Lecture 10: Loudness, Intensity

We have now discussed pitch/frequency and timbre/spectrum. The third attribute of a sound is loudness/intensity. We begin by discussing the physics side, intensity, which we will see is tightly related to (but not quite the same as) the perception of loudness.

The short version of this lecture, which you are not expected to understand at the beginning, is that

$$\textbf{Intensity} = \frac{\text{Power}}{\text{area}}$$

(measured in Watts per square meter, $W/\mathrm{m}^2 = \mathrm{kg}\ /\mathrm{s}^3$) describes the "strength" of a sound wave, and that it is related to the pressure and air movement in the sound wave by,

$$\text{Intensity} = \frac{1}{\rho v_{\text{sound}}} \langle (\Delta P)^2 \rangle = (\rho v_{\text{sound}}) \langle v_{\text{air}}^2 \rangle \,,$$

with ΔP and v_{air} the pressure and velocity change in the air due to the sound wave. Here $\langle\rangle$ means the average over the wave-form. For a sine wave, $\langle (\Delta P)^2 \rangle = (1/2)\Delta P_{\text{peak}}^2$, half the square of the peak pressure difference from atmospheric.

Now let's actually discuss intensity slowly with the intention of explaining.

Energy and Power

First recall that **energy** is "the stuff that makes things move." Heavy things are harder to get to move–so if they are moving, they contain more energy. Faster moving things also contain more energy. To be more specific, the faster something is already moving, the more energy it takes to increase its speed. Suppose you push on something which is moving. If you push with force F for a distance Δx, then the energy you impart on the object (and therefore use up yourself) is,

$$\Delta E = F \cdot \Delta x \,,$$

The dot here is instructions on the *sign* of the result: if you push in the direction the thing is moving (speeding it up), the energy is positive. If you push against its motion, the energy is negative. (When you push to slow down a car, you are taking energy out of the car.) [Technically the $\cdot$ means that this is a dot product of two vectors].

If each time you increased v it cost the same energy, then the energy would go as mv. However, since an object with a larger v takes more energy to have its v increased, the correct energy dependence on velocity is,

$$E_{\text{motion}} = \frac{1}{2} m v^2 \,.$$

This is not the only kind of energy; energy can also be stored in gravitational potential, chemical energy (think gasoline), heat, a pattern of compression and decompression, and so forth. The units of energy are the

$$\textbf{Joule}: \quad \mathrm{J} = \frac{\mathrm{kg\ m^2}}{\mathrm{s^2}}$$

which indeed has the same units as mv^2.

Power is the rate at which energy is delivered. I don't want my wall socket to give some amount of energy—I want it to give some amount of energy every second. The units of power are,

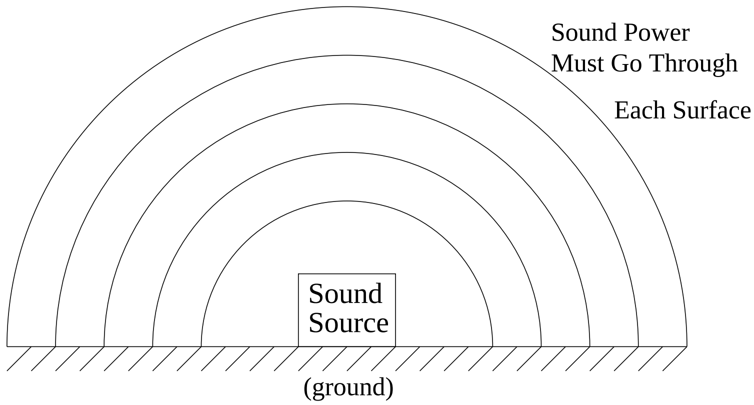
$$\text{Power} = \frac{\text{energy}}{\text{time}} : \qquad \textbf{Watt}: \quad \mathrm{W} = \frac{\mathrm{J}}{\mathrm{s}} = \frac{\mathrm{kg\ m^2}}{\mathrm{s^3}}$$

For most purposes, 1 Watt of power is not very much. Light bulbs famously absorb around 100 Watts of power (and return maybe 5 Watts of light). You, sitting in a chair, generate 100 Watts of heat, which is why you have to breathe and eat. One horsepower is (defined to be) 735 Watts.

