# Teilchenphysik:

## Lecture 5: Internal symmetries

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We will discuss two types of internal symmetries:

- Continuous internal symmetries. Specifically, baryon number, lepton number, and much more interesting, isospin
- discrete symmetries:
  - parity $P$ (actually a spacetime symmetry, but ...)
  - charge conjugation $C$
  - time reversal $T$ (also a spacetime symmetry)

  and their combination, $CPT$, which is *always* valid

## 2: Electric charge

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You already know this one. Every particle carries an electric charge:

- the electron-type particles $e^-$, $\mu^-$, $\tau^-$ carry charge $q = -1$
- The up-type quarks $u$, $c$, $t$ carry charge $q = +\frac{2}{3}$
- The down-type quarks $d$, $s$, $b$ carry charge $q = -\frac{1}{3}$
- The weak boson $W^+$ carries a charge $q = +1$
- Antiparticles carry the opposite charge: $e^+, \mu^+, \tau^+$ are $q = +1$, $\bar{u}, \bar{c}, \bar{t}$ are $q = -\frac{2}{3}$, $\bar{d}, \bar{s}, \bar{b}$ are $q = +\frac{1}{3}$, $W^-$ is $q = -1$
- The remaining particles $\gamma$, $g$, $Z$, $\nu_{e,\mu,\tau}$, $\bar{\nu}_{e,\mu,\tau}$ are neutral, $q = 0$

Symmetry transformation: rotate state $|\psi\rangle$ of charge $q$ ($\hat{Q}|\psi\rangle = q|\psi\rangle$) by a phase $e^{iq\theta}$ changes no physics.

State of mixed charge $\quad |\psi\rangle = c_1|\psi_1\rangle + c_2|\psi_2\rangle\,, \qquad Q|\psi\rangle = q_1 c_1|\psi_1\rangle + q_2 c_2|\psi_2\rangle$

$|\psi_1\rangle$ and $|\psi_2\rangle$ must be orthogonal and must always stay that way.
$q$ must be conserved by time evolution.

## 3: Electric charge: consequences

When particles interact or decay, initial $q$ must equal final $q$.

- Reaction $\pi^+ p \to \pi^0 n$ forbidden, but $\pi^0 p \to \pi^+ n$ allowed
- Decay $n \to p^+ e^- \bar{\nu}_e$ allowed, but $n \to p^+ \gamma$ forbidden
- Lightest charged particle $e^-$ has no energetically allowed decay, and must be *absolutely stable*
- Its heavier cousins need not be stable: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ and $\tau^- \to \nu_\tau \pi^-$ are allowed and in fact occur

Not only at beginning vs end of process:
also *everywhere within a Feynman diagram*
(We will see what this means before long ...)

# 4: Similar conserved charges: $B$ and $L$



Baryon number:

- every quark *uctdsb* has baryon number $B = +\frac{1}{3}$, antiquarks have $B = -\frac{1}{3}$
- all others have baryon number $B = 0$.

Lepton number:

- every lepton $e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau$ has lepton number $L = +1$, their antiparticles have $L = -1$
- All others have $L = 0$

All observations are consistent with each being conserved.[1]

- Decay $n_{udd} \to p^+_{uud} e^- \bar{\nu}_e$ allowed but $p^+_{uud} \to e^+ \pi^+_{u\bar{d}} \pi^-_{d\bar{u}}$ forbidden. Searched for, not observed: lifetime $\tau > 10^{32}$ years.
- Decay $\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu$ allowed but $\tau^- \to \bar{p}^- \pi^0$ forbidden

[1] We know that $B$ and $L$ are *anomalous* but this has no practical consequences. Neutrino oscillations *may* imply $L$ violation but this remains unproven.

## 5: More fun example: Isospin

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Consider up quark $u$, down quark $d$.

