Teilchenphysik:

Lecture 23: Neutrinos are massive!

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Neutrinos turn out *NOT* to be massless,
as they “should” be in the *most minimal* Standard Model.

- How do you look for masses “directly”?
- How do you look indirectly? Neutrino oscillations
- How neutrino oscillation works
- Dirac or Majorana?
- Sources and detectors for neutrinos
- What we know to date (2021)
- Open questions and How we will Answer them

2: Neutrino mass? Kurie plot

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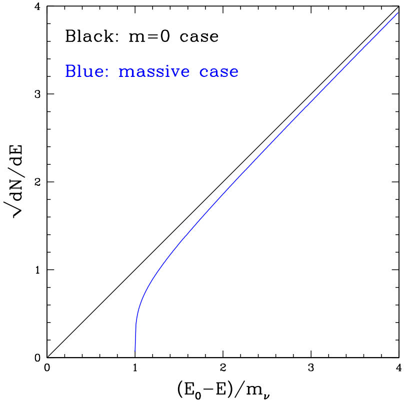
When $n \to e^- p^+ \overline{\nu}_e$, the $e^-, \overline{\nu}_e$ share the energy.

$$\frac{dN_{\text{decays}}}{dE_{e^-}} \propto (E + m_e)(E_0 - E)\sqrt{(E_0 - E)^2 - m_\nu^2}\,, \qquad E_0 = \text{total decay energy}$$



Max available energy smaller
And shape of curve is different when the neutrino is massive.
Only visible near “endpoint”
“Kurie plot”
(No not *that* Kurie)

3: Katrin experiment

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Katrin experiment studies this using tritium.
Precision requires very large apparatus
(Seen here during delivery to the lab)
Challenges: atomic + Condensed Matter effects
Tritium adsorbed onto surfaces ...

Current limit: $m_{\nu_e} < 1\,\text{eV}$

4: Cosmology?

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- Early Universe produced about 2/3 as many $\nu_{e,\mu,\tau}$ as γ
- If $m_\nu = 0$, now carry energy $\sim$ 1 meV each, and contain 10^{-5} of current energy of Universe
- If mass $m = 1$ eV, represent $>$ 1% of total energy budget!
- Move too fast to stick in galaxies and clusters
- $m \neq 0$ would leave "imprint" in microwaves + galaxy distribution

Analysis of galaxy distribution + (mostly) Microwave Sky bounds
$m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau} < 0.12\,\text{eV}$.

5: What if neutrinos are massive?

There is no reason that the mass basis and “flavor basis”
(what e, μ, τ couple to) should be the same.
They are *not* the same in quark sector: CKM matrix!
Expect the same thing:

- Flavor basis: $|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\nu_\tau\rangle$
- Mass basis: $|\nu_1\rangle$, $\nu_2\rangle$, $\nu_3\rangle$, masses $m_1 < m_2 < m_3$
- Matrix relation between them! Pontecorvo M aki N akagawa S akata matrix

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$\nu_{e,\mu,\tau}$ couple to definite leptons (eigenstates of flavor).
$\nu_{1,2,3}$ have definite masses (eigenstates of Hamiltonian)
U_{ai} converts from mass to flavor basis.

6: Neutrino oscillations

For simplicity imagine $U_{e3} = 0$. Then:

$$|\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle$$

When $p \to ne^+\nu_e$, the neutrino is in this exact state.

What state is it in a time t later?
Observe a distance $x = ct$ away, so evolves with phase $e^{i(px-Et)/\hbar} = e^{-i(E-cp)t}$.
What is that?

$$E_1 = \sqrt{c^2p^2 + m_1^2c^4} \simeq pc + \frac{m_1^2c^4}{2pc},$$

$$E_2 = \sqrt{c^2p^2 + m_2^2c^4} \simeq pc + \frac{m_2^2c^4}{2pc} = E_1 + \frac{(m_2^2 - m_1^2)c^4}{2pc}$$

A *relative phase* $e^{i\Delta m^2c^3t/2\hbar p}$ develops between neutrino mass-states

State after propagating a distance *L*:

$$\left|\nu(x = L)\right\rangle = e^{-im_1^2c^2L/2\hbar p}\left(U_{e1}\left|\nu_1\right\rangle + e^{-i\Delta m^2c^2L/2\hbar p}U_{e2}\left|\nu_2\right\rangle\right)$$

If I try to *detect* an *electron* neutrino, will I find it?

