Let's finish constructing the

Standard Model of Particle Physics

by investigating the **Higgs Mechanism**

- We saw what a gauge theory is
- We saw how a symmetry could be spontaneously broken
- What if both things happen at once? **Higgs mechanism**
- Gauge fields become massive, W^3, B mix into Z, A
- Fermions also become massive, L, R handed copies pair up

Weinberg 1967 (Nobel prize 1979)

2: Reminder: $SU(2)$ Gauge theory

Modify derivative of 2-component object:

$$\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \qquad D^\mu \phi = \left(\partial^\mu - ig \frac{\tau_a}{2} W_a^\mu \right) \phi$$

where τ_a are Pauli matrices and g the gauge coupling.
The W "rotate ϕ as you translate it in space"

Under symmetry transformation:

$$\phi(x) \to \left(1 + ig \frac{\tau_a}{2} \lambda_a(x) \right) \phi(x), \qquad W_a^\mu(x) \to W_a^\mu(x) + \partial^\mu \lambda_a(x) + g \epsilon_{abc} W_b^\mu \lambda_c$$

If ϕ is rotated, the W must also be rotated ($W\lambda$ term)
If ϕ rotation is space-varying, must add something to W to counteract this rotation when performing differentiation ($\partial^\mu \lambda$ term)

3: Spontaneous symmetry breaking

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Now suppose that the field ϕ has the following Lagrangian density:[1]

$$\mathcal{L} = D_\mu \phi^\dagger D^\mu \phi - V\,, \qquad V = \lambda(\phi^\dagger \phi - v^2/2)^2$$

The "vacuum" then has $|\phi^2| = 2\phi^\dagger\phi = v^2 \neq 0$. This is spontaneous symmetry breaking.

Question: Which direction should this vacuum value point?

[1] In my conventions, $\phi_1 = (\phi_{1r} + i\phi_{1i})/\sqrt{2}$ which is the standard conventions in the literature for (good) formal reasons. The squared length of ϕ is $\phi_{1r}^2 + \phi_{1i}^2 + \phi_{2r}^2 + \phi_{2i}^2 = 2\phi^\dagger\phi$ which is a little confusing.

4: Direction of the vacuum value

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The vacuum value of ϕ has $|\phi^2| = 2\phi^\dagger\phi = v^2$.
This has to point in some direction. Which direction?

It could point in different directions at different points in space. In the presence of gauge fields, it's not even clear that this is the wrong solution.

But I have a (gauge) symmetry to rotate independently at different points in space!

Let's *use up* our gauge symmetry by rotating $\phi \to \exp\left(ig\lambda_a\tau_a/2\right)\phi$ so that it always points in ϕ_{2r} direction:

$$\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{bmatrix} ,$$

Here v is the (expected) vacuum value and h are any remaining fluctuations. Both v and h are real.
We used up all 3 gauge freedoms to force ϕ_{2i} and both components of ϕ_1 to be zero.[2] Each zero component corresponds to one λ_a choice:

$$\sqrt{2}\phi = \begin{bmatrix} 0 \\ v \end{bmatrix} , \quad \left(1 + ig\frac{\tau_a}{2}\lambda_a\right)\sqrt{2}\phi = \begin{bmatrix} 1 + \frac{ig\lambda_3}{2} & \frac{g(\lambda_2+i\lambda_1)}{2} \\ \frac{g(-\lambda_2+i\lambda_1)}{2} & 1 - \frac{ig\lambda_3}{2} \end{bmatrix}\begin{bmatrix} 0 \\ v \end{bmatrix} = \begin{bmatrix} \frac{gv(\lambda_2+i\lambda_1)}{2} \\ v - \frac{igv\lambda_3}{2} \end{bmatrix}$$

[2] Warning: This works great until you start doing loop-level calculations and dealing with UV fluctuations. Then you have to do something more sophisticated. But it's fine at the level of this course.

6: Why a W field would normally be massless

What is a mass?

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2}\phi^2 + ...$$

It's an energy when ϕ takes the wrong value (here, $\neq 0$), when it's *not* varying in space and/or time.
This forces field to oscillate ($\partial_0 \phi \neq 0$) even when $\partial_i \phi = 0$.