- Strong interactions treat them identically.
- Not quite same mass, $m_d \simeq 5\text{MeV}$, $m_u \simeq 2\text{MeV}$, but both are small so difference is small effect
- Different electric charge but E&M has $\alpha_{\text{EM}} = 1/137$ small effect vs. strong force

Good approximate (*not* exact) symmetry:

$$\begin{bmatrix} u \\ d \end{bmatrix} \rightarrow \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix}$$

with $U = e^{i\theta_i \tau_i/2} \in SU(2)$, eg,[2] like rotations on spin-up, spin-down
Rotation $e^{i\theta_3\tau_3/2}$ *almost exact* (broken by weak force)

Rotations which mix $u, d$ ($e^{i\theta_1\tau_1/2}$, $e^{i\theta_2\tau_2/2}$) less-exact

[2] The $\tau_i$ are the Pauli matrices. Write $\sigma_i$ when using Paulis for true spin, $\tau_i$ for isospin

# 6: Isospin multiplets



The pair of quarks ($u$, $d$) are a *doublet* (like spin-$\frac{1}{2}$)
Antiparticles ($\bar{d}$, $\bar{u}$) also doublet.

$$\text{Combine quark+antiquark:} \quad \frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$$

Triplet is $\pi^+ = u\bar{d}$, $\pi^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$, $\pi^- = d\bar{u}$. Singlet is $\eta^0$ (heavier...)

$$\text{Combine three quarks:} \quad \frac{1}{2} \otimes \frac{1}{2} \times \frac{1}{2} = (1 \oplus 0) \otimes \frac{1}{2} = \frac{3}{2} \oplus \frac{1}{2} \oplus \frac{1}{2}$$

One of the $\frac{1}{2}$ is ($p$, $n$) proton+neutron. (The others come later)

Prediction: $m_{\pi^+} \simeq m_{\pi^0} \simeq m_{\pi^-}$ (139.57, 134.98, 139.57 MeV) and $m_p \simeq m_n$ (938.27, 939.56 MeV)

# 7: More predictions of Isospin



Scatter “a” pion from “a” neutron/proton: $1 \otimes \frac{1}{2} = \frac{3}{2} \oplus \frac{1}{2}$:

Consider $\quad p^+ + \pi^+ \to p^+ + \pi^+$

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle |1,1\rangle \to \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle |1,1\rangle$$

$$\left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle \to \left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle = \mathcal{M}_{3/2}$$

Whereas: $\quad p^+ + \pi^0 \to p^+ + \pi^0$

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle |1,0\rangle \to \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle |1,0\rangle$$

$$\frac{1}{\sqrt{3}}\left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \to \frac{1}{\sqrt{3}}\left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \frac{1}{3}\mathcal{M}_{3/2} + \frac{2}{3}\mathcal{M}_{1/2}$$

Also possible: $\quad p^+ + \pi^0 \to n + \pi^+$

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle |1,0\rangle \to \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle |1,1\rangle$$

$$\frac{1}{\sqrt{3}}\left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \to \frac{2}{\sqrt{3}}\left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \frac{\sqrt{2}}{3}\mathcal{M}_{3/2} - \frac{\sqrt{2}}{3}\mathcal{M}_{1/2}$$

# 8: Isospin scattering



Scattering theory: $\mathcal{M}_{3/2}, \mathcal{M}_{1/2}$ are functions of angle and CM energy.
QCD predicts them but QCD is really hard. Treat as unknown functions.
How does this help???

Eight independent processes: $(p, n) + (\pi^{+,0,-})$ including $p^+\pi^0 \leftrightarrow n\pi^+$ and $p^+\pi^- \leftrightarrow n\pi^0$.
But all expressible in terms of two "scattering amplitudes" $\mathcal{M}_{3/2}, \mathcal{M}_{1/2}$.
8 processes determined by only 2 (complex) functions! Several predictions!

Inter-relations are not exact but are good to a few %.