However, **for sound, 1 Watt is a lot!**. In ordinary conversation with one other person, your voice produces 10^{-5} W = .00001 W of sound power. When you breathe quietly, the sound production is more like 10^{-10} Watts. A trumpet, played by a professional at maximum dynamic (say, fff in the New World Symphony) can produce about 1 Watt of sound power–filling a large concert hall. Whatever the sound rating on your stereo equipment, the maximum it can produce, say, on a drumbeat when cranked to the maximum setting, is about 5% to 10% of the rated power (which is how much electricity goes in, not how much sound power goes out).

Intensity

The power of a sound source does not tell how loud a sound you will hear, because the sound energy spreads out as you move away. For a source on the ground, for instance,



The sound power produced by the source moves outwards, and has to go through each half-sphere. Since the area of the half-spheres gets larger, the sound is being "stretched out" over a larger and larger area, and will not sound as large.

Since what counts to you is how much sound energy enters your ear, and since your eardrum's size does not depend on the distance to the sound source, the relevant way to measure the strength of a sound wave is,

$$\textbf{Intensity} = \frac{\text{Power}}{\text{Area}}\,.$$

Multiply the intensity by the area of your eardrum to find out how much sound power actually enters your ear. The units of intensity are,

$$\text{units of }\ \textbf{Intensity}: \quad \frac{\text{W}}{\text{m}^2} = \frac{\text{J}}{\text{m}^2\ \text{s}} = \frac{\text{kg}}{\text{s}^3}$$

The last set of units looks rather strange, but that is what it turns out to be. The best thing to remember is Watts per square meter.

You might want to remember that the area of a hemisphere (relevant for a sound producer on the ground), and of a sphere (relevant for a sound producer suspended in the air) are,

$$\text{hemisphere}: \quad A = 2\pi R^2 \qquad \text{sphere}: \quad A = 4\pi R^2\,.$$

Here $\pi = 3.141592\ldots$ as usual.

Intensity, air speed, pressure

Next we need to relate the intensity to the physical description of the air which the sound wave is going through. Consider a bit of air, of length ℓ on a side. The volume of the bit of air is ℓ^3, so the mass, in terms of the density, is $m = \rho V = \rho \ell^3$. Therefore, the energy stored, as motion, in that bit of air is,

$$E_{\text{motion, in a box}} = \frac{1}{2} m v_{\text{air}}^2 = \frac{1}{2} \rho_{\text{air}} \ell^3 v_{\text{air}}^2\,.$$

In a sound wave, energy is also stored in the fact that the pressure is higher some places and lower other places (similar to how energy is stored in the stretching or compression of a spring). This turns out to be exactly equal to the amount of energy stored in the motion of the air, so I will just double the above:

$$E_{\text{in a box}} = \rho_{\text{air}} \ell^3 v_{\text{air}}^2\,.$$

So what is the intensity? It is the power per area–that is, the energy which *leaves the end* of the box, per unit time, per unit area of the end of the box. The sound energy moves with the sound wave; so the energy in the box moves out through the end of the box at the speed of sound. Therefore the power leaving the box out the end is,

$$P_{\text{leaving box}} = \frac{E_{\text{in box}}}{\text{time to leave box}} = \frac{E_{\text{in box}}}{\ell / v_{\text{sound}}} = \frac{\rho_{\text{air}} \ell^3 v_{\text{air}}^2}{\ell / v_{\text{sound}}} \rho_{\text{air}} v_{\text{sound}} \ell^2 v_{\text{air}}^2 \, .$$

The intensity is the power per unit area:

$$\text{Intensity } I = \frac{P_{\text{leaving box}}}{\text{Area of box}} = \frac{\rho_{\text{air}} v_{\text{sound}} \ell^2 v_{\text{air}}^2}{\ell^2}$$

or

$$I = (\rho_{\text{air}} v_{\text{sound}}) v_{\text{air}}^2 \, .$$

The (purely notional) box size ℓ has dropped out of the calculation, which is good–that indicates that the intensity I is something which depends on what the air is doing, not on how big a box of air you consider [intensity is an intensive quantity].