$$\begin{aligned} A &\equiv \rangle\nu_e|\ \left|\nu(x = L)\right\rangle \\ &= \left(U_{e1}^*\left\langle\nu_1\right| + U_{e2}^*\left\langle\nu_2\right|\right) e^{-im_1^2c^2L/2\hbar p}\left(U_{e1}\left|\nu_1\right\rangle + e^{-i\Delta m^2c^2L/2\hbar p}U_{e2}\left|\nu_2\right\rangle\right) \\ |A|^2 &= \left|U_{e1}^*U_{e1} + e^{-i\Delta m^2c^2L/2\hbar p}U_{e2}^*U_{e2}\right|^2 \end{aligned}$$

Call $U^*U_{e1} = \cos^2\theta_{1e}$, $U^*U_{e2} = \sin^2\theta_{1e}$:

$$|A|^2 = 1 - 2\cos^2\theta_{1e}\sin^2\theta_{1e}\left(1 - \cos\frac{\Delta m^2c^3L}{2\hbar E}\right) = 1 - \sin^2(2\theta_{1e})\sin^2\frac{\Delta m^2c^4t}{4\hbar E}$$

8: Two-flavor oscillations

$$P_{\nu_e \to \nu_e} = 1 - \sin^2(2\theta_{1e}) \sin^2 \frac{\Delta m^2 c^4 t}{4\hbar E}$$

At $t = 0$, "in-phase," 100% the same particle type as initially.
Amplitude of oscillation = $\sin^2(2\theta_{1e})$ max for $2\theta = \pi/2$.
Half-period (max to min) when $\frac{\Delta m^2 c^4 t}{4\hbar E} = \frac{\pi}{2}$
Sensitive to *smaller* Δm^2 when t is *larger* but E is *smaller*

9: Typical numbers for oscillations

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Setup/situation	Energy	Distance	Δm^2
Laboratory	1GeV	100m	$12\,\text{eV}^2$
Long-baseline	5GeV	500km	$1.2 \times 10^{-2}\text{eV}^2$
Atmospheric (above)	5 GeV	10 km	$0.6\,\text{eV}^2$
Atmospheric (below)	5 GeV	6000 km	$10^{-3}\,\text{eV}^2$
Reactor-neutrino	6 MeV	1 km	$7 \times 10^{-3}\text{eV}^2$
Long-baseline reactor	6 MeV	100 km	$7 \times 10^{-5}\text{eV}^2$
Solar	1 MeV	1.5×10^{11} km	10^{-11}eV^2

We will see that the physically relevant values are $\Delta m^2 = 2.7 \times 10^{-3}\,\text{eV}^2$ and $\Delta m^2 = 8 \times 10^{-5}\,\text{eV}^2$.

10: Reactor neutrinos

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Coulomb energy $E_{\text{coul}} \propto Z^2/A^{1/3}$ makes largest nuclei neutron-rich. When $^{236}U^*$ or $^{240}Pu^*$ split, daughters are highly *n*-rich.
Decays $n \to pe^-\overline{\nu}_e$ have broad spectrum up to 7 or 8 MeV.
Single reactor core produces $\sim$ 300 MW neutrino power!

Detector: Gadolinium doped hydrocarbon liquid scintillator.
$\overline{\nu}_e p \to e^+ n$ and $n + \text{Gd} \to \text{Gd}^* \to \text{Gd} + \gamma + \gamma$ 8MeV γ energy.
Clean: two signals e^+, γ in space+time coincidence.

100km (KamLAND) to 60km (JUNO) distance from reactors for $\Delta m^2 = 8 \times 10^{-5}$ eV^2, 1-2km for larger Δm^2.

Only sensitive to $\nu_e \to \nu_e$ (disappearance)

11: Beam neutrinos

Create beam of $\pi^+ \to \mu^+\nu_\mu$ or $\pi^- \to \mu^-\overline{\nu}_\mu$ at GeV to several GeV energy.
Old days: detector within 1km. Good for *discovery of neutrino*
Nowadays: detector 100's of km away. Detector has to be huge, multi-kiloton.

Look for $\nu\mu X \to \mu X'$ off nuclei. Disappearance experiment.
e^- hard (not impossible) to detect. τ harder, but now also done.

Arrange for beam energy + distance go give specific Δm^2.

12: Solar neutrinos

Sun is powered by nuclear reactions:

$$p + p \to D_{pn} + e^{+} + \nu_e \qquad {}^3\text{He} + {}^4\text{He} \to {}^7\text{Be} + \gamma$$

$$D + p \to {}^3\text{He}_{ppn} + \gamma \qquad {}^7\text{Be} \to {}^7\text{Li} + \nu_e + e^{+}$$

$${}^3\text{He} + {}^3\text{He} \to {}^4\text{He} + p + p \qquad {}^7\text{Li} + p \to 2\,{}^4\text{He}$$

$$p + p + e^{-} \to D + \nu_e \qquad {}^7\text{Be} + p \to {}^8\text{B} \to 2\,{}^4\text{He} + e^{+} + \nu_e$$

Neutrino energies: *pp* low < 0.3MeV, *Be* medium ~ 1MeV, *B* high < 17MeV.