# 9: Isospin, Deuteron, Dineutron



A neutron and proton can stick together into $D^+$.
Why no bound *nn* or *pp* states? They are both unstable.

$$pp = \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \left|1,1\right\rangle \qquad \text{but} \qquad pn = \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left|1,0\right\rangle + \frac{1}{\sqrt{2}}\left|0,0\right\rangle$$

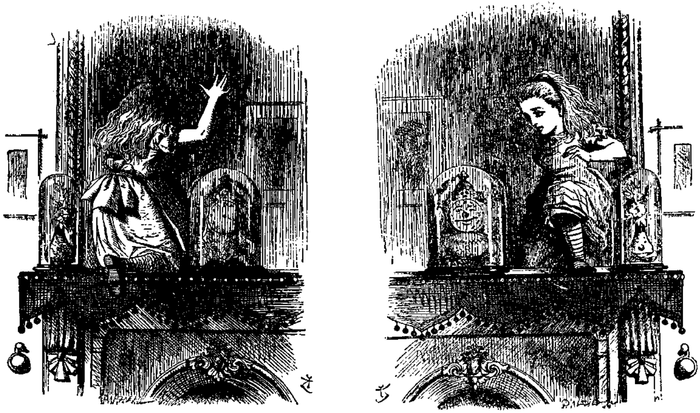
So *pp* is always isospin-0 but *pn* can be isospin-1 or isospin-0.
Isospin-0 is deeper bound. (Why? That's tricky QCD.)

*prediction*: *p*, *n* are fermions. Total wave function must be odd.
Isospin-0 is odd $\frac{|pn\rangle - |np\rangle}{\sqrt{2}}$, so spin state must be even: spin-1. ✓

# 10: Discrete symmetry: parity

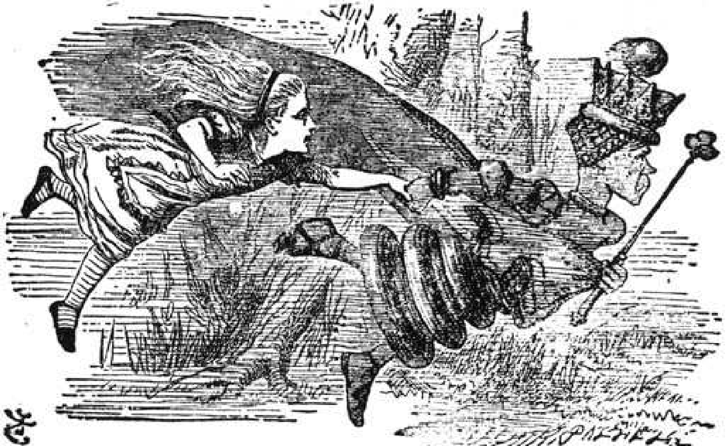


When you look in the mirror, are the laws of physics you see correct?

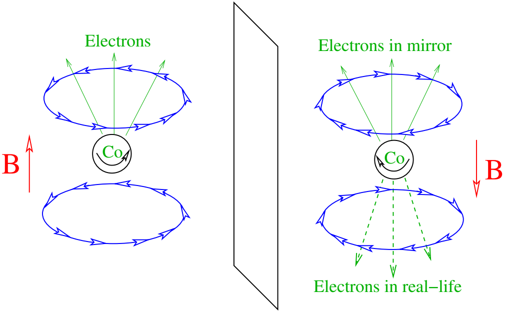


# 11: Explicit parity violation

No they are not!



# 12: Explicit parity violation




$^{60}_{27}$Co is unstable,

$$^{60}_{27}\mathrm{Co} \rightarrow\, ^{60}_{28}\mathrm{Ni} + e^{-} + \bar{\nu}_e$$

and has large spin.
Align spin with $B$ field.
$e^{-}$ comes out preferentially in spin direction, which violates parity

# 13: So why talk about parity at all?

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The *weak force* violates parity. But it's *weak.*
*Strong*, *Electromagnetic*, *gravitational* forces respect parity.
They are 99.9% of the physics we deal with daily.

Standard to define parity not as mirror-reflection, but as reflection of *all three* axes.
(Mirror reflection = parity + rotation). Apply twice: get back where you started.

Parity operator $\hat{P}$ $\quad \hat{P}\hat{x} = -\hat{x}\hat{P}$. Double operation $\hat{P}\hat{P} = \mathbf{1}$

Scalar: $\hat{P}s = s\hat{P}$

Pseudoscalar: $\hat{P}p = -p\hat{P}$

Vector: $\hat{P}\vec{v} = -\vec{v}\hat{P}$

Pseudovector: $\hat{P}\vec{a} = \vec{a}\hat{P}$

Odd-parity states only mix with odd-parity states, and even with even.
Two odd-parity particles are net even-parity, just as $(-1)^2 = 1$.

