Teilchenphysik:

Lecture 18: Charged weak interactions

TECHNISCHE
UNIVERSITÄT
DARMSTADT

What we have learned:

- Matter-particle content
- Dominant ways particles interact: EM and Strong interactions
- How to compute scattering, decay, bound states
- Bound states of strongly interacting particles (hadrons)

What's missing:

- Too many conserved quantities so far: each fermionic species number is conserved (u, d, s, c, b, t and e, μ, τ)
- No discussion of neutrinos and how they couple
- So far, P, C and T are exact symmetries.
- What happens at very high energies, of order 100 GeV?

We are missing the physics of the *weak interactions*
Today we will *start* to discuss the weak interactions,
with the *weak charged-current* interactions

2: Conservation of up-quark number

Suppose u quark enters a Feynman diagram.
Only vertices: $\overline{u}\gamma_\mu A^\mu u$ and $\overline{u}\gamma_\mu G^\mu u$
Any vertex a u enters, it also exits and vice versa
u enters diagram – must exit.

Number entering: initial u + final $\overline{u}$
Number exiting: initial $\overline{u}$ + final u. So $u_i + \bar{u}_f = \bar{u}_i + u_f$, or
$u_i - \bar{u}_i = u_f - \bar{u}_f$ initial number = final number
Same is true for d, s, c, b, t and for e^-, μ^-, τ^-

3: What “should be” stable

According to our arguments so far, the following are stable:

- The e^-, μ^-, τ^- (lightest particles with each number)
- Lightest particle with u-number: π^+
- Lightest particle with net quark-number: proton $p^+ = (uud)$
- Lightest s, c, b-species: K^-, D^0, B^-
- Anything lighter than its possible decay daughters: $n, K^0, \Lambda^0, \Xi, \Omega, D^+, B^0, \Lambda_c, \Lambda_b$ ($n \to p^+\pi^-$, $\Lambda^0 \to p^+K^-$, etc)

Which particles are *actually* stable? e^-, p^+ (and neutrinos, γ)

We are missing an interaction

4: The interaction we are missing is weak

Particles with allowed strong-interaction *I*-respecting decays:
ρ, Δ, ϕ with widths of order $\Gamma \sim 100\,\text{MeV}$, lifetime $\sim 10^{-23}$ s

Particles with *I*-forbidden, high-order, or EM decays:
J/ψ, η_c, π^0, η, etc. Widths order KeV to MeV, lifetimes 10^{-16} to 10^{-21} s

Particles whose decays are forbidden by rules so far, but with available decay energy of 100's of MeV:
π^+, K, D, B, Λ, Ξ, Ω, Λ_c, Λ_b, μ^-, τ^- Lifetimes $\sim 10^{-10}$ to 10^{-6} s

Particles with so-far-forbidden decays and *MeV* scale decay energy: *n*, nuclei.
Lifetimes $\sim 10^3$ s to 10^{17} s

Decays occur via very weak and highly energy-dependent interaction.

5: The W boson

What we are missing is another spin-1 particle $W^{\pm}$ with

- a big **mass** $M_W \simeq 80.4\,\text{GeV}$ (not MeV, GeV)
- a coupling $g_w \simeq 0.65$, $\alpha_w = \frac{g_w^2}{4\pi} \simeq \frac{1}{30}$
- Coupling between a fermion and a partner with charge-difference of ± 1

in-line	$\epsilon^{\mu}_{\lambda}, \ \lambda = 1, 2, 3$
out-line	$\epsilon^{*\mu}_{\lambda}, \ \lambda = 1, 2, 3$
Propagator	$\dfrac{-i(g_{\mu\nu} - q_{\mu}q_{\nu}/(M^2c^2))}{q^2 - M^2c^2}$
Vertex	$\dfrac{-ig_w}{2\sqrt{2}}\gamma^{\mu}(1-\gamma^5)$

Vertex between (e, ν_e); (μ, ν_{μ}); (τ, ν_{τ}); (u, d'); (c, s'); (t, b')

6: Vertex between charged objects

There is a W^+ and W^-. Which is which depends on which way you “follow the propagator”
On one end, charge goes down; on the other it goes up.

If $\mu^-, \overline{\nu}_\mu$ on one end, e^+, ν_e on other...

7: Neutrinos finally interact!

Consider an electron neutrino (5% of Sun's luminosity is in ν_e)
How can it interact with atomic nucleus?

