same way, leading to spin-$3/2$ $I = 3/2$ (the $\Delta^{++,+,0,-}$) and spin-$1/2$ $I = 1/2$ (N, P)
- Including strangeness, $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$
- Spin-$3/2$ 10 and spin-$1/2$ 8 (lighter)
- Possible to fit masses, magnetic moments with a few parameters