# 14: Charge conjugation



Turn all matter into antimatter and vice versa.

$$\hat{C} n = \bar{n} \hat{C} \qquad \hat{C} \pi^{+} = \pi^{-} \hat{C} \qquad \hat{C} \pi^{0} = (\pm 1) \pi^{0} \hat{C}$$

For a particle which is its own antiparticle, $C$ takes the particle to itself – up to an overall phase.
But since $\hat{C}\hat{C} = \mathbf{1}$, phase must be +1 or $-1$.
If phase is +1, particle is $C$-even. If $-1$, it's $C$-odd.
We will see that the $\pi^0$ is $C$-even but $P$-odd.

Weak interactions break $C$, but strong, EM, gravity respect it.

# 15: Combining C with P

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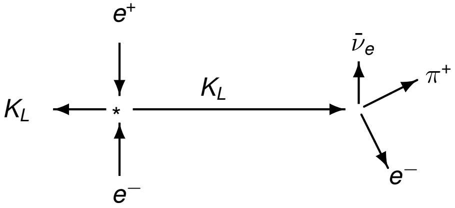
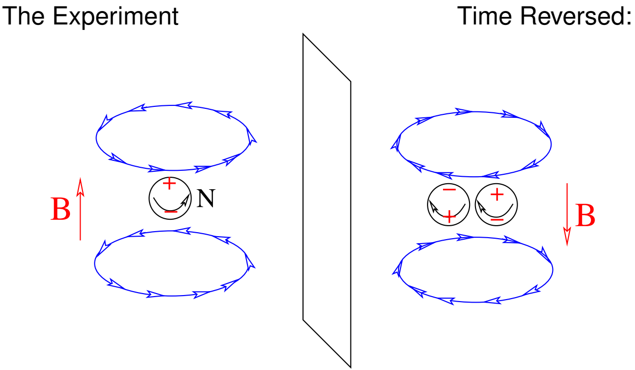
The combined transformation is *almost* a symmetry of weak interactions. But not quite: imagine colliding $e^+e^-$ at $> 1$ GeV energy:




Can create a pair of $K_L$ particles, mass 497 MeV.
$K_L$ is unstable: 40% of decays are to $\pi^+e^-\bar{\nu}_e$ or $\pi^-e^+\nu_e$.
But $\pi^-e^+\nu_e$ occurst 0.3% more often than $\pi^+e^-\bar{\nu}_e$.

Initial state is $C$ and $P$ symmetric. But I end up with more $\pi^-$ than $\pi^+$. That Violates C and CP! *But the violation is tiny and only occurs in special reactions*

Run a movie backwards: is the *microphysics* right?




Neutron in *B*-field aligns spin. Look for *electric* dipole.

If it exists – same physical law would give *opposite* dipole if the movie runs backward. So far: no dipole observed, *T* respected (dipole $< 10^{-13}\, e\,\mathrm{fm}$)

# 17: CPT theorem



Deep theoretical result (theorem): the combined symmetry transformations $C$, $P$, $T$ together ($CPT$ = $CTP$ = $PTC$ = ...) are *always* a symmetry.

Counterexamples searched/tested for but never found.

Implies that $T$ is *not* a valid symmetry, by the same tiny effects which break $CP$.

# 18: Summary



- Electric charge arises from a $U(1)$ symmetry and gives charge conservation
- Similar charges – $B$ and $L$ – exist in particle physics
- Isospin is a more interesting example.
  - Not exact but approximate $SU(2)$ symmetry
  - Organizes particles into “multiplets”
  - Gives precise relations for masses *and* interactions
- Parity – mirror reflection – violated only by weak interactions
- Charge conjugation – matter $\leftrightarrow$ antimatter – also broken weakly
- CP and T nearly valid (tiny weak-interaction violations), and CPT is exact