Neutrino ν_e must turn into $e^- W^+$. The W^+ can merge with an n to give a p^+

8: A case we can calculate

Replace ν_e with ν_μ (from atmosphere or beam), n, p with e^-, ν_e

$$\mathcal{M} = \frac{g_w^2}{8} \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \bar{u}_\mu \gamma^\mu (1 - \gamma^5) u_{\nu_\mu} \bar{u}_{\nu e} \gamma^\nu (1 - \gamma^5) u_e$$

If e^- at rest, then unless ν_μ super-high energy, $|q^2| \ll M_W^2$ and replace propagator with

$$\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \to -\frac{g_{\mu\nu}}{M_W^2}$$

$$|\overline{\mathcal{M}}|^2 = \frac{1}{2} \left(\frac{g_w^2}{8 M_W^2} \right)^2 \operatorname{Tr} \gamma^\mu (1 - \gamma^5) \not{p}_1 \gamma^\nu (1 - \gamma^5) (\not{p}_3 + m_\mu) \\ \times \operatorname{Tr} \gamma_\mu (1 - \gamma^5) \not{p}_2 \gamma_\nu (1 - \gamma^5) \not{p}_4$$

Neglecting m_e, using $m_\nu = 0$. Wait – why spin-average of $1/2$?

9: $1-\gamma^5$ is a Left-projector

Let's look harder at $1-\gamma^5$:

$$1-\gamma^5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Apply to particle moving on z axis, R or L handed:

$$(1-\gamma^5)u_R = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{E+m} \\ 0 \\ \frac{p}{\sqrt{E+m}} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{E+m-p}{\sqrt{E+m}} \\ 0 \\ \frac{p-E-m}{\sqrt{E+m}} \\ 0 \end{bmatrix}$$

$$(1-\gamma^5)u_L = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \sqrt{E+m} \\ 0 \\ \frac{-p}{\sqrt{E+m}} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{E+m+p}{\sqrt{E+m}} \\ 0 \\ \frac{-p-E-m}{\sqrt{E+m}} \end{bmatrix}$$

Square-length of $(1-\gamma^5)u_R$ is $2(E-p)$ and for u_L it's $2(E+p)$.

$(1-\gamma^5)$ "kills off" R-handed part for $E \gg m$. Recall–$m_\nu = 0$, exact for neutrinos.

10: But what about parity?

Parity: mirror reflection should keep physics the same.
Turns $R \leftrightarrow L$. So production of L and of R should be equally likely, if P is a good symmetry.

Well, it's **NOT!** The W-boson interactions break P as badly as they could!

- Parity flips sign on γ^5.
- If interaction were γ^μ I would not flip sign
- If interaction were $\gamma^\mu\gamma^5$ I would flip sign once at each vertex. Two vertices – no net sign flip.
- Because it's $\gamma^\mu(1-\gamma^5)$, parity fundamentally changes interaction and is **not** a symmetry.
  Specifically, it's cross-terms with one γ^5 factor which matter.
- Also destroy C symmetry, but preserve combination CP.

11: Calculations with γ^5

In performing traces, what does γ^5 do?

$$\mathrm{Tr}\,\gamma^5 = \mathrm{Tr}\,\gamma^\mu\gamma^\nu\gamma^5 = 0 \qquad \mathrm{Tr}\,\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\gamma^5 = 4i\epsilon^{\mu\nu\alpha\beta}$$

Therefore

$$\mathrm{Tr}\,\gamma^\mu(1-\gamma^5)\not{p}_1\gamma^\nu(1-\gamma^5)\not{p}_3 = 8(p_1^\mu p_3^\nu + p_3^\mu p_1^\nu - g^{\mu\nu} p_1\cdot p_3 - i\epsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{3\beta}$$

We also need

$$\epsilon_{\mu\nu\alpha\beta}\left(p_1^\alpha p_3^\beta + p_3^\alpha p_1^\beta - g^{\alpha\beta}p_1\cdot p_3\right) = 0$$

$$\epsilon_{\mu\nu\alpha\beta}\epsilon^{\mu\nu\kappa\sigma} = -2\left(g_\alpha^\kappa g_\beta^\sigma - g_\alpha^\sigma g_\beta^\kappa\right)\ .$$

Use these to do all the algebra....

12: Scattering: results

Using these methods one finds:

$$|\overline{M}|^2 = 2\left(\frac{g_w^2}{M_W^2}\right)^2 (p_1 \cdot p_2)(p_3 \cdot p_4)\,, \quad \sigma = \frac{1}{8\pi}\left(\frac{g_w^2}{M_W^2}\right)^2 E_\nu^2 \left(1 - \frac{m_\mu}{2E}\right)^2$$

Key observations:

- Result depends on g_w^2/M_W^2, but not on g_w^2, M_W^2 separately. Low-energy phenomena are only sensitive to this ratio.
- Cross-sections usually scale as E^{-2}. This scales as E^{+2}
- That means, weak phenomena rapidly get less important at low-E.
- Scale where weak, EM phenomena have equal σ is of order M_W.