What about gauge fields: does a *uniform* gauge field cost energy?
Consider W_a^μ constant, *without* scalar field.

$$\mathcal{L} = \frac{1}{4}(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)(\partial^\mu W_a^\nu - \partial^\nu W_a^\mu)$$

vanishes if W_a^μ is uniform (up to terms of order W^4)
This is why W^μ is massless: only derivatives cost energy,
constant values cost no energy / action / whatever.

7: Constant W and ϕ vacuum value

Now consider W_a^μ nonzero when a scalar ϕ has a vacuum value! $F_{\mu\nu}F^{\mu\nu}$ is still 0, but:

$$
\begin{aligned}
D^\mu \phi &= \left(\partial^\mu - ig\frac{\tau_a}{2} W_a^\mu\right)\phi \\
&= \left[\begin{array}{cc} \partial^\mu - igW_3^\mu/2 & g(-W_2^\mu - iW_1^\mu)/2 \\ g(W_2^\mu - iW_1^\mu)/2 & \partial^\mu + igW_3^\mu/2 \end{array}\right]\left[\begin{array}{c} 0 \\ \frac{v+h}{\sqrt{2}} \end{array}\right] \\
&= \left[\begin{array}{c} \frac{g(-W_2^\mu - iW_1^\mu)(v+h)}{2\sqrt{2}} \\ \frac{\partial^\mu h}{\sqrt{2}} + i\frac{gW_3^\mu(v+h)}{2\sqrt{2}} \end{array}\right] \\
D_\mu \phi^\dagger D^\mu \phi &= \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\,\frac{g^2(v+h)^2}{4}\left((W_1^\mu)^2 + (W_2^\mu)^2 + (W_3^\mu)^2\right)
\end{aligned}
$$

Nonvanishing v gives rise to masses: $M_W^2 = g^2 v^2/4$ or $M_W = gv/2$.
(Also gives rise to couplings to h, in proportion to mass.)

8: A little history

What we just described is called the **Higgs mechanism.**
Or, the *London Nambu Anderson Braut Englert Higgs Guralnik Hagen Kibble mechanism*

- Founding idea for E&M in nonrelativistic systems from Fritz London and Yoichiro Nambu (Nobel 2008) circia 1960
- Field theory formulation for E&M ($U(1)$) in nonrelativistic setting by Philip Anderson (Nobel 1977) in 1962
- Relativistic case, still $U(1)$: Braut Englert (Nobel 2013) and Higgs (Nobel 2013), who pointed out particle associated with h fluctuations in 1964
- $SU(2)$ case: Guralnik Hagen and Kibble 1964 or 1965.
- Application to Standard Model: Weinberg (Nobel 1979) 1967.

Somehow the "Higgs" name stuck, perhaps because he predicted the boson.

9: Degrees of freedom?

How doe the *counting of Degrees of Freedom* work here?

Consider first the case where ϕ has no vacuum value.

- ϕ has 4 independent components $\phi_{1r}, \phi_{1i}, \phi_{2r}, \phi_{2i}$
- Each W field is massless, with 2 polarization states: $2 \times 3 = 6$ states

What if ϕ develops a vacuum value and I use unitary gauge?

- The ϕ field has 1 independent component h
- Each W field is massive, with 3 polarization states: $3 \times 3 = 9$ states

Either way there are a total of 10 physical states.
The 3 components of the scalar get “eaten” to turn into longitudinal polarization states of the W bosons

10: Electromagnetism

We're mostly there! But we haven't included Electromagnetism or fermions.
Let's start with electromagnetism. A $U(1)$ gauge field B^μ, with ϕ having charge 1/2:

$$D^\mu\phi = \left(\partial^\mu - ig_w\frac{\tau_a}{2}W_a^\mu - i\frac{g'}{2}B^\mu\right)\phi$$
$$= \begin{bmatrix} \partial^\mu - i\frac{g_wW_3^\mu + g'B^\mu}{2} & g_w(-W_2^\mu - iW_1^\mu)/2 \\ g_w(W_2^\mu - iW_1^\mu)/2 & \partial^\mu - i\frac{-g_wW_3^\mu + g'B^\mu}{2} \end{bmatrix}\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$

Again writing $\phi_1 = 0$ and $\sqrt{2}\phi_2 = v$ we find:

$$D_\mu\phi^\dagger D^\mu\phi = \frac{1}{2}\left(\frac{g_w^2v^2}{4}((W_1^\mu)^2 + (W_2^\mu)^2) + \frac{v^2}{4}(g'B^\mu - g_wW_3^\mu)^2\right)$$

Here W_1 and W_2 get masses $M_W = gv/2$ as before.
But a *linear combination* of W_3 and B become massive...