Middle-energy: chlorine detector (disappearance) 1960's
Low-energy: Gallium detectors 1980's. Disappearance experiments.
High-energy: Deuterium detector ~ 2000: disappearance and conversion to ν_μ, ν_τ.

13: MSW effect (time permitting)

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14: What neutrino phenomena do we observe?

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- Solar, Long-baseline reactor: $\Delta m^2_{12} = 7.53(18) \times 10^{-5}\,\text{eV}^2$
- Beams, atmosphere: $\Delta m^2_{13} = 2.44(6) \times 10^{-3}\,\text{eV}^2$
- Not known: are close-together masses lighter? or heavier? (normal / inverted hierarchy)

What are the mixing angles?

$$U_{ai} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- $\theta_{12} = 33.4(8)^\circ$ measured in solar, reactor experiments
- $\theta_{23} = 49(1)^\circ$ measured with beams, cosmic ray/atmosphere
- $\theta_{13} = 8.57(12)^\circ$ measured with short-baseline reactor (Daya Bay, Reno)
- δ not very well constrained

Surprise is how big these angles are!

15: Majorana vs Dirac?

Massless neutrinos: ν_e is L-handed, $\overline{\nu}_e$ is R-handed.
Massive: you can “run and catch up with, then pass” a neutrino.
If you catch up with and pass a L-handed ν_e, do you see..

- A R-handed *neutrino*?
- A R-handed *antineutrino*?

If it’s a R-handed neutrino (**Dirac neutrinos**), then there are more degrees of freedom (total types of particles)

If it’s a R-handed antineutrino (**Majorana neutrinos**), then there are no new degrees of freedom. But lepton number is violated!

These possibilities actually have *exactly the same* predictions for all oscillation phenomena, and for cosmology.

16: Why Majorana neutrinos are suspect

Coupling ν to $\overline{\nu}$ with a mass violates $SU(2)$ and $U_B(1)$ symmetry.
In the Standard Model, one must instead couple to the Higgs field – twice

$$\mathcal{L} \supset K_{ij} \overline{L}_i \epsilon \phi^* \ \phi^\top \epsilon L_j$$

The mass is $v^2 K_{ij}$. We need $K_{ij} \sim 1/(10^{14}\ \mathrm{GeV})$ to get the right masses.

This is a *nonrenormalizable operator* and we were told not to use those.
But theoretically, it's OK *IF* there are new particles with masses less than 10^{14} GeV. That seems possible.

17: Why Dirac neutrinos are suspect

Dirac neutrinos involve adding a new *R*-handed field *N* to the SM.
This allows a new Higgs coupling

$$\mathcal{L} \supset Y_{nij} \, \overline{N}_i \phi^\top \epsilon L_j$$

analogous to the way the up quark u_R couples to Q.
This generates the right masses if the eigenvalues of Y_n are $\sim 10^{-13}$.

Why so much smaller than all other masses? No explanation.
Worse, Lagrangian can also contain $M_{ij}\overline{N}_i N_j$.
Ruins all predictions unless $M < 10^{-10}$eV (or smaller?)

18: Dirac or Majorana? Do experiment?

Due to the weirdness of nuclear physics, there are nuclei which cannot undergo β decay, but can undergo *double* beta:

$$^{136}\mathrm{Xe}^{54} \to {}^{136}\mathrm{Ba}^{56} + 2e^- + 2\overline{\nu}_e \qquad \text{But not} \qquad ^{136}\mathrm{Xe}^{54} \to {}^{136}\mathrm{Cs}^{55} + e^- + \overline{\nu}_e$$

If ν_e is its own antiparticle, process can occur with no ν!

Measurement: look for $e^- e^-$ right at Q-threshold.
Several ongoing experiments, approaching required sensitivity.

19: Summary

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Neutrinos are massive!

- Direct detection of neutrino masses still only provides limits
- If mass, flavor bases are different, neutrinos can oscillate
- Requires MeV and kilometers or Gev and thousand kilometers
- Several oscillation phenomena observed: neutrinos from Sun, reactors, beams, cosmic ray showers.
- Two splittings $\Delta m_{13} = 2.4 \times 10^{-3}$ eV^2 and $\Delta m_{12} = 7.5 \times 10^{-5}$ eV^2
- mixing angles are large! Not like CKM at all!
- We don't know if *R*-neutrino is its own species, or antiparticle of *L*-neutrino (Dirac or Majorana?)
- Neutrinoless double beta experiments can resolve this.

Adds 7 parameters to Standard Model. 9 for Majorana. 5 are already measured.