13: Muon decay

Move around which lines are incoming and outgoing to get muon decay process! Also get matrix element:

$$|\overline{\mathcal{M}}|^2 = 2\left(\frac{g_w^2}{M_W^2}\right)^2 p_1 \cdot p_2 \; p_3 \cdot p_4$$

Note: dot is between e^-, ν_μ, and between $\mu, \bar{\nu}_e$ momenta.

$$\text{width } \Gamma = \frac{1}{2m_\mu}\int \frac{d^3p_2 d^3p_3 d^3p_4}{(2\pi)^9 2p_1 2p_2 2p_3} |\overline{\mathcal{M}}|^2 (2\pi)^4 \delta^4(p_1 - p_2 - p_3 - p_4)$$

14: Three-body final phase space

The book discusses this final phase space in some detail.

- It's ugly, too ugly to show in a lecture, but
- It's not *that* ugly, you can understand it with patience
- Each individual momentum can range from $p = 0$ up to $p = m_\mu/2$
- Total cross-section *and* distribution of electron-momentum can be computed with reasonable effort

$$\Gamma = m_\mu^4 \left(\frac{g_w^2}{M_W^2} \right)^2 \frac{m_\mu}{12(8\pi)^3}$$

Usually expect $\Gamma \propto m$, but now extra m^4/M_W^4 factor.
Due to incredibly unfortunate historial accident, the combination $G_F \equiv g_w^2/(4\sqrt{2}\, M_W^2)$ is often used, in which case $\Gamma = G_F^2 m_\mu^5/192\pi^3$.
Corresponding lifetime is $\tau = 2.197 \times 10^{-6}$ seconds.

15: Muon lifetime: Prediction?

The muon lifetime *is not* a prediction of W-interaction model.
It only serves as a *measurement* of G_F.
Energy distribution, helicity of e^- *is* a prediction, and ...

Consider any other weak process, now that G_F is measured.
Now the decay width *is* a prediction.

$$\Gamma_{\tau^-} = \Gamma_{\mu^-} \times \frac{m_\tau^5}{m_\mu^5} \times \text{number of available final species}$$

For μ^-, decay must be to $\nu_\mu e^- \bar{\nu}_e$.
But τ^- has energy to make $e^- \bar{\nu}_e$, $\mu^- \bar{\nu}_\mu$, $d\bar{u}$ (3 colors). 5 available species.
Also predicts: 3/5 of τ decays should be to hadrons.
Works out well, up to calculable $\mathcal{O}(\alpha_s/2\pi)$ corrections.

16: Free neutron decay?

Let's pretend that *W* couples to (p, n) the same as to (ν, e^-) or (u, d). Then I can compute the decay $n \to p^+ e^- \bar{\nu}_e$ same as μ^- decay.

What's different? The n, p have nearly same mass: $m_n = 939.57\,\text{MeV}$, $m_p = 938.28\,\text{MeV}$, $m_n - m_p = 1.29\,\text{MeV}$. Electron mass $m_e = 0.511\,\text{MeV}$ not negligible.
Phase space is different, with final-state *p* carrying almost no kinetic energy.

17: Predictions

- Distribution of e^--energy E is approximately

$$\frac{d\Gamma}{dE} \propto E\sqrt{E^2 - m_e^2}\left((m_n - m_p) - E\right)^2$$

- Decay lifetime is $\tau \sim 1318$ seconds. Oops (expt: 886 sec)

Energy distribution fairly close except near $E = 0$.
Lifetime right order, but not close in detail. Why not?

- (n, p) DO NOT couple to W "just" like (d, u).
  Turns out, must replace $\gamma^\mu(1 - \gamma^5) \rightarrow \gamma^\mu(1 - c_A\gamma^5)$, and $\Gamma \propto 1 + 3c_A^2$
  (But c_A separately measurable)
- p^+, e^- are charged. So the approximation that e^- flies out as free particle is wrong – especially at low energy, it has distorted wave-function which enhances emission.

18: Other nuclei?

Many nuclei energetically allow either $n \to p^+ e^- \bar{\nu}_e$ or $p^+ \to n e^+ \nu_e$ (or electron capture: $p^+ e^- \to n \nu_e$)
Generically, eg, for ${}^{239}\mathrm{U}^{92} \to {}^{239}\mathrm{Np}^{93} e^- \bar{\nu}_e$, final nucleus different from initial: overlap of old nucleus, with one n switched to p, onto new nucleus.
For *superallowed* processes, nuclei before/after match up!
Sometimes nuclei have very different spins, e, ν must carry off angular momentum, with impact on wave-function (forbidden decays)
e^- wave-function distortion large as nuclear charge gets big

This is turning into nuclear physics and we won't say anything more.

19: Summary

There is another force-carrier $W^{\pm}$, but this time it's super heavy!

- The W interactions change species-type: $e^- \to \nu_e$ etc.
- This breaks many would-be conservation laws.
  e^- number *not* conserved, only e^- number *plus* ν_e number (e-type lepton number)
- Also maximally break P and C symmetry, but respect CP
- Large W mass makes effects suppressed by E^4/M_W^4 factor
- Explains instability, but large lifetime, of numerous particles
- Role in nuclear physics is important, somewhat confusing

Next time: look in more detail at W interactions with quarks, weak meson decays....