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I have two fields, B and W_3. The Lagrangian is

$$-\mathcal{L} = \frac{1}{4}(\partial^\mu B^\nu - \partial^\nu B^\mu)(\partial_\mu B_\nu - \partial_\nu B_\mu) + \frac{1}{4}(\partial^\mu W_3^\nu - \partial^\nu W_3^\mu)(\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3) + \frac{1}{2}\frac{v^2}{4}(g' B^\mu - g_w W_3^\mu)(g' B_\mu - g_w W_\mu^3)$$

What to do? Pick new linear combinations of the fields!
Let's apply a rotation, angle θ_W, between B, W_3:

$$\begin{bmatrix} Z^\mu \\ A^\mu \end{bmatrix} = \begin{bmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{bmatrix} \begin{bmatrix} W_3^\mu \\ B^\mu \end{bmatrix}$$

We want $Z^\mu \propto g' B^\mu - g_w W_3^\mu$. Therefore choose

$$\cos\theta_W = \frac{g_w}{\sqrt{g_w^2 + g'^2}}, \qquad \sin\theta_W = \frac{g'}{\sqrt{g_w^2 + g'^2}}, \qquad \cos^2\theta_W + \sin^2\theta_W = 1$$

12: Electroweak Mixing II

$$Z^\mu = \frac{g_w}{\sqrt{g_w^2 + g'^2}} W_3^\mu - \frac{g'}{\sqrt{g_w^2 + g'^2}} B^\mu \qquad W_3^\mu = \frac{g_w}{\sqrt{g_w^2 + g'^2}} Z^\mu + \frac{g'}{\sqrt{g_w^2 + g'^2}} A^\mu$$

$$A^\mu = \frac{g'}{\sqrt{g_w^2 + g'^2}} W_3^\mu + \frac{g_w}{\sqrt{g_w^2 + g'^2}} B^\mu \qquad B^\mu - \frac{g'}{\sqrt{g_w^2 + g'^2}} Z^\mu + \frac{g_w}{\sqrt{g_w^2 + g'^2}} A^\mu$$

Massive combination: $\quad \frac{v^2}{4}(-g_w W_3^\mu + g' B^\mu)^2 = \frac{v^2}{4}(g_w^2 + g'^2) Z^\mu Z_\mu$

What is my Lagrangian after this transformation?

$$\begin{aligned} -\mathcal{L} &= \frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ &\quad + \frac{1}{4}(\partial^\mu Z^\nu - \partial^\nu Z^\mu)(\partial_\mu Z_\nu - \partial_\nu Z_\mu) + \frac{M_Z^2}{2} Z_\mu Z^\mu , \\ M_Z^2 &= \frac{(g_w^2 + g'^2)v^2}{4} = \left(1 + \frac{g'^2}{g_w^2}\right) M_W^2 , \quad M_Z = \frac{M_W}{\cos\theta_W} \end{aligned}$$

13: What about fermions?

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First, leptons. Fields are:

$$L = \begin{bmatrix} \nu \\ e_L \end{bmatrix}, \quad e_R$$

Their covariant derivatives are:

$$D^\mu L = \left(\partial^\mu - i g_w \frac{\tau_a}{2} W_a^\mu + \frac{i g'}{2} B^\mu \right) L, \qquad D^\mu e_R = \left(\partial^\mu + i g' B^\mu \right) e_R$$

The factors on B^μ mean the charges are $-1/2$ and -1 respectively.
I can respect all gauge symmetries by putting in a coupling:

$$Y_{e,ij} \overline{e}_{R,i} \phi^\dagger L_j + \text{h.c.}$$

The B-charges add up to $+1 - \frac{1}{2} - \frac{1}{2} = 0$ and the W-transformations of $\phi^\dagger$ and L cancel.
$Y_{e,ij}$ is a 3×3 matrix over the (e, μ, τ) index.

14: Electron mass etc

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What's this combination?

$$Y_e \overline{e}_R \phi^\dagger L = \overline{e}_R \begin{bmatrix} 0 & \frac{v+h}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \nu_e \\ e_L \end{bmatrix} = \frac{Y_e(v+h)}{\sqrt{2}} \overline{e}_R e_L$$

is a mass for electrons with mass value $Y_e v/\sqrt{2}$, and a coupling to Higgs bosons which is proportional to mass.

