Teilchenphysik:

Lecture 19: Weak hadronic decays

Last time: weak interactions with leptons.
New but “straightforward”
This time: weak interactions with quarks.

- Quarks start out in bound states
- Quarks need to end up in bound states.
- Final state possibilities: no hadrons, one hadron, several hadrons
- Will require more information about hadronic bound states etc
- Which quark couples to which, via W boson?? Complicated.

Related issues: doubly-weak processes:

- Some decays are doubly weak.
- Doubly-weak oscillation phenomena
- Box diagrams, CP violation

postponed for later lectures

2: Heavy quarks are easier

There are three *energy ranges* to consider:

1. Once a heavy quark decays to a light one, plenty of energy available to create multiple hadrons.
2. Once a heavier quark decays, enough energy to make a few extra hadrons
3. Initial meson doesn't have enough energy to create *any* final-state mesons; decay must be leptonic

Case 1: heavy quark decays as if it's alone in vacuum.
Easy to analyze.

Case 2: quark decay mixed up with final state, particle-rearrangement in complex way.

Case 3: q and $\bar{q}$ in meson must annihilate into W.
Only one process, but need information about meson to compute it.

Let's start with case 1.

3: Heavy quark decay

This case isn't really discussed in the book, but it's easy so let's do it.
Consider $B^- = b\overline{u}$
The b can decay
Energy available:
$m_b \sim 5\,\mathrm{GeV}$
Binding energies:
$\sim 300\,\mathrm{MeV}$
Binding energy negligible
Decays as if in vacuum,
Decay products figure out
how to assemble into
mesons afterwords.

Prediction: decay width almost same for $\overline{B}^0 = b\overline{d}$, $B^- = b\overline{u}$, $\Lambda_b = udb$
Prediction: e, μ, τ similar probability; ud, cs each $3\times$ that likely.

4: Do predictions work?

Yes! $\tau_{\overline{B}^0} = 1.638 \times 10^{-12}\,\mathrm{s}, \quad \tau_{B^-} = 1.519 \times 10^{-12}\,\mathrm{s}, \quad \tau_{\Lambda_b} = 1.471 \times 10^{-12}\,\mathrm{s}$

In each case, about 11% decays each to $e^-\overline{\nu}_e X$, $\mu^-\overline{\nu}_\mu X$, $\tau^-\overline{\nu}_\tau X$,
with X any hadrons. $\sim$ 67% of decays are $W \to d\overline{u}$ or $s\overline{c}$, producing only hadrons.

But wait a minute: value of lifetime is all wrong.
It "should" be $\tau = \tau_{\mu^-}(m_\mu/m_B)^5/9$ but is much larger.
And shouldn't b only couple to the t quark, which is too heavy???

5: CKM matrix

Think first about leptons and neutrinos. There are 3 neutrino states. The *W* couples each lepton to its own neutrino state. The ν_e is *defined* as the neutrino which the *W* boson couples to an e^-; similarly the ν_μ and μ^-, the ν_τ and τ^-.

Now consider quarks. There are 3 charge-$-1/3$ quarks. We can name the one which *u* couples to d', the one *c* couples to s', and the one *t* couples to b'. These are automatically orthogonal.

But who's to say that these are the *same* as the mass eigenstates *d*, *s*, *b*? If "whoever gives masses" *doesn't talk to* "whoever sets *W*-couplings" then these could be different.

And they are different. But they happen to be close for reasons we don't understand.

6: CKM matrix

The mass-eigenstates (d, s, b) are not the flavor eigenstates (d', s', b') which the W couples to the (u, c, t) respectively. Instead,

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = V_{ij} \begin{bmatrix} d \\ s \\ b \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

Here V_{ij} the CKM matrix is a 3×3 *complex, unitary* matrix.
Meaning: V_{ij} is how strongly $i \in \{u, c, t\}$ couples to $j \in \{d, s, b\}$
Standard phase conventions, not showing a few tiny imaginary parts:

$$V_{ij} = \begin{bmatrix} 0.9743 & 0.2253 & .0013 - .0033i \\ -.2252 & 0.9734 & 0.0412 \\ .0081 - .0033i & -.0404 & 0.9991 \end{bmatrix}$$

It *happens* that V_{ij} is *nearly* diagonal.
b-quark decay proportional to $|V_{cb}|^2 + |V_{ub}|^2 \sim 1/600$ explains why τ_b so big.