We should do the same thing for pressure. There is a shortcut, though. By thinking about how a pressure difference causes the air to move, one can show that

$$P - P_{\text{atmos}} = (\rho_{\text{air}} v_{\text{sound}}) v_{\text{air}} \qquad \text{or} \qquad v_{\text{air}} = \frac{P - P_{\text{atmos}}}{\rho_{\text{air}} v_{\text{sound}}}$$

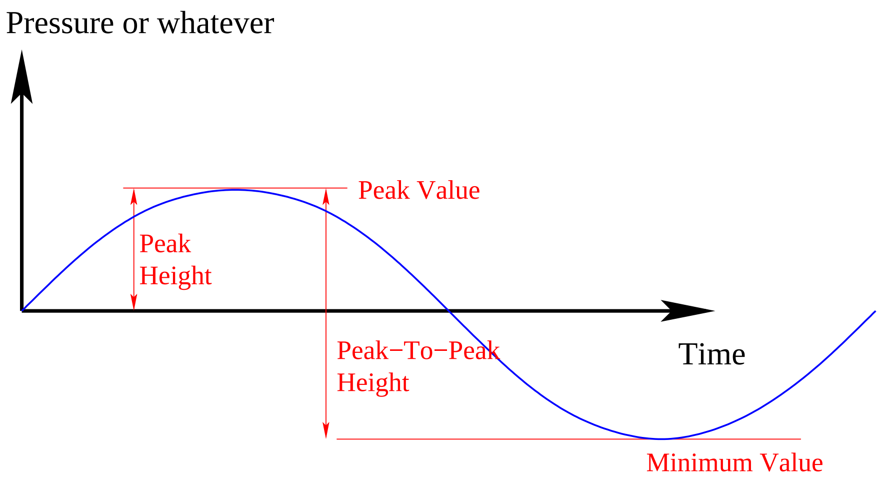
for a forward moving sound wave. That means that the intensity is related to the pressure via,

$$I = \frac{1}{\rho_{\text{air}} v_{\text{sound}}} (P - P_{\text{atmos}})^2 \, .$$

These expressions are correct for a sound wave as a whole if we interpret $(v_{\text{air}})^2$ or $(P - P_{\text{atmos}})^2$ to mean the average over the sound wave. Most waves have places where v_{air} is larger and places where it is smaller or near zero. For a sine wave, it turns out that the average is exactly half of the peak value:

$$(P - P_{\text{atmos}})^2_{\text{average}} = \frac{1}{2} (P - P_{\text{atmos}})^2_{\text{peak}} \quad \text{for a sine wave.}$$

For those unfamiliar with peak values for a sine wave,



The last question for this lecture is, how much does the air itself move back and forth? An estimate is that the air should move back and forth by

$$\Delta x_{\text{air displacement}} = v_{\text{air}} \times t \, .$$

But what should t be? The period? And what should v_{air} be? The peak value? Clearly not, since the air is only moving forward for half the period (in a sine wave), and most of that time it is slower than the peak value. Also, we probably want the peak air movement (the difference between the furthest forward the air gets, and the middle or average location) rather than the peak-to-peak. This shaves off another factor of 2. So the right answer is,

$$\Delta x_{\text{air, peak}} = \frac{v_{\text{air, peak}}}{2\pi f} \qquad \text{for a sine wave.}$$

The peak-to-peak value, which means the furthest forward the air gets minus the furthest backward the air gets, is twice this:

$$\Delta x_{\text{air, peak to peak}} = \frac{v_{\text{air, peak}}}{\pi f} \, .$$