So e_R and e_L are right, left components of electron.
What are couplings to gauge fields? Only e_L couples to $W^\pm$, and

$$ig' B^\mu \frac{1+\gamma^5}{2} e + i \frac{g_w W_3^\mu + g' B^\mu}{2} \frac{1-\gamma^5}{2} e = i \frac{g' g_w}{\sqrt{g_w^2 + g'^2}} A^\mu e + (...) Z^\mu e$$

Coupling to A^μ is P, C respecting with coupling $g_e = g' g_w / \sqrt{g_w^2 + g'^2} = g' \cos\theta_W$
Coupling to Z^μ is ... what we presented previously.

15: Quark sector

Same thing works for quarks, but with one extra trick.

$$\text{fields } Q = \begin{bmatrix} u_L \\ d_L \end{bmatrix}, \quad d_R\,, \quad u_R$$

$$\text{Derivs } D^\mu Q = \left(\partial^\mu - ig_w \frac{\tau_a}{2} W_a^\mu - ig_s \frac{\lambda_\alpha}{2} G_\alpha^\mu - i\frac{g'}{6} B^\mu\right) Q\,,$$

$$D^\mu d_R = \left(\partial^\mu - ig_s \frac{\lambda_\alpha}{2} G_\alpha^\mu + i\frac{g'}{3} B^\mu\right) d_R \;...$$

$$\text{Higgs-couplings } -\mathcal{L} \supset Y_{dij}\overline{d}_{Ri}\phi^\dagger Q_j + Y_{uij}\overline{u}_{Ri}\phi^\top (i\sigma_2) Q_j + \text{h.c.}$$

Up quark couples differently to get the right total (hyper)charge.
$i\sigma_2$ flips top, bottom entries so ϕ couples to up, not down.

Again, electric charges are right and couplings to Z are what we saw before....

The Higgs induces a *matrix* mass between flavors:

$$\frac{vY_{eij}}{\sqrt{2}} e_{Ri} e_{Lj} = \begin{bmatrix} e_R & \mu_R & \tau_R \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} e_L \\ \mu_L \\ \tau_L \end{bmatrix}$$

9 complex numbers = 18 free real entries.
BUT I can change *basis* by a $U(3)$ rotation on e_L *and* a $U(3)$ rotation on e_R: $9 \times 2 = 18$ freedoms. The *magnitudes of eigenvalues* unchanged, but matrix made real positive and diagonal.

Almost same story in quark sector: two matrices Y_{uij}, Y_{dij} have 36 freedoms. $U(3)$ rotation on Q, u_R, d_R removes 27 freedoms leaving 9 – actually[3] 10. Of these, 6 are eigenvalues – masses $m_u, m_c, m_t, m_d, m_s, m_b$ and 4 are angles – the CKM matrix.

[3] One of our unitary transformations is to rotate everyone by the same phase. That doesn't do anything so it doesn't help.

17: Standard Model: What is predicted

Parameters I need:

- Gauge couplings g_s, g_w and g' and vacuum value v (4)
- *each* fermion mass value, as well as Higgs mass value (3+3+3+1=10)
- Four CKM angles (4). 18 total parameters!

What that predicts:

- With three numbers: g_w, g', v, we get predictions for M_W, M_Z, g_e, g_w, g_Z, and couplings between (W, Z) and Higgs, as well as values and pattern of Z-coupling strengths
- The fermion masses also tell the strength of their Higgs couplings
- Higgs mass predicts strength of Higgs self-interactions

The only unmeasured couplings as of 2021 are Higgs self-couplings and the smallest of the Higgs-fermion couplings.

18: Summary

Take two pieces of physics from last time:

- Nonabelian gauge theory
- Scalars with spontaneous symmetry breaking

Stick them together and get the Higgs Anderson Braut Englert Guralnik Hagen Kibble mechanism
Add in Electromagnetism-like field B^μ
Higgs effect re-mixes $B^\mu, W_3^\mu \to A^\mu, Z^\mu$
Pattern of masses, couplings, mixings match all experimental particle physics.

So wait – has Particle Physics been "finished" since 2012?
Arguably, yes, this is a settled field. But not really.

- In this description, neutrinos are massless. But they aren't.
- Dark matter exists but it is not explained in the Standard Model.

And maybe there's some surprises still waiting!

19: Scratch space