7: CKM matrix: examples

Consider B^- decay again. Then consider $D^+ \to \overline{K}^0 e^+ \nu_e$ and finally, $n \to p e^- \overline{\nu}_e$.

8: Pion decay

For *b*-decay I just ignored binding into hadrons. 10% corrections.
For D, Λ_c decays, it's more like 30% corrections
For π^+ decay? It's 100% correction, because there is no energy to make *any final-state hadrons at all.* How does decay even happen?

The $u\overline{d}$ must annihilate off completely!

9: Pion decay constant

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Diagram again:

The u, d part is: $\bar{v}_d \gamma^\mu (1-\gamma^5) u_u$ times some wave-function information about $(u, \overline{d})$ which I really don't know.
But I know it has $^\mu$ index! Only available 4-vector is pion 4-momentum p:

$$\mathcal{M} = \frac{V_{ud} g_w^2}{8 M_W^2} \overline{u}_\nu \gamma^\mu (1-\gamma^5) v_\ell \; F^\mu \,, \qquad F^\mu = f_\pi p_{\pi^+}^\mu$$

Here f_π is some constant of proportionality, units of energy, describing how much $u, \overline{d}$ overlap in pion. OK, let's compute $|\mathcal{M}|^2$:

$$|\mathcal{M}|^2 = ... = \frac{|V_{ud}|^2 f_\pi^2 g_w^4}{8 M_W^4} \big(2(p_\pi \cdot p_\ell)(p_\pi \cdot p_\nu) - p_\pi^2 (p_\ell \cdot p_\nu) \big)$$

10: Finishing pion decay

Since it's 1 → 2 there is no kinematics to do! And $p_\nu^2 = 0$, $p_\ell^2 = m_\ell^2$, $p_\pi^2 = m_\pi^2$,

$$p^2 = (p_\ell + p_\nu)^2 = p_\ell^2 + 2p_\ell \cdot p_\nu + p_\nu^2 \quad \Rightarrow \quad 2p_\ell \cdot p_\nu = m_\pi^2 - m_\ell^2$$

Similarly $p_\pi \cdot p_\ell = (p_\ell + p_\nu) \cdot p_\ell = m_\ell^2 + p_\nu \cdot p_\ell$ etc, and we can compute everything! After super-simple phase-space integral, some algebra, we find:

$$\Gamma_{\pi^+} = \frac{|V_{ud}|^2 f_\pi^2}{\pi m_\pi^3} \frac{g_w^4}{256 M_W^4} m_\ell^2 (m_\pi^2 - m_\ell^2)^2$$

Result: $f_\pi = 130\,\text{MeV}$.

Prediction:

$$\frac{\Gamma(\pi^+ \to \mu^+ \nu_\mu)}{\Gamma(\pi^+ \to e^+ \nu_e)} = \frac{m_e^2 (m_\pi^2 - m_e^2)^2}{m_\mu^2 (m_\pi^2 - m_\mu^2)} = 1.28 \times 10^{-4}$$

far, far more likely to decay to μ, ν_μ than e, ν_e.

11: Why won't pions decay to electrons?

This is about spins. The W couples to L-handed matter, R-handed antimatter:
For $\pi^+ \to e^+\nu_e$, the e^+ is R-handed, ν_e L-handed. But π is spin-0.

Suppressed by $E - p$ which we discussed last lecture!
This turns into m^2/m_π^2 factor. Limit of small mass–decay doesn't happen.

12: Kaon decay: K^+

The same decay as π^+ can happen: $u\bar{s} \to W^+ \to \mu^+\nu_\mu$

- Somewhat different decay constant: $f_K = 1.19 f_\pi$ (not surprising)
- Since $m_K^2/m_\mu^2 \simeq 22$, now also mass-suppressed
- Suppressed by $|V_{us}|^2 = 1/20$ (as are all decays!)

But now enough energy for new decays:

$K^+ \to \pi^+\pi^0$ (20% of time) or even 3 pions (7%).
Also $K^+ \to \ell^+\nu_\ell\pi^0$ occur, 8% of decays.
D: semileptonic $\sim f_D^2 m_c^3$ but total $\sim m_c^5$ ... almost always have final-state hadrons

13: What about K^0?

The π^0 decays straight to photons. But $K^0 = d\bar{s}$ cannot: EM conserves strangeness.

How does it decay?
Obvious ways,
$K^0 \to \pi\pi$
Also $K^0 \to \pi^- e^+ \nu_e$
and $K^0 \to \pi\pi\pi$
(just enough energy)

Also, interesting way: $K^0 \to \mu^+\mu^-$

14: Doubly weak processes

Look at diagram for $K^0 \to \mu^+\mu^-$

Surely this is $\propto 1/M_W^8$, and negligible?
Actually, no. Loop momentum, $q \sim M_W$ dominates.
Suppressed only by factor $\alpha_w/\pi \sim 1/100$. And $V_{ud}V_{us}^*$

15: We are forgetting something!!

Draw diagram again, but remember all quark types!

$$\mathcal{M} = \mathcal{M}_u V_{ud} V_{us}^* + \mathcal{M}_c V_{cd} V_{cs}^*$$

Unitarity of V: $V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0$ (and $V_{td} V_{ts} \simeq .0004$)
Cancel if $\mathcal{M}_u \simeq \mathcal{M}_c$ (GIM). Corrections $\propto m_c^2/M_W^2$, *pre*dicted m_c!!!

16: Where else does this happen?

Think about the neutral kaon again: $\overline{K}^0 = s\overline{d}$ and $K^0 = d\overline{s}$:

The K^0 can turn into $\overline{K}^0$ and *vice versa*
The correct eigenstates are K^S and K^L, the *CP* even/odd states

$$K^S = \frac{d\overline{s} - s\overline{d}}{\sqrt{2}}, CP \text{ even}, \qquad K^L = \frac{d\overline{s} + s\overline{d}}{\sqrt{2}}, CP \text{ odd}$$

CP-even can decay to $\pi\pi$, while *CP*-odd must decay to 3π, slower.

17: Why doesn't it decay first?

The effect we are talking about is suppressed by α_w and small CKM angles / m_c^2/M_W^2.
How can it compete with annihilation? Won't K^0 decay before this can play any role?

Key: decay involves square of matrix element $|\langle\psi_f|H_I|\psi_I\rangle|^2 \propto 1/M_W^4$
Oscillation involves matrix element $\langle\psi_2|H_I|\psi_1\rangle \propto 1/M_W^2$

Naively, oscillation phenomena should be far faster than decay.
It's only because of strong suppression of oscillation that they are even comparable.

18: CP violation!

The mixing involves enough CKM elements to possess phases

Phases flip under *CP*. Phase means the mixing is not to exact *CP* states
Every now and then a K^L decays to $\pi\pi$. (Observed 1964, Nobel 1980)

19: *CP* in other neutral systems

The same *should* happen in $D^0/\overline{D}^0$. Effects are small.

Similar things *do* happen in $B^0/\overline{B}^0$.

Exquisitely well studied in this system.

Several distinct *CP* violating effects observed in B^0 and in B_s systems.

Motivated “factories” producing large number of $B^0, \overline{B}^0$ pairs.

But we don’t have time to discuss this further.

20: Summary

- Decay of heavy quarks: quark decays, final-state particles stick together
- Which quarks couple to which: CKM matrix V_{ij}
- Decay of light quarks: initial, final states play essential role
- Pions decay by annihilation to leptons. f_π.
- Kaons can decay leptonically, semileptonically, hadronically
- Doubly-weak processes are less suppressed than you might guess
  But involve nontrivial cancellations between heavy/light quarks
- Lead to $\Delta s = 2$ oscillation phenomena
- *CP* is not conserved, but violation involves these oscillations.